



UNIT - II

Differential Equation:

An eqn. involving differential coefficients or derivatives is called differential eqn.

Ordinary differential eqn.

A differential eqn. which depends on only one independent variable is called ordinary differential eqn.

Order and degree:

* The order of the highest derivative occurring in the gvn. eqn is called the order of a differential eqn.

* The degree of the highest derivative occurring in the gvn. eqn. is called the degree of a differential eqn.

Second order linear ODE with constant coefficients:

The general linear ODE with constant coefficients is of the form

$$a_0 \frac{d^2 y}{dx^2} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n y = f(x) \quad \rightarrow (1)$$

where a_0, a_1, \dots, a_n are constants and

$f(x)$ is a function of x .

When $f(x) = 0$ in (1) is called homogeneous ODE &

If $f(x) \neq 0$ in (1) is called non-homogeneous ODE.



This eqn. can be written as,

$$[a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \dots + a_n] y = f(x)$$

$$\text{where } D = \frac{d}{dx}$$

$$\text{Solution} = \text{CF} + \text{PI}$$

= Complementary function + Particular integral

To find CF:

Roots

i). Roots are real & different

$$m_1 \neq m_2 \neq \dots$$

ii). Roots are real & same

$$m_1 = m_2 = m$$

iii). Roots are imaginary.
(or complex)

$$m = \alpha \pm i\beta$$

To find PI:

$$\text{PI} = \frac{1}{f(D)} f(x)$$

CF

$$A e^{m_1 x} + B e^{m_2 x}$$

$$(A + Bx) e^{mx}$$

$$e^{\alpha x} [A \cos \beta x + B \sin \beta x]$$

$$\text{RHS} = 0$$

J. Solve $(D^2 - 5D + 6) = 0$

Soln.

The auxiliary eqn. is

$$m^2 - 5m + 6 = 0$$

$$(m-3)(m-2) = 0$$

$$m = 2, 3$$

∴ The roots are real and different.



$$CF = Ae^{2x} + Be^{3x}$$

$$\therefore y = CF = Ae^{2x} + Be^{3x}$$

2]. Solve $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0$

Soln.

$$(D^2 - 6D + 9)y = 0$$

The Auxiliary eqn is

$$m^2 - 6m + 9 = 0$$

$$(m-3)^2 = 0$$

$$m = 3, 3$$

The roots are real and same.

$$CF = (A + Bx)e^{3x}$$

$$\therefore y = CF = (A + Bx)e^{3x}$$

3]. Solve $(D^2 + 1)^2 y = 0$

Soln.

The Auxiliary eqn. is $(m^2 + 1)^2 = 0$

Taking square root on both sides

$$m^2 + 1 = 0$$

$$m^2 = -1$$

$$m = \pm i$$

The roots are $\pm i$ and $\pm i$ imaginary

$$\text{Here } \alpha = 0, \beta = 1$$

$$\therefore CF = e^0 (A \cos x + B \sin x)$$

$$= A \cos x + B \sin x$$

$$\therefore y = CF = A \cos x + B \sin x$$

4]. Solve $(D^4 - 1)y = 0$

Soln.

The auxiliary eqn. is $m^4 - 1 = 0$



$$(m^2)^2 - 1^2 = 0$$

$$(m^2 + 1)(m^2 - 1) = 0$$

$$\left. \begin{array}{l} m^2 + 1 = 0 \\ m^2 = -1 \\ m = \pm i \end{array} \right\} \begin{array}{l} m^2 - 1 = 0 \\ m^2 = 1 \\ m = \pm 1 \end{array}$$

$$\therefore \text{CF} = Ae^{\alpha} + Be^{-\alpha} + C \cos \alpha + D \sin \alpha .$$