



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-I VECTOR CALCULUS

SOLENOIDAL AND IRROTATIONAL

Solenoidal & Irrotational vector:

Solenoidal vector: $\nabla \cdot \vec{F} = 0$

Irrotational vector: $\nabla \times \vec{F} = \vec{0}$

Problems

1] Find 'a' such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.

Soln.

Given $\vec{F} = (3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$
and $\nabla \cdot \vec{F} = 0$

$$\frac{\partial}{\partial x}(3x - 2y + z) + \frac{\partial}{\partial y}(4x + ay - z) + \frac{\partial}{\partial z}(x - y + 2z) = 0$$

$$3 + a + 2 = 0$$

$$\boxed{a = -5}$$

2] Show that $\vec{F} = x\vec{i} + x\vec{j} + y\vec{k}$ is solenoidal.

Soln.

Given $\vec{F} = x\vec{i} + x\vec{j} + y\vec{k}$

To prove $\nabla \cdot \vec{F} = 0$

$$\nabla \cdot \vec{F} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (x\vec{i} + x\vec{j} + y\vec{k}) =$$

$$= \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(y)$$

$$\nabla \cdot \vec{F} = 0$$

$\therefore \vec{F}$ is solenoidal

3] Show that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.

Soln.

Given $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$

To prove $\nabla \times \vec{F} = \vec{0}$



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Now,

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$= \vec{i} \left[\frac{\partial}{\partial y} (zy) - \frac{\partial}{\partial z} (zx) \right] - \vec{j} \left[\frac{\partial}{\partial x} (zy) - \frac{\partial}{\partial z} (yx) \right] + \vec{k} \left[\frac{\partial}{\partial x} (zx) - \frac{\partial}{\partial y} (yz) \right]$$

$$= \vec{i} [z - z] - \vec{j} [y - y] + \vec{k} [z - z]$$

$$= 0\vec{i} + 0\vec{j} + 0\vec{k}$$

$$= \vec{0}$$

$\therefore \vec{F}$ is irrotational.

AJ. If \vec{A} and \vec{B} are irrotational. Prove that $\vec{A} \times \vec{B}$ is solenoidal.

Soln.

Given \vec{A} & \vec{B} are irrotational.

i.e., $\nabla \times \vec{A} = \vec{0}$ and $\nabla \times \vec{B} = \vec{0}$

WKT $\nabla \cdot (\vec{A} \times \vec{B}) = (\nabla \times \vec{A}) \cdot \vec{B} - (\nabla \times \vec{B}) \cdot \vec{A}$

$$= \vec{0} \cdot \vec{B} - \vec{0} \cdot \vec{A}$$

$$= 0 - 0$$

$$= 0$$

Hence $\vec{A} \times \vec{B}$ is solenoidal.

QJ. Find the values of a, b, c so that the vector $\vec{F} = (x+y+az)\vec{i} + (bx+2y-z)\vec{j} + (-x+cy+2z)\vec{k}$ may be irrotational.

Soln.

Given \vec{F} is irrotational.

i.e., $\nabla \times \vec{F} = \vec{0}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y+az & bx+2y-z & -x+cy+2z \end{vmatrix} = \vec{0}$$





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$\vec{i}(c+1) - \vec{j}(-1-a) + \vec{k}(b-1) = 0\vec{i} + 0\vec{j} + 0\vec{k}$
 Equating the coefficients of like terms,

$$\begin{aligned}
 c+1 &= 0, & -1-a &= 0, & b-1 &= 0 \\
 c &= -1 & a &= -1 & b &= 1
 \end{aligned}$$

6]. Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential.

Soln.

Given $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$

Now

$$\begin{aligned}
 \nabla \times \vec{F} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 + 2xz^2 & 2xy - z & 2x^2z - y + 2z \end{vmatrix} \\
 &= \vec{i}[-1+1] - \vec{j}[4xz-4xz] + \vec{k}[2y-2y] \\
 &= 0\vec{i} - 0\vec{j} + 0\vec{k} \\
 \nabla \times \vec{F} &= \vec{0}
 \end{aligned}$$

Hence \vec{F} is irrotational.

$$\Rightarrow \vec{F} = \nabla \phi$$

$$\begin{aligned}
 (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k} \\
 = \vec{i} \frac{\partial \phi}{\partial x} + \vec{j} \frac{\partial \phi}{\partial y} + \vec{k} \frac{\partial \phi}{\partial z}
 \end{aligned}$$

Equating the coefficients of \vec{i} , \vec{j} & \vec{k} , we get

$$\frac{\partial \phi}{\partial x} = y^2 + 2xz^2 \quad \left| \quad \frac{\partial \phi}{\partial y} = 2xy - z \quad \left| \quad \frac{\partial \phi}{\partial z} = 2x^2z - y + 2z
 \right.$$

Integrating partially w.r. to x, y, z ,

$$\phi = xy^2 + x^2z^2 + f_1(y, z) \rightarrow (1)$$

$$\phi = xy^2 - yz + f_2(x, z) \rightarrow (2)$$

$$\phi = x^2z^2 - yz + z^2 + f_3(x, y) \rightarrow (3)$$

Comparing (1), (2), and (3), we get

$$\phi = xy^2 - yz + x^2z^2 + z^2 + c, \text{ where } c \text{ is the arbitrary constant}$$



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