SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore – 35

DEPARTMENT OF MATHEMATICS

UNIT – V DESIGN OF EXPERIMENTS

ANALYSIS OF VARIANCE (ANOVA):

ANOVA is a technique that will enable us to test the significance of the difference among more than two sample mean.

ASSUMPTION:

1) The observations are random.
2) The observations are independent.
3) The samples are drawn from normal populations.
4) Population variances are equal.

BASIC PRINCIPLES:

1) Randomisation
2) Replication
3) Local control.

BASIC DESIGN:

* Completely randomised design (CRD) One-way classifi
* Randomised Block design (RBD) Two-way classificat
* Latin square design (LSD) three-way classificati
* Two square factorial design

Hint :- F- Ratio : $F = \frac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$

procedure to find :-

2) Sum of all the terms (T) & Total no of Sample size (N)

3) Correction factor (C·F), $C \cdot F = \frac{T^2}{N}$

4) TSS : Total sum of squares
= (sum of the squares of all the terms) − C·F.

5) SSC : Sum of squares between samples

6) SSE : Error sum of squares
= TSS − SSC

7) Annova table

8) Conclusion :

1) Formulating H_0 & H_1

1) A completely randomised design experiment with 10 plots and 3 treatments gave the following result.

plot No. :	1	2	3	4	5	6	7	8	9	10
treatment :	A	B	C	A	C	C	A	B	A	B
yield :	5	4	3	7	5	1	3	4	1	7

Analyse the result for treatment effects.

Treatment	Yield			
(x_1) A	5	7	3	1
(x_2) B	4	4	7	-
(x_3) C	3	5	1	-

x_1	x_2	x_3	Total	x_1^2	x_2^2	x_3^2
5	4	3	12	25	16	9
7	4	5	16	49	16	25
3	7	1	11	9	49	1
1	-	-	1	1	-	-
16	15	9	40	84	81	35
Σx_1	Σx_2	Σx_3		Σx_1^2	Σx_2^2	Σx_3^2

Step 1: Formulating H_0 & H_1:

H_0: there is no significance difference between the treatments.

H_1: There is significance difference between the treatments

Step 2: To find T & N:

$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3$

$= 16 + 15 + 9 = 40$

$N = n_1 + n_2 + n_3$

$= 4 + 3 + 3 = 10$

Step 3: Correction Factor, C.F.

$$C.F = \frac{T^2}{N} = \frac{40^2}{10}$$

$$= 160$$

Step 4: $TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 - C.F$

$$= 84 + 81 + 35 - 160$$

$$= 40$$

Step 5: $SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - C.F.$

$$= \frac{16^2}{4} + \frac{15^2}{3} + \frac{9^2}{3} - 160$$

$$= 6$$

Step 6: $SSE = TSS - SSC$

$$= 40 - 6 = 34$$

Step 7: Annova table:

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-Rat[illegible]
Between samples (Column)	SSC : 6	$C-1 = 3-1$ $= 2$	$MSC : 6/2$ $: 3$	$F_c : \frac{4.[illegible]}{3}$ $: 1.[illegible]$
Within samples (Error)	SSE : 34	$N-c = 10-3$ $= 7$	$MSE : 34/7$ $: 4.9$	F_a([illegible] $: [illegible]$

Step 8: Conclusion:

$F_c = 1.61 < 19.35 = F_\alpha$, Ho is accepted

(i.e.) There is no significance difference between the treatments.

2) The following table shows the lives in hours of four brands of electric lamps.

A : 1610 1610 1650 1680 1700 1720 1800

B : 1580 1640 1640 1700 1750

C : 1466 1550 1600 1620 1640 1660 1740 1820

D : 1510 1520 1530 1570 1600 1680

Perform an analysis of variance and test the homogeneity of the means lives of the 4 brands of lamps. Origin: $\frac{x_{ij} - 1640}{10}$ Avg(Min, Max)

x_1	x_2	x_3	x_4	Total	x_1^2	x_2^2	x_3^2	x_4^2
-3	-6	-18	-13	-40	9	36	324	169
-3	0	-9	-12	-24	9	0	81	144
1	0	-4	-11	-14	1	0	16	121
4	6	-2	-7	1	16	36	4	49
6	11	0	-4	13	36	121	0	16
8	-	2	4	14	64	-	4	16
16	-	10	-	26	256	-	100	-
-	-	18	-	18	-	-	324	-
29	11	-3	-43	-6	391	193	853	515
Σx_i	Σx_2	Σx_3	Σx_4		Σx_1^2	Σx_2^2	Σx_3^2	Σx_4^2

Step 1: Formulating H_0 and H_1

H_0: There is no significance difference between the 4 brands of electric bulbs.

H_1: There is significance difference between the 4 brands of electric bulbs.

Step 2 : To find T & N

$$T = \Sigma x_1 + \Sigma x_2 + \Sigma x_3 + \Sigma x_4$$
$$= 29 + 11 - 3 - 43 = -6$$

$$N = n_1 + n_2 + n_3 + n_4$$
$$= 7 + 5 + 8 + 6 = 26$$

Step 3 : Correction factor.

$$C.F = \frac{T^2}{N} = \frac{-6^2}{26} = 1.3846$$

Step 4 : $TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - C.F$

$$= 391 + 193 + 853 + 515 - 1.3846$$
$$= 1950.61$$

Step 5 : $SSC = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - C.F.$

$$= \frac{29^2}{7} + \frac{11^2}{5} + \frac{-3^2}{8} + \frac{-43^2}{6} - 1.3846$$
$$= 120.14 + 24.2 + 1.125 + 308.16 - 1.3846$$
$$= 452.2404$$

Step 6 : $SSE = TSS - SSC$

$$= 1950.61 - 452.240$$
$$= 1498.375$$

Step 7 : Anova table :

Source of Variations	Sum of squares	Degrees of freedom	Mean squares	F-Ratio
Between samples (c)	SSC 452.2404	c−1 = 4−1 = 3	MSC $= \frac{452.2404}{3}$ = 150.7468	$F = \frac{150.7468}{68.10}$ = 2.2136
Within Samples (E)	SSE 1498.375	N−C = 26−4 = 22	MSE $= \frac{1498.375}{22}$ = 68.10	$F_\alpha(3,22)$ = 3.05

Step 8 : Conclusion :

$F = 2.2136 < 3.05 = F_\alpha$, H_0 is accepted.

i.e. there is no significance difference between the 4 brands of electric bulbs.