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## 19MAT204 - PROBABILITY AND STATISTICS

## PART-B

1. A company has 2 plants. Plant I manufactures $25 \%$ of the items. Plant II manufactures $75 \%$ of the items. $3 \%$ and $5 \%$ of the items manufactured by plant I and II are known to be defective. What is the chance that it was generated by plant II?
2. A Firm has three machines $\mathrm{A}, \mathrm{B}, \mathrm{C}$ which generate items in the proportion $2: 6: 3.50 \%, 70 \%$ and $90 \%$ of the items generated by A, $B, C$ respectively are known to have standard quality. An item selected at random from a day's production is known to have standard quality. What is the chance that it comes from machine C.
3. The contents of bags I,II and III are as follows; 1 white, 2 black and 3 red; 2 white , 1 black and 1 red; 4 white, 5 black and 3 red. One bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from bag I.
4. 

If $A$ and $B$ are independent events, prove that (i) $A$ and $\bar{B}$ are independent (ii) $\bar{A}$ and $B$ are independent (iii) $\bar{A}$ and $\bar{B}$ independent.
5. A random variable $X$ has the following probability distribution.

$$
\begin{array}{lllllllll}
X & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7
\end{array}
$$

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$\begin{array}{lllllllll} \\ P(x) & 0 & k & 2 k & 2 k & 3 k & k^{2} & 2 k^{2} & 7 k^{2}+k\end{array}$

## Find

(1)The value of $\boldsymbol{k}$
(2)Evaluate $\mathrm{P}(\mathrm{X}<6), \mathrm{P}(0<\mathrm{X}<5)$
(3) The smallest value of a for which $P(X \leq a)>\frac{1}{2}$.
(4)The Cumulative distribution function.
6. A random variable X has the following probability function

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | A | 3 a | 5 a | 7 a | 9 a | 11 a | 13 a | 15 a | 17 a |

Find (i) Determine the value of ' $a$ '
(ii) Find $\mathrm{P}(\mathrm{X}<3), \mathrm{P}(\mathrm{X} \geq 3), \mathrm{P}(0<\mathrm{X}<5)$
(iii) Find the distribution function of $X$.
7. A random variable $X$ has the following probability distribution

| X | -2 | -1 | 0 | 1 | 2 | 3 |
| :--- | :---: | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{P}(\mathrm{x})$ | 0.1 | K | 0.2 | 2 K | 0.3 | 3 K |

Find (1) The value of $\mathrm{K}(2)$ Evaluate $\mathrm{P}(\mathrm{X}<2)$ and $\mathrm{P}(-2<\mathrm{X}<2)$
Find the Cumulative distribution of $X$ (4) Find the mean of $X$.
8. If the Random variable $X$ takes the value $1,2,3,4$ such that $2 \mathrm{P}(\mathrm{X}=1)=3 \mathrm{P}(\mathrm{X}=2)=\mathrm{P}(\mathrm{X}=3)=5 \mathrm{P}(\mathrm{X}=4)$. Find the probability distribution.

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9. A continuous R.V $X$ has the p.d.f $f(x)=3 x^{2}, 0 \leq x \leq 1$. Find the value of a, such that $P(X \leq a)=P(X>a)$. Find the value b such that $\mathrm{P}(\mathrm{X}>\mathrm{b})=0.05$.
10. A continuous R.V. $X$ has the p.d.f.

$$
f(x)=\left\{\begin{array}{cc}
\frac{k}{1+x^{2}} & -\infty<x<\infty \\
0 & \text { otherwise }
\end{array}\right.
$$

. Find
(1) The value of $\boldsymbol{k}$
(2) Distribution function of $X$
(3) $\boldsymbol{P}(\boldsymbol{X} \geq 0)$
11. The probability function of an infinite discrete distribution is given by $P(X=j)=\frac{1}{2^{x}}(x=1,2,3, \ldots)$
(1) Mean and variance of $\boldsymbol{X}$
(2) M.G.F
(3) $\quad P(X$ is even $)$
12. A random variable has the pdf

$$
(x)=\left\{\begin{array}{ll}
2 e^{-2 x} & x \geq 0 \\
0 & \text { otherwise }
\end{array}\right. \text {. Obtain the mgf and first }
$$

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four moments about the origin. Also find the mean and variance.
13. Find the M.G.F of the random variable with the probability law $P(X=x)=q^{x-1} p, x=1,2,3, \ldots$. Find the mean and variance.
14. A continuous Random variable X has the distribution function $F(x)=\left\{\begin{array}{cc}0 & x \leq 1 \\ k(x-1)^{4} & 1 \leq x \leq 3 . \\ 1 & x>3\end{array}\right.$.
(1) Find K
(2) p.d.f $f(x)$
(3) $\mathrm{P}(\mathrm{X}<2)$.
15. The diameter of an electric cable say X , is assumed to be a continuous Random variable with P.d.f

$$
f(x)=6 x(1-x), 0 \leq x \leq 1
$$

(i) Check that the above is a P.d.f
(ii) Determine a and b such that $P(X<b)=P(X>b)$
(iii) Find the distribution function of $X$
(iv) Find $P\left(X \leq \frac{1}{2} / \frac{1}{3}<X<\frac{2}{3}\right)$
16. If the probability density of $X$ is given by

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$f(x)=\left\{\begin{array}{cc}2(1-x) & 0<x<1 \\ 0 & \text { otherwise }\end{array}\right.$
Find its $r$ th moment. Hence evaluate $E\left[(2 x+1)^{2}\right]$
17. If the cumulative distribution function of X is given
by $F(x)= \begin{cases}1-\frac{4}{x^{2}}, & x>2 \\ 0 & , x \leq 2\end{cases}$
Find (i) $\mathrm{P}(\mathrm{X}<3)$ (ii) $\mathrm{P}(4<\mathrm{X}<5)$
(iii) $P(X \geq 3)$.
18. Experience has shown that walking in a certain park, the time X (in mins), between seeing two people smoking has a density function of the form
$f(x)= \begin{cases}\lambda x e^{-x} & x>0 \\ 0 & \text { elsewhere }\end{cases}$
(a) Calculate the value of $\lambda$
(b) Find the distribution function of X
(c) What is the probability that a person who has just seen a person smoking will see another person smoking in 2 to 5 minutes? In atleast 7 minutes?
19. The density function of a random variable X is given by

$$
f(x)=\left\{\begin{array}{c}
k x(2-x)^{2} \quad 0<x<2 \\
0 \quad \text { otherwise }
\end{array} \text { find (i) } \mathrm{k}\right. \text { (ii) Mean and }
$$

variance of the distribution.

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20. Find the M.G.F for the distribution

$$
f(x)=\left\{\begin{array}{ll}
\frac{x}{4} & e^{-\frac{x}{2}} x>0 \\
0 & \text { otherwise }
\end{array}\right. \text { find (i) M.G.F (ii) First Four }
$$

moments obout the origin
21. A random variable has the p.d.f given by
$f(x)=\left\{\begin{array}{cc}2 e^{-2 x} & x \geq 0 \\ 0 & x<0\end{array}\right.$
Find (a) The moment generating function
(b) First four moments about the origin.

