SNS COLLEGE OF TECHNOLOGY
(An Autonomous Institution)

Coimbatore-641035.

UNIT-II ORDINARY DIFFERENTIAL EQUATIONS Homogeneous Linear ODE with constant coefficients

$$
\text { coff }-\bar{x}
$$

Difberentral Equation:
As eqn. Trolving diffoiontial cocticicionte ors derivatios is called difiocontial agn.
oedinacy ditbecontial uan.
A deffercontial can. which depends on
only one fropepondent vareiabie is called olderarey
diforiontial eqn.
Oeder and dogree:

* The aeder of the bignose descirative cacuring in the 9 rm . agm is called the oldor of a deffeiential oqr.
* The degree of the bighest desivative occuring in the grn. eqn. Is called the degree of a differentral eqn.

Second oscter iprear ODE wifs constant coetbicients: The genclal linoar $O D E$ urtes constant coefficients is of the form
$a_{0} \frac{d^{n} y}{d x^{n}}+a_{1} \frac{d^{n-1} y}{d x^{n-1}}+a_{2} \frac{d^{n-2} y}{d x^{n-2}}+\cdots+a_{n} y=f(x)$
where $a_{0}, a_{1}, \ldots a_{n}$ are constants and ${ }^{d}$ (1).
$f(x)$ is a function of $x$. when $f(x)=0$ in $(1)$ is called bomogeneocis ODE \&

If $f(x) \neq 0$ in (i) is called nonbomogeneous OOE.
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$$
\begin{aligned}
& \text { This eq. can be written as, } \\
& {\left[a_{0} D^{n}+a_{1} D^{n-1}+a_{2} D^{n-2}+\cdots+a_{n}\right) y=f(x)} \\
& \text { where } D=\frac{d}{d x} \\
& \text { Solution }=C F+P I \\
& \text { = Complement ard function + Particular isteg } \\
& \text { To find EF: } \\
& \text { Roots CF } \\
& \text { 1). Roots are leal is difenant } \\
& m_{1} \neq m_{2} \geqslant \ldots . \\
& \text { in) Roots are leal of same } \\
& m_{1}=m_{2}=m \\
& \text { iii) Roots are imaginary. } \\
& m=\alpha \pm i \beta \\
& \text { To fend PI: } \\
& P I=\frac{1}{f(D)} f(x)
\end{aligned}
$$

RH $=0$
D. Solve $\left(D^{2}-5 D+6\right)=0$
solo.
The auxplary eat. is
$\left(m^{2}-5 m+6=0\right.$
$(m-3)(m-2)=0$
$m=2,3$
$\therefore$ The roots are real and alifberent.
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$$
\begin{aligned}
& C F=A e^{8 x}+A e^{8 x} \\
& \therefore y=C F=A e^{2 x}+B 0^{3 x} \\
& \text { BJ. Solve } \frac{d^{2} y}{d x^{2}}-6 \frac{d y}{d x}+a y=0
\end{aligned}
$$

solis.

$$
\left(D^{2}-6 D+9\right) y=0
$$

The Auxiliary ear is

$$
m^{5}-6 m+9=0
$$

$$
(m-3)^{2}=0
$$

$$
m=3,3
$$

The loots ar eccl and same.
$C F=(A \neq B x) e^{3 x}$
$\therefore y=C F=(A+B x) e^{3 x}$
3]. Solve $\left(D^{2}+1\right)^{2} y=0$
Sold. The Auralarg eqn. is $m^{2}+s^{2}=0$
takary square root on both sides $m^{2}+1=0$ $m^{2}=-1$ $m= \pm 1$
The loots are $=0.2 .1$ imaginary
Here $\alpha=0, \beta=1$
$\therefore C F=e^{0}(A \cos x+B \sin x)$
$=A \cos x+B \sin x$

$$
\therefore y=C F=A \cos x+B \sin x
$$

4]. Solve $\left(D^{4}-1\right) Y=0$
Sols. The auritary eau. is $m^{4} \rightarrow=0$

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$$
\begin{aligned}
& \left(m^{2}\right)^{2}-1^{2}=0 \\
& \left(m^{2}+1\right)\left(m^{2}-1\right)=0 \\
& \begin{array}{c|c}
m^{2}+1=0 & m^{2}-1=0 \\
m^{2}=-1 & m^{2}=1 \\
m= \pm i & m= \pm 1
\end{array} \\
& \therefore C F=A e^{x}+B e^{-x}+C \cos x+D \sin x \text {. }
\end{aligned}
$$

