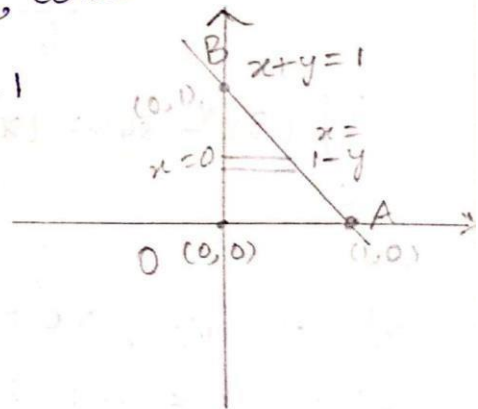




4]. Use Green's theorem & evaluate
 $\int_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$, where C is the
curve with $x=0, y=0, x+y=1$

Soln.

Let $x=0, y=0, x+y=1$
when $x=0, y=1 \Rightarrow (0, 1)$
 $y=0, x=1 \Rightarrow (1, 0)$



By Green's theorem,

$$\int_C [M dx + N dy] = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] dx dy$$

$$\text{Here } M = 3x^2 - 8y^2 \quad \left| \quad N = 4y - 6xy\right.$$
$$\frac{\partial M}{\partial y} = -16y \quad \left| \quad \frac{\partial N}{\partial x} = -6y\right.$$

RHS

$$\iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] dx dy = \int_0^1 \int_0^{1-y} [-6y + 16y] dx dy$$



$$\begin{aligned}
 &= \int_0^1 \int_0^{1-y} 10y \, dx \, dy \\
 &= 10 \int_0^1 \int_0^{1-y} y \, dx \, dy = 10 \int_0^1 y [x]_0^{1-y} \, dy \\
 &= 10 \int_0^1 y [1-y-0] \, dy \\
 &= 10 \int_0^1 [y - y^2] \, dy \\
 &= 10 \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1 = 10 \left[\frac{1}{2} - \frac{1}{3} \right] \\
 &= 10 \left[\frac{3-2}{6} \right] \\
 &= \frac{10}{6}
 \end{aligned}$$

$$\therefore \int_C (3x^2 - 8y^2) \, dx + (4y - 6xy) \, dy = \frac{5}{3}$$

5]. verify Green's theorem $\int_C (x^2 - xy^3) \, dx + (y^2 - 2xy) \, dy$
where C is the square with vertices $(0,0)$, $(2,0)$, $(2,2)$, $(0,2)$.

Soln.

$$\text{Given } \int_C (x^2 - xy^3) \, dx + (y^2 - 2xy) \, dy$$

to verify

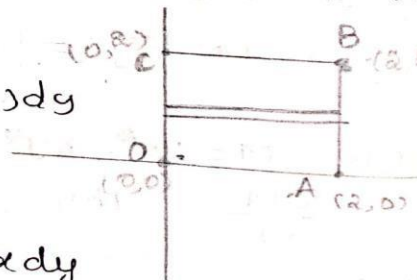
$$\int_C M \, dx + N \, dy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] \, dx \, dy$$

$$\text{Here } M = x^2 - xy^3$$

$$\frac{\partial M}{\partial y} = -3xy^2$$

$$N = y^2 - 2xy$$

$$\frac{\partial N}{\partial x} = -2y$$





RHS

$$\begin{aligned} \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right] dx dy &= \int_0^2 \int_0^2 [-2y + 3xy^2] dx dy \\ &= \int_0^2 \left[-2yx + 3 \frac{x^2}{2} y^2 \right]_{x=0}^2 dy \\ &= \int_0^2 \left[-4y + \frac{3}{2}(4)y^2 \right] dy \\ &= \int_0^2 [-4y + 6y^2] dy \\ &= \left[-\frac{4y^2}{2} + \frac{6y^3}{3} \right]_0^2 \\ &= [-2(4) + 2(8)] - 0 \\ &= 8 \end{aligned}$$

LHS
To evaluate $\int [M dx + N dy]$, we shall take C in the different paths.

- i). Along OA [$y=0$]
- ii). Along AB [$x=2$]
- iii). Along BC [$y=2$]
- iv). Along CD [$x=0$]

Along OA [$y=0 \Rightarrow dy=0$]

$$\begin{aligned} \int_{OA} (x^2 - xy^2) dx + (y^2 - 2xy) dy \\ &= \int_0^2 [x^2 - 0] dx + [0 - 0](0) \\ &= \int_0^2 x^2 dx = \left[\frac{x^3}{3} \right]_0^2 \end{aligned}$$



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Along AB ($x=2 \Rightarrow dx=0$)

$$\int_{AB} (x^2 - xy^3) dx + (y^2 - 2xy) dy$$
$$= \int_0^2 (-4 - 2y^3)(0) + (y^2 - 4y) dy$$
$$= \int_0^2 [y^2 - 4y] dy$$
$$= \left[\frac{y^3}{3} - \frac{4y^2}{2} \right]_0^2$$
$$= \left(\frac{8}{3} - 2(4) \right) - 0 = \frac{8}{3} - 8$$
$$= \frac{8 - 24}{3}$$
$$= \frac{-16}{3}$$

Along BC ($y=2 \Rightarrow dy=0$)

$$\int_{BC} (x^2 - xy^3) dx + (y^2 - 2xy) dy$$
$$= \int_2^0 (x^2 - x(8)) dx + 0$$
$$= \int_2^0 [x^2 - 8x] dx$$
$$= \left[\frac{x^3}{3} - 8 \frac{x^2}{2} \right]_2^0$$
$$= 0 - \left(\frac{8}{3} - 4(4) \right) = - \left[\frac{8 - 48}{3} \right]$$
$$= \frac{40}{3}$$



Along C_0

$$x=0 \Rightarrow dx=0$$

$$\int_{C_0} (x^2 - xy^3) dx + (y^2 - 2xy) dy$$

$$= \int_2^0 [0 + (y^2 - 0) dy]$$

$$= \int_2^0 y^2 dy$$

$$= \left[\frac{y^3}{3} \right]_2^0$$

$$= 0 - \frac{8}{3}$$

$$= -\frac{8}{3}$$

$$\therefore \int_C (x^2 - xy^3) dx + (y^2 - 2xy) dy = \frac{-8 \cdot 16 + 40 + 8}{3}$$

$$= \frac{-24}{3}$$

$$= 8$$

\therefore LHS = RHS

Hence verified.

