CHINESE REMAINDER THEOREM

The system of n Congruences $x_i = a_i \pmod{m_i}$ $i = 1, 2, 3, \dots n$. Where $GCD(m_i, m_j) = 1$ if $i \neq j$ has unique solution modulo $(m_1 m_2 m_3 \cdots m_n)$

Problem:

$X = 2 \mod 3$

$X = 3 \mod 5$

$X = 2 \mod 7.$

Here $a_1 = 2$ $a_2 = 3$ $a_3 = 2$

$m_1 = 3$ $m_2 = 5$ $m_3 = 7$

Step 1:

$m = m_1 \times m_2 \times m_3$

$= 3 \times 5 \times 7$

$= 105$

Clearly $(m_1, m_2) = 1$ $(m_2, m_3) = 1$ $(m_3, m_1) = 1$

Step 2:

$\frac{m}{m_1} = 7 \times 5 = 35$

$\frac{m}{m_2} = 3 \times 7 = 21$

$\frac{m}{m_3} = 3 \times 5 = 15$

Step 3:

$\frac{m}{m_1} x_1 = 1 \pmod{m_1}$, $\frac{m}{m_2} x_2 = 1 \pmod{m_2}$, $\frac{m}{m_3} x_3 = 1 \pmod{m_3}$

$35 x_1 = 1 \pmod 3$, $21 x_2 = 1 \pmod 5$, $15 x_3 = 1 \pmod 7$

$x_1 = 2$ $x_2 = 1$ $x_3 = 1$

$2x_1 \equiv 1$, $x_2 \equiv 1$, $x_3 \equiv 1$

Required solution.

$$X \equiv \left(\frac{m}{m_1}x_1 a_1 + \frac{m}{m_2}x_2 a_2 + \frac{m}{m_3}x_3 a_3\right) \pmod{m_1 m_2 m_3}$$

$$\equiv (35 \times 2 \times 2 + 21 \times 1 \times 3 + 15 \times 1 \times 2)$$

$$\equiv (140 + 63 + 30) \pmod{105}$$

$$\equiv 233 \bmod 105$$

$$\equiv 23.$$

$$X \equiv 23$$