

## UNIT - 2 BEAMS & THEORY OF BENDING

### Over hanging Beam:

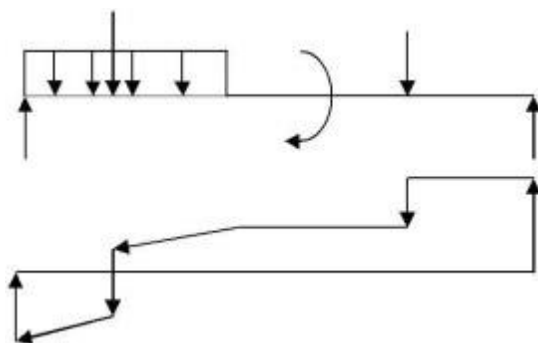
1. Find the reactions at the supports.
2. When taking moment to find the reactions consider even the pure moment in the beam, be careful with the direction of the moment. Then follow the SF and BM diagram procedure to complete the figure.
3. Simply add the load from right to find the shear force at various points. Upward SF minus downward SF will give SF at a point it may be +ve or -ve SF.
4. Multiply the load with distance to find the moment at various points. Anti clockwise BM minus clockwise BM will BM at a point it may be +ve or -ve SF.
5. The moment changes the sign from positive to negative such point is known as point of contraflexure. To find the point of contraflexure find the section where MB is zero equate clockwise moments to anti clockwise moment to distance x from a support.
6. Moments are zero at the supports where there is no overhanging, and at the over hanging end.

### Drawing Shear force diagram:

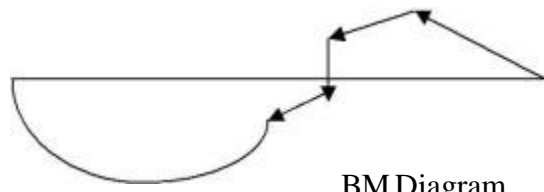
1. Draw a reference line equal to length of the beam to scale.
2. Move the line up if SF is pointing upward or move the line down if SF is pointing downward.
3. When there is no load between loads draw horizontal line parallel to reference line.
4. Point load is represented by vertical line.
5. udl is represented by inclined line.
6. Uniformly varying load is represented by parabola line.
7. Ignore moment for shear force diagram.

### Drawing Bending Moment diagram:

1. Draw a reference line equal to length of the beam to scale.
2. Locate a point to find BM, clockwise is taken as negative and anti clockwise is taken as positive.
3. Draw an inclined line to the point if the moment is due to point load only between sections.
4. Draw a parabolic line to the point if the moment is due to udl load between sections.
5. Draw a vertical line for pure moment on the beam, downward if it is clockwise moment and upward if it is anti clockwise moment.



SF Diagram



BMD Diagram

**ending Stress**

$M = WL/4$  Simply support beam point load at mid span

$M = WL$  Cantilever beam load at distance  $L$  from the support  $\frac{M = wL^2}{2}$

$M = wL^2 / 2$  Cantilever beam of udl throughout the span

Stress is zero at centroid (NA) that is at distance  $y$  from the xx-axis and maximum at the top and bottom

We know,

$$\frac{M}{I} = \frac{\zeta}{y} = \frac{E}{R}$$

**M** – Bending moment or Moment may vary depending on the load  
example **I**– Moment of Inertia.

$\zeta$  – Stress due to bending moment. To find  $\zeta_c$  then  $y = y_c$  and to find  $\zeta_t$  then  $y = y_t$  - Centroid of the section about xx axis (NA). To find  $\zeta_c$  then  $y = y_c$  and to find  $\zeta_t$  then  $y = y_t$

**E** – Modulus of Elasticity or Young’s modulus.

**R**- Radius of curvature due to bending.

For symmetric section value of  $\zeta_c = \zeta_t$  because  $y_c = y_t$  example, rectangle, circular, and symmetric I section. That is N.A will be at mid point. The value  $y_c = y$  from the bottom to NA for beam under compression and  $y_t = y$  from the top to NA for beam under tension. To find the safe Load or moment find the value of  $\zeta_c/y_c$  and  $\zeta_t/y_t$  and take the least value for safe design.

$I = bd^3$

$I = \pi(D_o^4 - D_i^4)/64$  for hollow pipe and solid rod  $y = D_o/2$  for solid pipe  $D_i = 0$

Centroid (NA) of total section  $y = \text{sum of (area of each section x centroid of each section from xx axis)} / \text{sum of (area of each section)}$  Ref: figure

$$y = \frac{a_1y_1 + a_2y_2 + \dots + a_ny_n}{a_1 + a_2 + \dots + a_n}$$

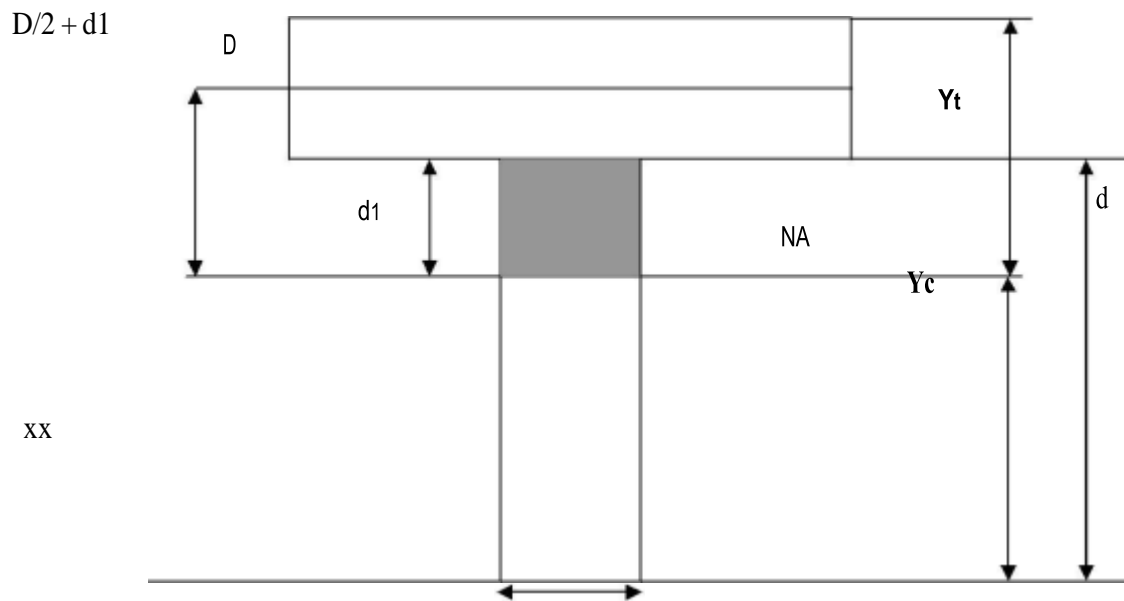
**Substitute the value y in the moment of inertia equation.**

Stress is caused due to Shear force or load. The shear load is right angle to the section. Shear Stress is zero at the top and bottom of the section and it is the maximum at centroid (NA) distance  $y$  from the xx-axis.

$$\eta = \frac{FAy}{Ib}$$

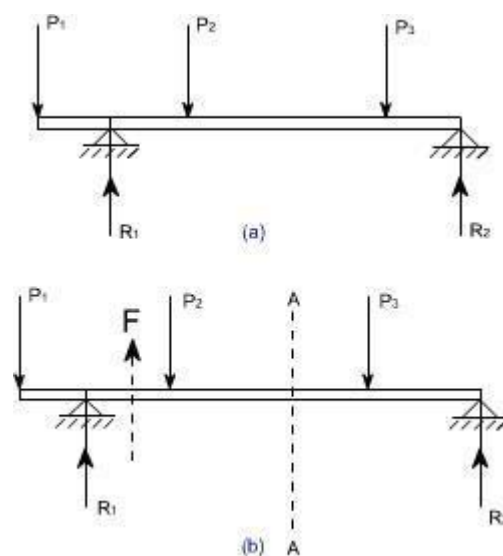
$\eta$ – Shear stress at a point  $F$ – Shear load  $A$  – Area of the section considered.

$y$  – Centroid distance of the section considered from the Neutral axis of the whole section.  $I$ – Inertia of the whole section  $b$ – Width of the section considered.



### Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms



**Fig 1**

Now let us consider the beam as shown in fig 1(a) which is supporting the loads  $P_1$ ,  $P_2$ ,  $P_3$  and is simply supported at two points creating the reactions  $R_1$  and  $R_2$  respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume that the resultant of loads and reactions to the left of AA is „F'

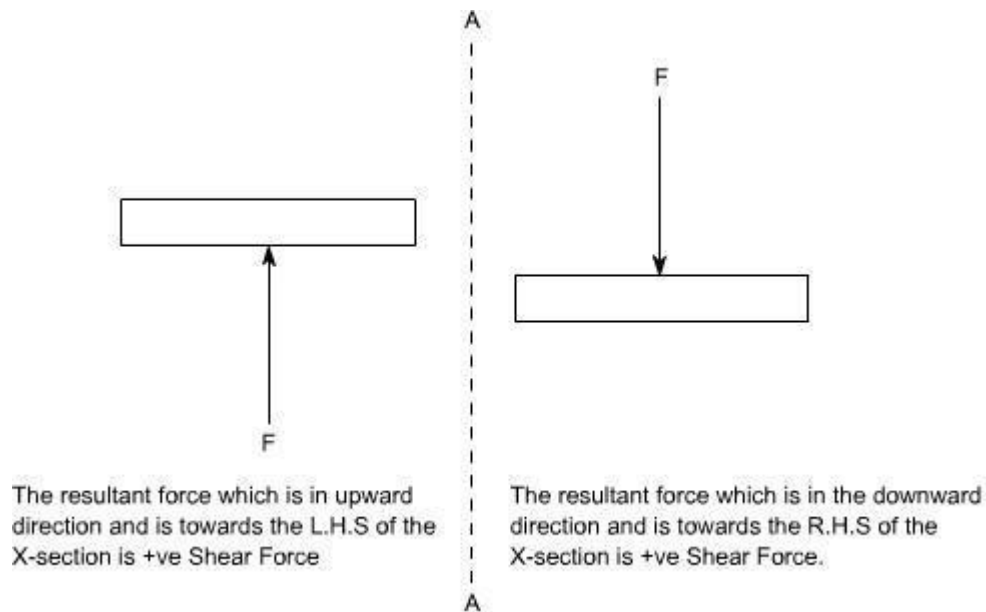
vertically upwards, and since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be  $F$ , acting downwards. This forces „ $F$ ' is as a shear force. The shearing force at any  $x$ - section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

Therefore, now we are in a position to define the shear force „ $F$ ' to as follows:

At any  $x$ -section of a beam, the shear force „ $F$ ' is the algebraic sum of all the lateral components of the forces acting on either side of the  $x$ -section.

### Sign Convention for Shear Force:

The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.



**Fig 2: Positive Shear Force**

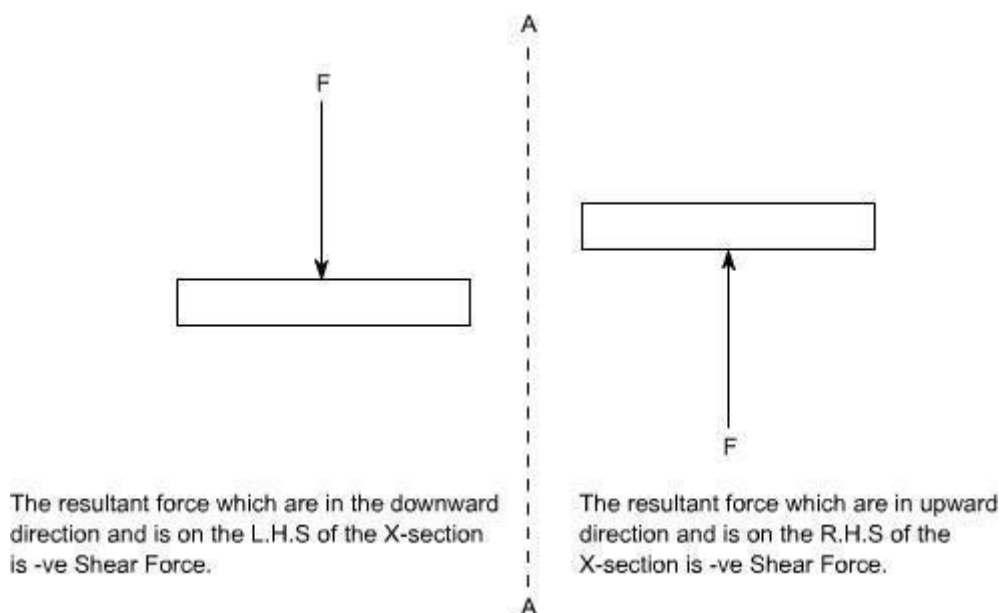
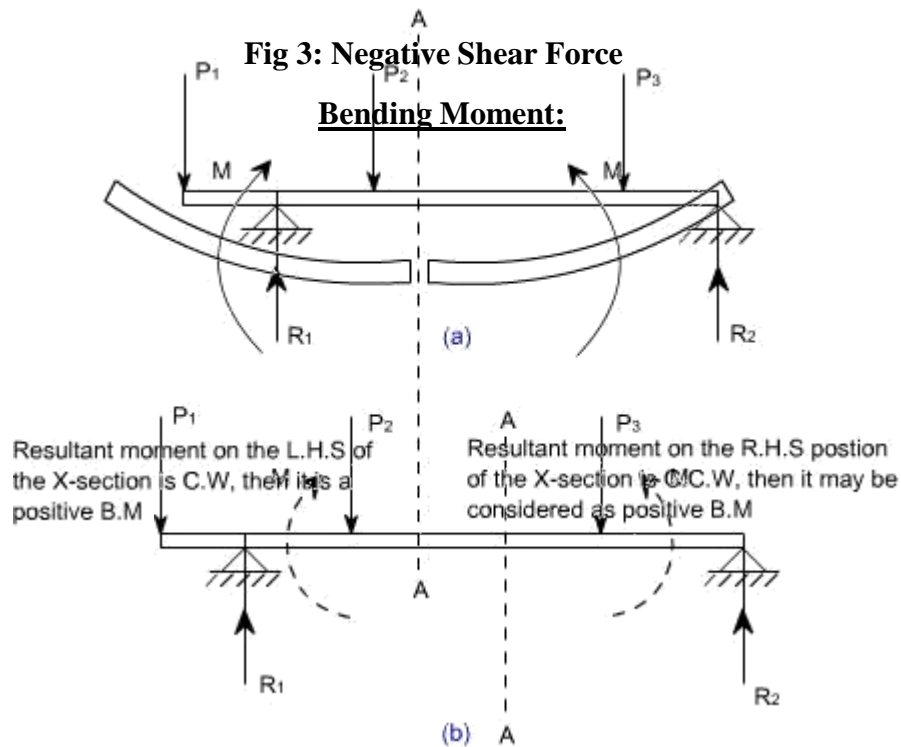


Fig 4



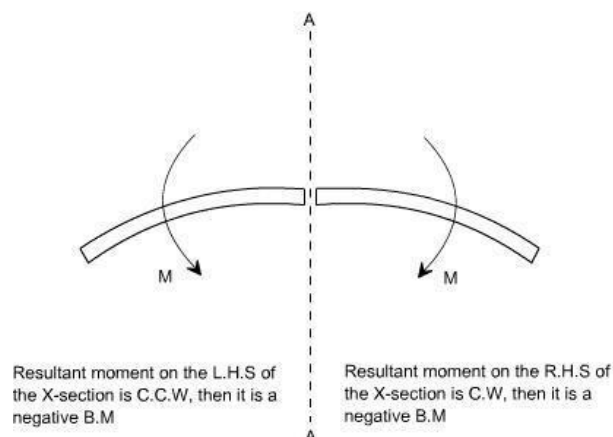
Let us again consider the beam which is simply supported at the two prints, carrying loads  $P_1$ ,  $P_2$  and  $P_3$  and having the reactions  $R_1$  and  $R_2$  at the supports Fig 4. Now, let us imagine that the beam is cut into two portions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is  $M$  in C.W direction, then moment of forces to the right of x-section AA must be „ $M'$ “ in

C.C.W. Then „ $M'$ “ is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section

**Sign Conventions for the Bending Moment:**

For the bending moment, following sign conventions may be adopted as indicated in Fig 5 and Fig 6.

**Fig 5: Positive Bending Moment**



Some times, the terms „Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

**Bending Moment and Shear Force Diagrams:**

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

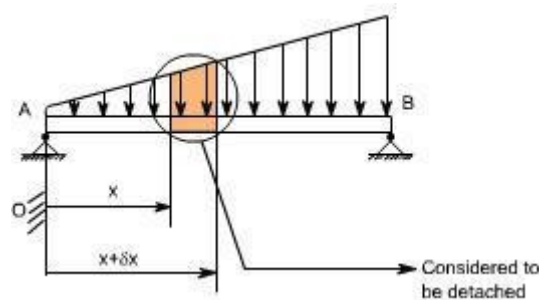
Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force „F' varies along the length of beam. If x denotes the length of the beam, then F is function x i.e. F(x).

Similarly a bending moment diagram is a graphical plot which depicts how the internal bending moment „M' varies along the length of the beam. Again M is a function x i.e. M(x).

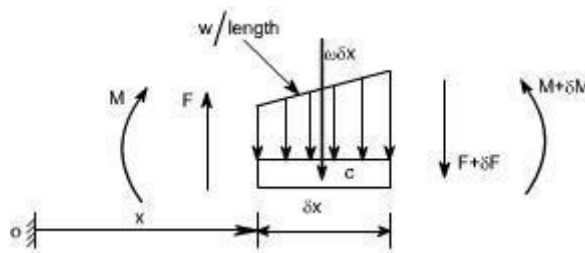
**Basic Relationship Between The Rate of Loading, Shear Force and Bending Moment:**

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established.

Let us consider a simply supported beam AB carrying a uniformly distributed load w/length. Let us imagine to cut a short slice of length dx cut out from this loaded beam at distance „x' from the origin „O'.



Let us detach this portion of the beam and draw its free body diagram.



The forces acting on the free body diagram of the detached portion of this loaded beam are the following

- The shearing force F and F+  $\delta F$  at the section x and x +  $\delta x$  respectively.
- The bending moment at the sections x and x +  $\delta x$  be M and M +  $dM$  respectively.

- Force due to external loading, if „w' is the mean rate of loading per unit length then the total loading on this slice of length  $\delta x$  is  $w \cdot \delta x$ , which is approximately acting through the centre „c'. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre „c'.

This small element must be in equilibrium under the action of these forces and couples. Now let us take the moments at the point „c'. Such that

$$\begin{aligned}
 &M + F \cdot \frac{\delta x}{2} + (F + \delta F) \cdot \frac{\delta x}{2} = M + \delta M \\
 \Rightarrow &F \cdot \frac{\delta x}{2} + (F + \delta F) \cdot \frac{\delta x}{2} = \delta M \\
 \Rightarrow &F \cdot \frac{\delta x}{2} + F \cdot \frac{\delta x}{2} + \delta F \cdot \frac{\delta x}{2} = \delta M \quad [\text{Neglecting the product of} \\
 &\hspace{15em} \delta F \text{ and } \delta x \text{ being small quantities}] \\
 \Rightarrow &F \cdot \delta x = \delta M \\
 \Rightarrow &F = \frac{\delta M}{\delta x}
 \end{aligned}$$

Under the limits  $\delta x \rightarrow 0$

$$\boxed{F = \frac{dM}{dx}} \quad \dots\dots\dots (1)$$

Re solving the forces vertically we get

$$\begin{aligned}
 &w \cdot \delta x + (F + \delta F) = F \\
 \Rightarrow &w = - \frac{\delta F}{\delta x} \\
 &\text{Under the limits } \delta x \rightarrow 0
 \end{aligned}$$

$$\Rightarrow w = - \frac{dF}{dx} \text{ or } - \frac{d}{dx} \left( \frac{dM}{dx} \right)$$

$$\boxed{w = - \frac{dF}{dx} = - \frac{d^2M}{dx^2}} \quad \dots\dots\dots (2)$$