UNIT - 2 BEAMS & THEORY OF BENDING

CANTILEVER WITH A POINT LOAD AT ITS FREE END

Consider a *cantilever *AB* of length *l* and carrying a point load *W* at its free end *B* as shown in Figure 2 (*a*). We know that shear force at any section *X*, at a distance *x* from the free end, is equal to the total unbalanced vertical force. *i.e.*,

$$F_x = -W \qquad \text{(Minus sign due to right downward)}$$

and bending moment at this section,

$$M_x = -W.\,x \qquad \text{(Minus sign due to hogging)}$$

From the equation of shear force, we see that the shear force is constant and is equal to – *W* at all sections between *B* and *A*. And from the bending moment equation, we see that the bending moment is zero at *B* (where *x* = 0) and increases by a straight-line law to – *Wl*. at (where *x* = *l*). Now draw the shear force and bending moment diagrams as shown in Figure 2 (*b*) and (*c*) respectively.

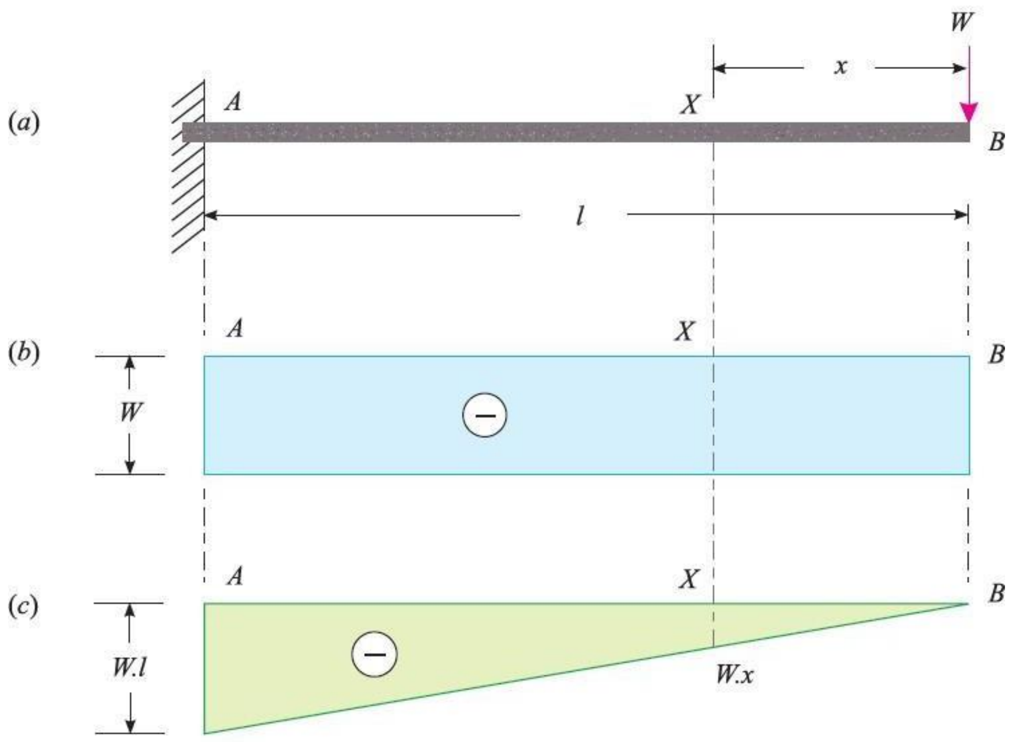

Figure 2. Cantilever with a point load

1 Draw shear force and bending moment diagrams for a cantilever beam of span 1.5 m carrying point loads as shown in Figure 3.

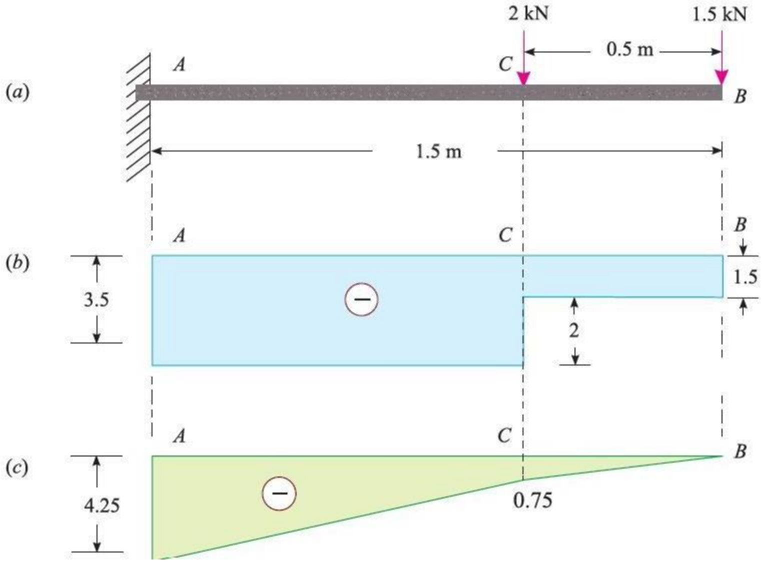

Figure 3.

Given Data: Span (l) = 1.5 m ; Point load at B ($W1$) = 1.5 kN and point load at C ($W2$) = 2 kN.

Shear force diagram

The shear force diagram is shown in Figure 3 (*b*) and the values are tabulated here:

$F_B = - W1 = - 1.5$ kN

$F_C = - (1.5 + W2) = - (1.5 + 2) = - 3.5$ kN

$F_A = - 3.5$ kN

Bending moment diagram

The bending moment diagram is shown in Figure 3 (*c*) and the values are tabulated here:

$M_B = 0$

$M_C = - [1.5 \times 0.5] = - 0.75$ kN-m

$M_A = - [(1.5 \times 1.5) + (2 \times 1)] = - 4.25$ kN-m

CANTILEVER WITH A UNIFORMLY DISTRIBUTED LOAD

Consider a cantilever AB of length l and carrying a uniformly distributed load of w per unit length, over the entire length of the cantilever as shown in Figure 4 (*a*). We know that shear force at any section X, at a distance x from B, $F_x = - w. x$ ... (Minus sign due to right downwards) Thus we see that shear force is zero at B (where $x = 0$) and increases by a straight-line law to $-wl$ at A as shown in Figure 4 (*b*).

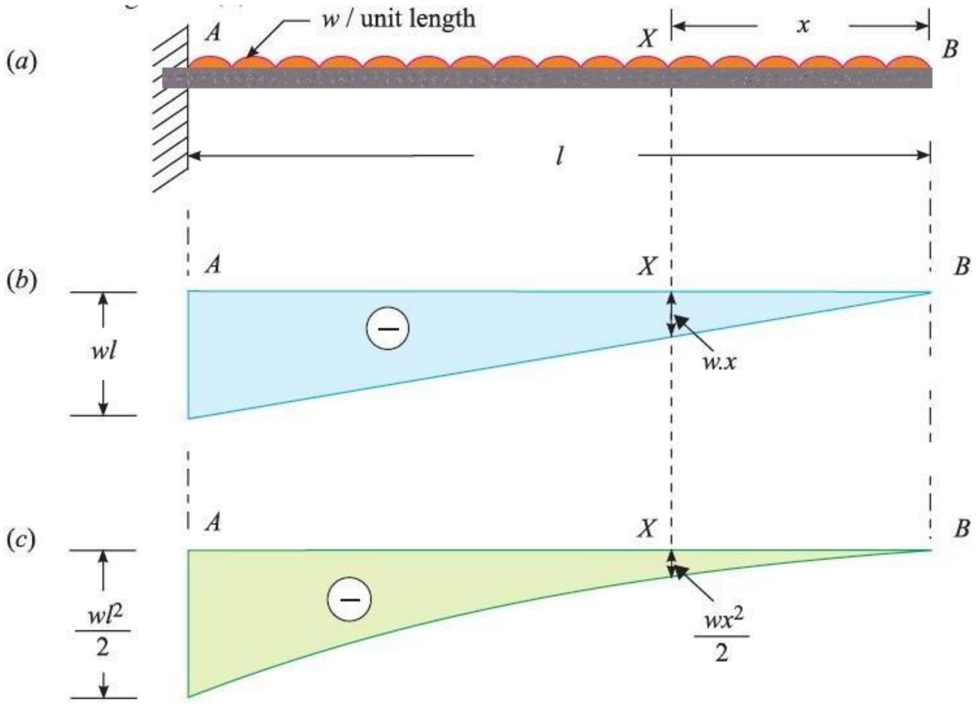

Figure 4.

We also know that bending moment at X,

$$M_x = -wx \cdot \frac{x}{2} = -\frac{wx^2}{2} \quad \text{...(Minus sign due to hogging)}$$

Thus we also see that the bending moment is zero at B (where $x = 0$) and increases in the form of a parabolic curve to $-\frac{wl^2}{2}$ at B (where $x = l$)

as shown in figure 4 (c)

1. ***A cantilever beam AB, 2 m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw shear force and bending moment diagrams for the beam.***

Given: span (l) = 2 m; Uniformly distributed load (w) = 1.5 kN/m and length of the cantilever CB carrying load (a) = 1.6 m.

Shear force diagram

The shear force diagram is shown in Figure 5 (b) and the values are tabulated here:

$$F_B = 0$$
$$F_C = -w \cdot a = -1.5 \times 1.6 = -2.4 \text{ kN}$$
$$F_A = -2.4 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Figure 5 (c) and the values are tabulated here:

$$M_B = 0$$
$$M_C = -\frac{wa^2}{2} = \frac{1.5 \times (1.6)^2}{2} = -1.92 \text{ kN-m}$$
$$M_A = -\left[(1.5 \times 1.6)\left(0.4 + \frac{1.6}{2}\right)\right] = -2.88 \text{ kN-m}$$

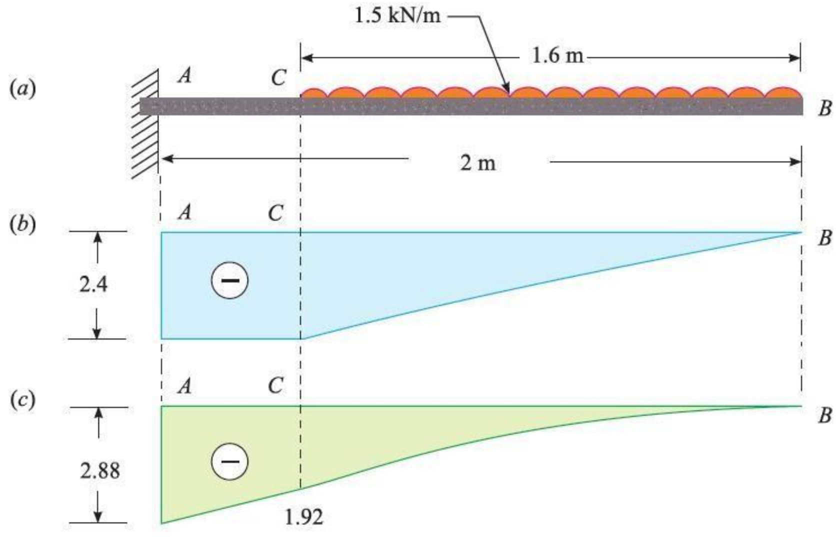

Figure 5.

2. ***A cantilever beam of 1.5 m span is loaded as shown in Figure 6 (a). Draw the shear force and bending moment diagrams.***

Given: Span (l) = 1.5 m; Point load at B (W) = 2 kN; Uniformly distributed load (w) = 1 kN/m and length of the cantilever AC carrying the load (a) = 1 m.

Shear force diagram

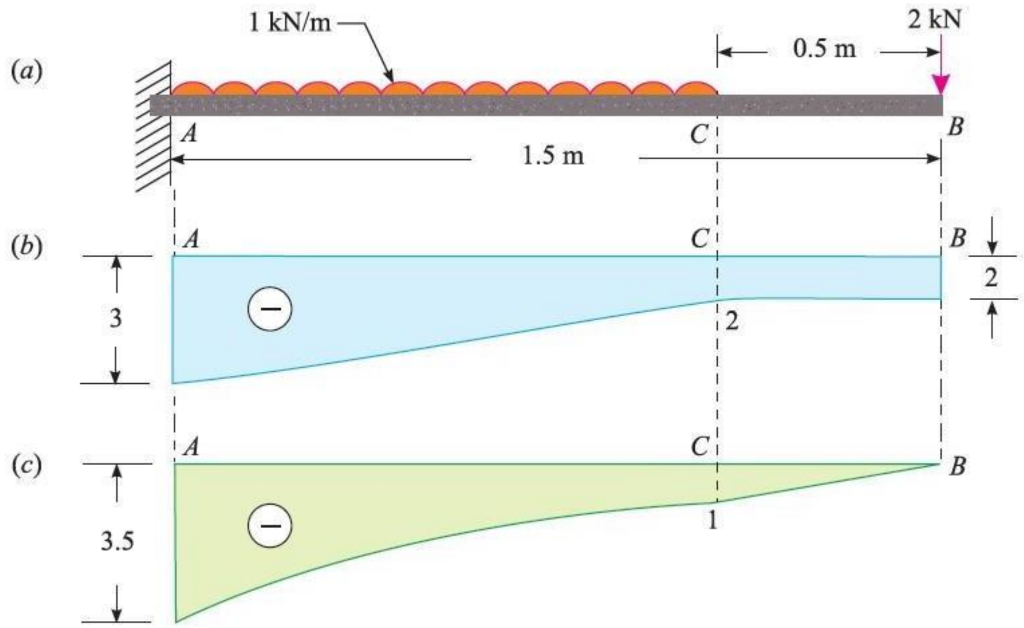

Figure 6.

$$F_B = -W = -2 \text{ kN}$$
$$F_C = -2 \text{ kN}$$
$$F_A = -[2 + (1 \times 1)] = -3 \text{ kN}$$

Bending moment diagram

$$M_B = 0$$
$$M_C = -[2 \times 0.5] = -1 \text{ kN-m}$$
$$M_A = -\left[(2 \times 1.5) + (1 \times 1) \times \frac{1}{2}\right] = -3.5 \text{ kN-m}$$

3. A simply supported beam AB of span 2.5 m is carrying two-point loads as shown in Figure. Draw the shear force and bending moment diagrams.

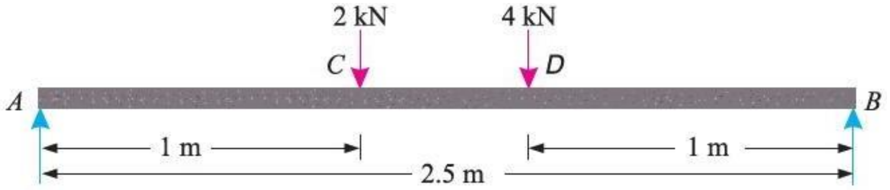

Figure 7.

Given: Span (l) = 2.5 m; Point load at C (W_1) = 2 kN and point load at B (W_2) = 4 kN.

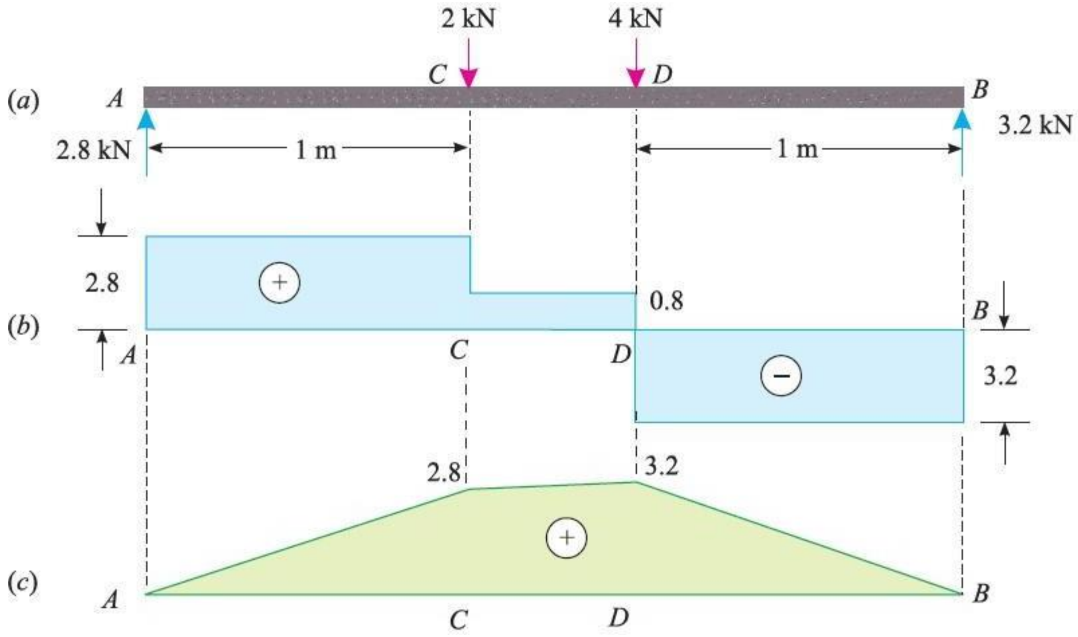

Figure 7.1

First of all let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 2.5 = (2 \times 1) + (4 \times 1.5) = 8$$

$$R_B = 8/2.5 = 3.2 \text{ kN}$$

and

$$R_A = (2 + 4) - 3.2 = 2.8 \text{ kN}$$

Shear force diagram

The shear force diagram is shown in Figure 7.1 (b) and the values are tabulated here:

$$F_A = +R_A = 2.8 \text{ kN}$$

$$F_C = +2.8 - 2 = 0.8 \text{ kN}$$

$$F_D = 0.8 - 4 = -3.2 \text{ kN}$$

$$F_B = -3.2 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Figure 7.1 (c) and the values are tabulated here:

$$M_A = 0$$
$$M_C = 2.8 \times 1 = 2.8 \text{ kN-m}$$
$$M_D = 3.2 \times 1 = 3.2 \text{ kN-m}$$
$$M_B = 0$$

4. A simply supported beam 6 m long is carrying a uniformly distributed load of 5 kN/m over a length of 3 m from the right end. Draw the S.F. and B.M. diagrams for the beam and also calculate the maximum B.M. on the section.

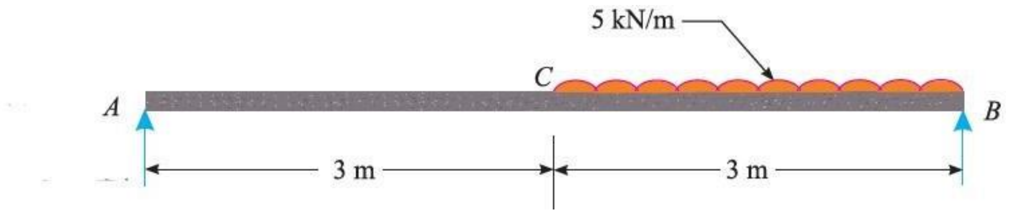

Figure 8.

Given: Span (l) = 6 m; Uniformly distributed load (w) = 5 kN/m and length of the beam *CB* carrying load (a) = 3 m.

First of all, let us find out the reactions R_A and R_B. Taking moments about *A* and equating the same,

$$R_B \times 6 = (5 \times 3) \times 4.5 = 67.5$$

$\therefore$ $$R_B = \frac{67.5}{6} = 11.25 \text{ kN}$$

and $$R_A = (5 \times 3) - 11.25 = 3.75 \text{ kN}$$

Shear force diagram

The shear force diagram is shown in Figure 8.1 (b) and the values are tabulated here:

$$F_A = +R_A = +3.75 \text{ kN}$$
$$F_C = +3.75 \text{ kN}$$
$$F_B = +3.75 - (5 \times 3) = -11.25 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Figure 8.1 (c) and the values are tabulated here:

$$M_A = 0$$
$$M_C = 3.75 \times 3 = 11.25 \text{ kN}$$
$$M_B = 0$$

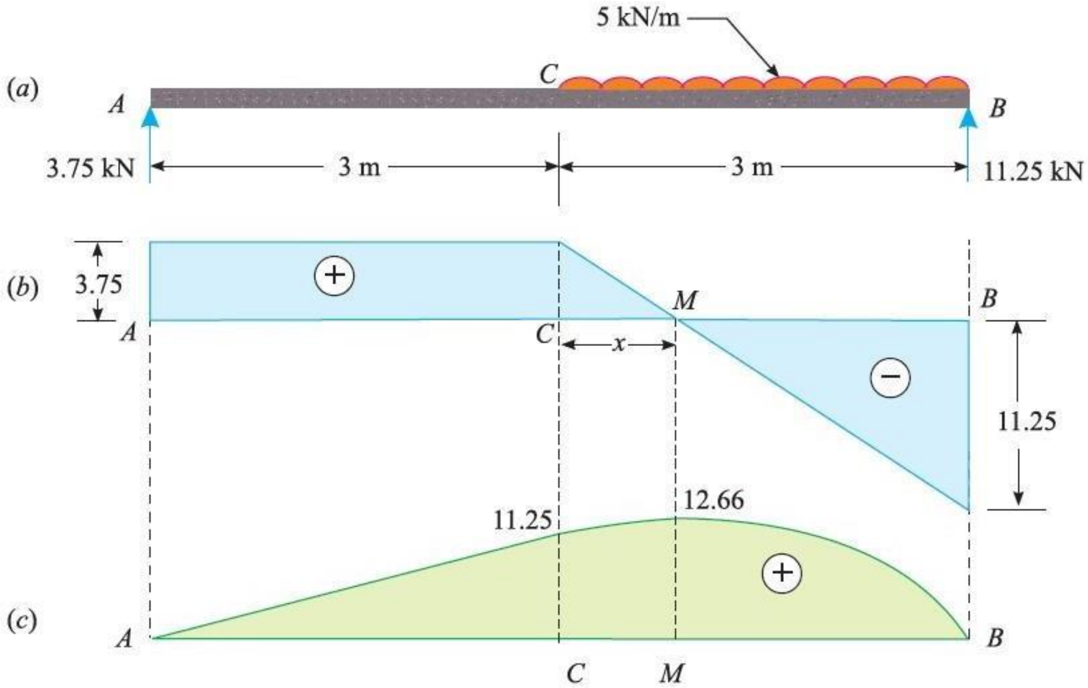

Figure 8.1.