UNIT - 1 STRESS, STRAIN DEFORMATION OF SOLIDS

GRAPHICAL SOLUTION – MOHR'S STRESS CIRCLE

The transformation equations for plane stress can be represented in a graphical form known as Mohr's circle. This grapical representation is very useful in depending the relationships between normal and shear stresses acting on any inclined plane at a point in a stresses body.

To draw a Mohr's stress circle consider a complex stress system as shown in the figure

The above system represents a complete stress system for any condition of applied load in two dimensions

The Mohr's stress circle is used to find out graphically the direct stress < and sheer stress<< on any plane inclined at < to the plane on which < $_x$ acts.The direction of < here is taken in anticlockwise direction from the BC.

STEPS:

In order to do achieve the desired objective we proceed in the following manner

(i) Label the Block ABCD.

(ii) Set up axes for the direct stress (as abscissa) and shear stress (as ordinate)

(iii) Plot the stresses on two adjacent faces e.g. AB and BC, using the following sign convention.

Direct stresses<< tensile positive; compressive, negative

Shear stresses – tending to turn block clockwise, positive

– tending to turn block counter clockwise, negative

[i.e shearing stresses are +ve when its movement about the centre of the element is clockwise]

This gives two points on the graph which may than be labeled as respectively to denote stresses on these planes.

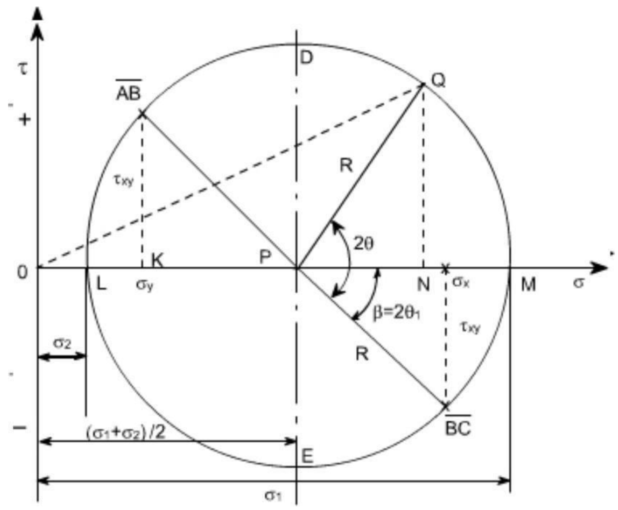
(iv) Join $\overline{AB}$ and $\overline{BC}$.

(v) The point P where this line cuts the s axis is than the centre of Mohr's

stress circle and the line joining $\overline{AB}$ and $\overline{BC}$ is diameter. Therefore the circle can now be drawn.

Now every point on the circle then represents a state of stress on some plane through C.

Proof:



Consider any point Q on the circumference of the circle, such that PQ makes an angle 2<< with BC, and drop a perpendicular from Q to meet the s axis at N.Then OQ represents the resultant stress on the plane an angle < to BC. Here we have assumed that $<_x << <_y$

Now let us find out the coordinates of point Q. These are ON and QN.

From the figure drawn earlier

ON = OP + PN

OP = OK + KP

If we examine the equation (1) and (2), we see that this is the same equation which we have already derived analytically

Thus the co-ordinates of Q are the normal and shear stresses on the plane inclined at < to BC in the original stress system.

N.B: Since angle PQ$\overline{BC}$ is 2< on Mohr's circle and not < it becomes obvious that angles are doubled on Mohr's circle. This is the only difference, however, as They are measured in the same direction and from the same plane in both figures.

Further points to be noted are

(1) The direct stress is maximum when Q is at M and at this point obviously the sheer stress is zero, hence by definition OM is the length representing the maximum principal stresses $<_1$ and $2<_1$ gives the angle of the plane $<_1$ from BC. Similar OL is the other principal stress and is represented by $<_2$

(2) The maximum shear stress is given by the highest point on the circle and is represented by the radius of the circle.

This follows that since shear stresses and complimentary sheer stresses have the same value; therefore the centre of the circle will always lie on the s axis midway between $<_x$ and $<_y$. [since $+<_{xy}$ & $<<_{xy}$ are shear stress & complimentary shear stress so they are same in magnitude but different in sign.]

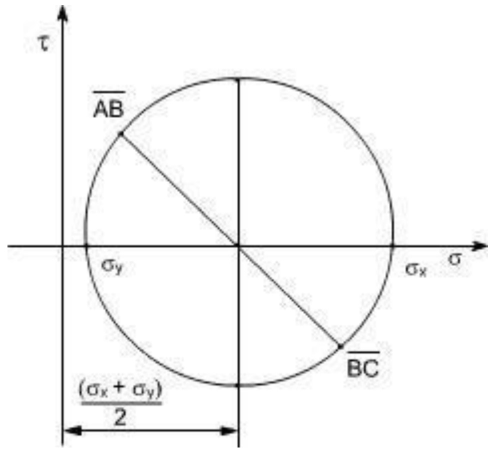
(3) From the above point the maximum sheer stress i.e. the Radius of the Mohr's stress circle would be

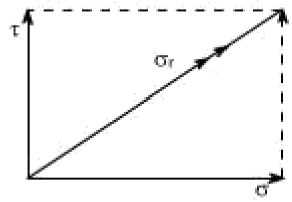
$$\frac{(\sigma_x - \sigma_y)}{2}$$

While the direct stress on the plane of maximum shear must be mid – may between $<_{x}$ and $<_{y}$

i.e

$$\frac{(\sigma_x + \sigma_y)}{2}$$





zero.

(4) As already defined the principal planes are the planes on which the shearcomponents are

Therefore are conclude that on principal plane the sheer stress is zero.(5)Since

the resultant of two stress at 90^0 can be found from the parallogram of vectors as shown in the diagram.Thus, the resultant stress on the plane at q to BC is given by OQ on Mohr's Circle.

(6) The graphical method of solution for a complex stress problems using Mohr's circle is a very powerful technique, since all the information relating to any plane within the stressed element is contained in the single construction. It thus, provides a convenient and rapid means of solution. Which is less prone to arithmetical errors and is highly recommended.

Numericals:

Let us discuss few representative problems dealing with complex state of stress to be solved either analytically or graphically.

Q2:

For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows:

(a) 85 MN/m^2 tensile

(b) 25 MN/m^2 tensile at right angles to (a)

(c) Shear stresses of 60 MN/m^2 on the planes on which the stresses (a) and

(b) act; the sheer couple acting on planes carrying the 25 MN/m^2 stress is clockwise in effect. Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged

Solution:

The problem may be attempted both analytically as well as graphically. Let us first obtain the analytical solution

σ_1 and σ_2

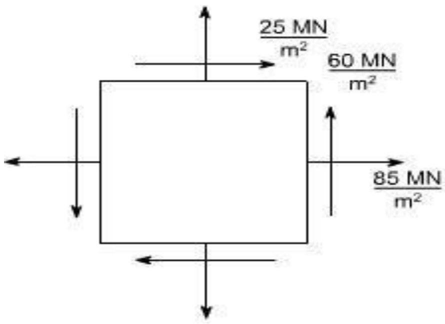
$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$= \frac{1}{2}(85 + 25) \pm \frac{1}{2}\sqrt{(85 + 25)^2 + (4 x 60^2)}$$

$$= 55 \pm \frac{1}{2}.60\sqrt{5} = 55 \pm 67$$

$$\Rightarrow \sigma_1 = 122 \ MN/m^2$$

$$\sigma_2 = -12 \ MN/m^2 (compressive)$$

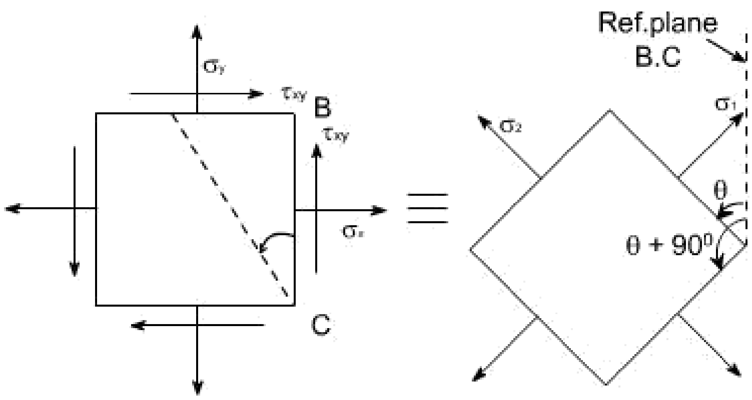


$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

The principle stresses are given by the formula

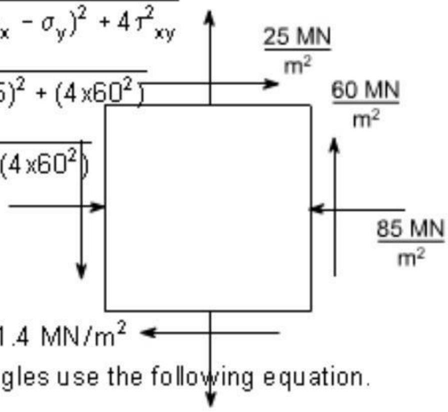
For finding out the planes on which the principle stresses act us the equation

The solution of this equation will yield two values < i.e they < 1 and < 2 giving < 1= 31071' & < 2= 121071'



(b) In this case only the loading (a) is changed i.e. its direction had been changed. While the other stresses remains unchanged hence now the block diagram becomes.

Again the principal stresses would be given by the equation.



$$\sigma_1 \,\&\, \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$= \frac{1}{2}(-85 + 25) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= \frac{1}{2}(-60) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= -30 \pm \frac{1}{2}\sqrt{12100 + 14400}$$

$$= -30 \pm 81.4$$

$$\sigma_1 = 51.4\ \mathrm{MN/m^2};\ \sigma_2 = -111.4\ \mathrm{MN/m^2}$$

Again for finding out the angles use the following equation.

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$= \frac{2 \times 60}{-85 - 25} = +\frac{120}{-110}$$

$$= -\frac{12}{11}$$

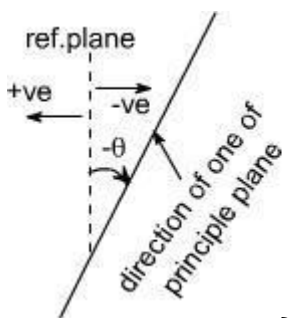
$$2\theta = \tan\left(-\frac{12}{11}\right)$$

$$\Rightarrow \theta = -23.74^0$$

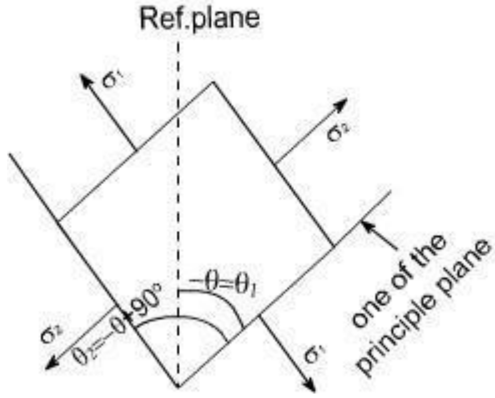
Thus, the two principle stresses acting on the two mutually perpendicular planes i.e principle planes may be depicted on the element as shown below:

So this is the direction of one principle plane & the principle stresses acting on this would be $_0$ < $_1$ when is acting normal to this plane, now the direction of other principal plane would be 90^0 + < because the principal planes are the two mutually perpendicular plane, hence rotate the another plane
< + 90^0 in the same direction to get the another plane, now complete the material element if < is negative that means we are measuring the angles in the opposite direction to the reference plane BC .

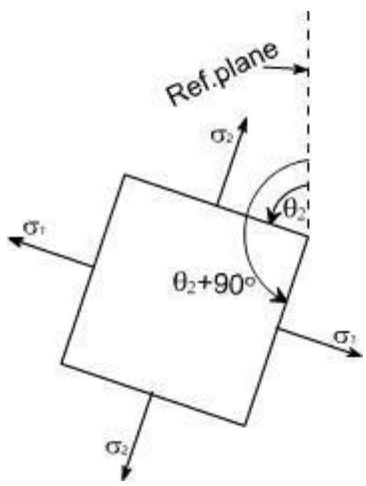
Therefore the direction of other principal planes would be {<< + 90} since the angle << is always less in magnitude then 90 hence the quantity (<<< + 90) would be positive therefore the Inclination of other plane with reference plane would be positive therefore if just complete the Block.



It would appear as



If we just want to measure the angles from the reference plane, than rotate this block through 180^0 so as to have the following appearance.



So whenever one of the angles comes negative to get the positive value, first Add 90^0 to the value and again add 90^0 as in this case < = < $23^0 74'$

so < $_1$ = < $23^0 74'$ + 90^0 = $66^0 26'$ .Again adding 90^0 also gives the direction of other principle planes

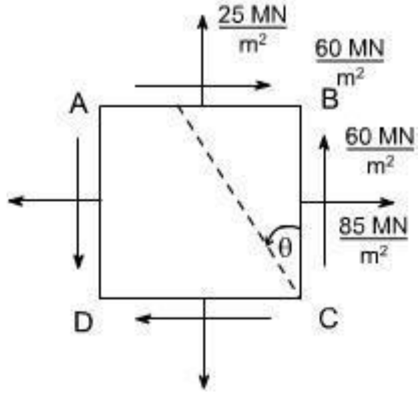
i.e < $_2$ = $66^0 26'$ + 90^0 = $156^0 26'$

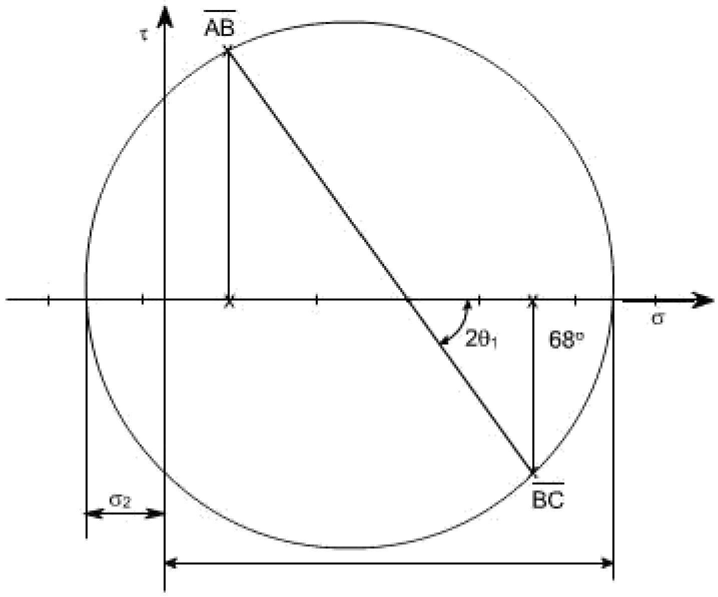
This is how we can show the angular position of these planes clearly.

GRAPHICAL SOLUTION:

Mohr's Circle solution: The same solution can be obtained using the graphical solution i.e the Mohr's stress circle,for the first part, the block diagram bec

Construct the graphical construction as per the steps given earlier.





Taking the measurements from the Mohr's stress circle, the various quantities computed are

$<_1$ = 120 MN/m^2 tensile

$<_2$ = 10 MN/m^2 compressive

$<_1$ = 34^0 counter clockwise from BC

$<_2$ = 34^0+ 90 = 124^0 counter clockwise from BC

Part Second : The required configuration i.e the block diagram for this case is shown along with the stress circle. By taking the measurements, the various quantites computed are given as

$<_1$ = 56.5 MN/m^2 tensile

$<_2$ = 106 MN/m^2 compressive

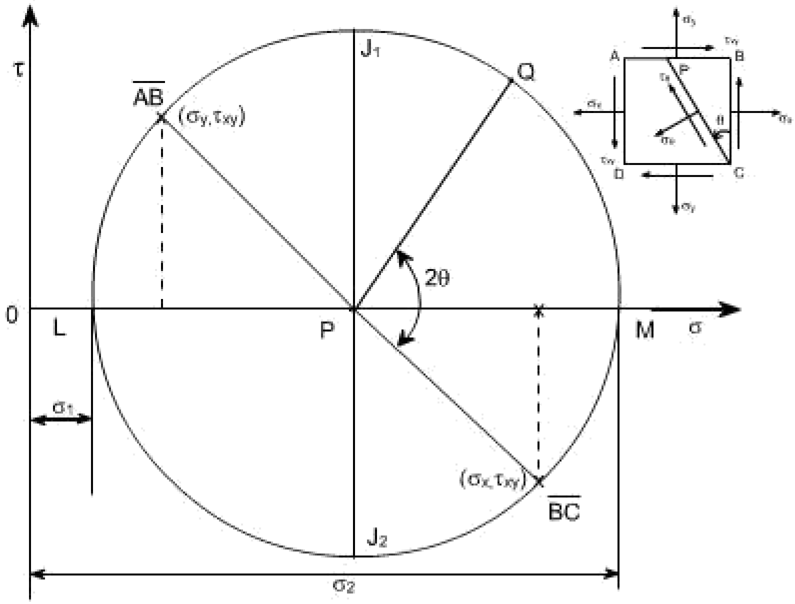
$<_1$ = $66^0$15' counter clockwise from BC

$<_2$ = $156^0$15' counter clockwise from BC

Salient points of Mohr's stress circle:

1. complementary shear stresses (on planes 90^0 apart on the circle) are equal in magnitude
2. The principal planes are orthogonal: points L and M are 180^0 apart on the circle (90^0 apart in material)
3. There are no shear stresses on principal planes: point L and M lie on normal stress axis.
4. The planes of maximum shear are 45^0 from the principal points D and E are 90^0 , measured round the circle from points L and M.
5. The maximum shear stresses are equal in magnitude and given by points D and E
6. The normal stresses on the planes of maximum shear stress are equal i.e. points D and E both have

   normal stress co-ordinate which is equal to the two principal stresses.



know that the circle represents all possible states of normal and shear stress on any plane through a stresses point in a material. Further we have seen that the co-ordinates of the point „Q' are seen to be the same as those derived from equilibrium of the element. i.e. the normal and shear stress components on any plane passing through the point can be found using Mohr's circle. Worthy of note:

1. The sides AB and BC of the element ABCD, which are 90^0 apart, are represented

   on the circle by $\overline{AB}$ P and $\overline{BC}$ P and they are 180^0 apart.

2. It has been shown that Mohr's circle represents all possible states at a point. Thus, it can be seen at a point. Thus, it, can be seen that two planes LP and PM, 180^0 apart on the diagram and therefore 90^0 apart in the material, on which shear stress < < is zero. These planes are termed as principal planes and normal stresses acting on them are known as principal stresses.

   Thus , $<_1$ = OL

   $<_2$ = OM

3. The maximum shear stress in an element is given by the top and bottom points of the

circle i.e by points J_1 and J_2 ,Thus the maximum shear stress would be equal to the radius of i.e. $<_{max}= 1/2(<< _1 <<< _2)$,the corresponding normal stress is obviously the

distance OP = $1/2 (<< _x+ < _y)$, Further it can also be seen that the planes on which the shear stress is maximum are situated 90^0 from the principal planes (on circle), and 45^0 in the material.

4. The minimum normal stress is just as important as the maximum. The algebraic minimum stress could have a magnitude greater than that of the maximum principal stress if the state of stress

were such that the centre of the circle is to the left of

orgin. i.e. if $< _1 = 20$ MN/m^2 (say)

$< _2 = < 80$ MN/m^2 (say)

Then $< _{max}{}^m = (< _1 < < < _2 / 2) = 50$ MN/m^2

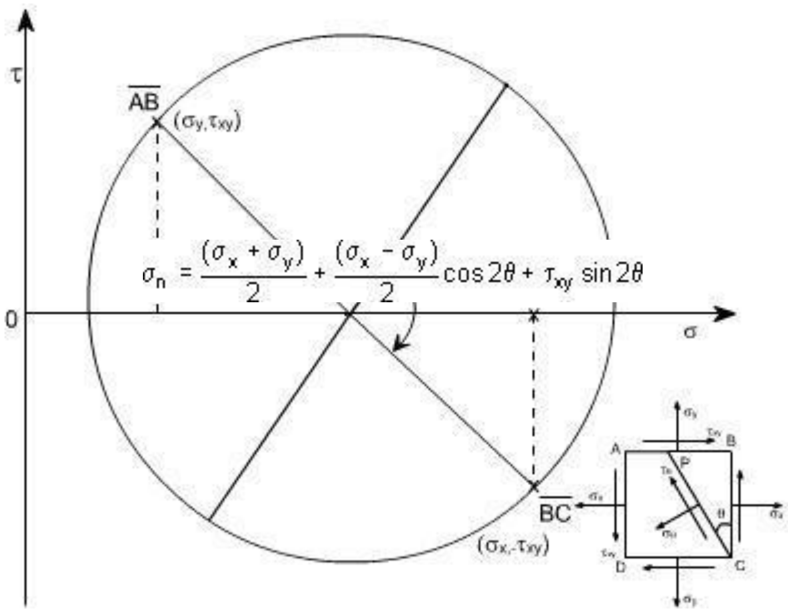
If should be noted that the principal stresses are considered a maximum or minimum mathematically e.g. a compressive or negative stress is less than a positive stress, irrespective or numerical value.

5. Since the stresses on perpendular faces of any element are given by the co- ordinates of two

diametrically opposite points on the circle, thus, the sum of the two normal stresses for any and all orientations of the element is constant, i.e. Thus sum is an invariant for any particular state of stress.

Sum of the two normal stress components acting on mutually perpendicular planes at a point in a state of plane stress is not affected by the orientation of these planes.

This can be also understand from the circle Since AB and BC are diametrically oppositethus,



what ever may be their orientation, they will always lie on the diametre or we can say that their sum won't change, it can also be seen from analytical relations

We know

on plane BC; < = 0

$<_{n1} = <_x$

on plane AB; < = 270^0

< n2 = < y

Thus $<_{n1} + <_{n2} = <_x + <_y$

6. If $<_1 = <_2$, the Mohr's stress circle degenerates into a point and no shearing stresses are developed on xy plane.

7. If $<_x + <_y = 0$, then the center of Mohr's circle coincides with the origin of < < < < < co-ordinates.

SUMMARY

Principal Plane: - It is a plane where shear force is zero is called principal plane.

Principal Stress: - The normal stress on the principal plane is called principal stress.

Obliquity: - It is angle between the resultant stress and normal stress. **Mohr's circle: -** It is a graphical (circle) method to find the stresses and strains on a plane. **Principal Plane and Stresses can be solved by**

1. Analytical Method – Solving horizontal and vertical stresses to find the normal stress and shear stress using trigonometry method.
2. Graphical Method – Mohr‟s circle method

Analytical Method:

The equation is solved assuming **ζx** and **ζy** as tensile stresses as positive and **ηxy** shear stress clockwise as positive to major principal stress. Simply change the sign if stresses are opposite.

Graphical Method - Drawing Rules of Mohr's Circle:

1. Fix the origin (0,0) that is (x,y) at convenient place in the graph.
2. X – axis to locate axial stress for both x and y directions.
3. Y – axis to locate shear stress for clockwise and anti clockwise shear.
4. Tensile stress is positive along x axis right of origin.
5. Compressive stress is negative along x axis left of origin.
6. Clockwise Shear stress is positive along y axis upward of origin.
7. Anti clockwise shear stress is negative along y axis downward of origin..
8. When there is no shear force (η xy= 0) draw Mohr‟s circle from axial stresses. Thecentre of the Mohr‟s circle bisects axial stresses (ζx,0) and (ζy,0).
9. When there is shear force draw Mohr‟s circle from axial stresses and shear stress. The centre of the Mohr‟s circle bisects the line between (ζx, ηxy) and (ζy, ηxy).
10. Angle of inclination is to be drawn from point (ζy, ηxy) at centre of Mohr‟s to angle 2θ in clockwise direction.
11. Normal stress, and maximum and minimum principal stresses are taken from theorigin along the x-axis of the Mohr‟s circle.
12. Maximum shear stress is the radius of the Mohr‟s circle, and shear stresses are taken along the y-axis of the Mohr‟s circle.
13. The angle between the resultant stress and normal stress in angle of oblique.