# UNIT - 1 STRESS, STRAIN DEFORMATION OF SOLIDS

## TYPES OF STRESSES

Though there are many types of stresses, yet the following two types of stresses are important from the subject point of view:

1. Tensile stress. 2. Compressive stress.

## TENSILE STRESS

When a section is subjected to two equal and opposite pulls and the body tends to increase its length, as shown in Fig.1, the stress induced is called tensile stress. The corresponding strain is called tensile strain. As a result of the tensile stress, the *cross-sectional area of the body gets reduced.

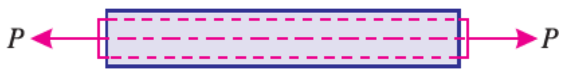



***Fig. 1. Tensile stress***

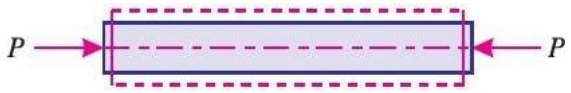



***Fig.2. Compressive stress***

## COMPRESSIVE STRESS

When a section is subjected to two equal and opposite pushes and the body tends to shorten its length, as shown in Fig. 2, the stress induced is called compressive stress. The corresponding strain is called compressive strain. As a result of the compressive stress, the cross-sectional area of the body gets increased.

## ELASTIC LIMIT

We have already discussed that whenever some external system of forces acts on a body, it undergoes some deformation. If the external forces, causing deformation, are removed the body springs back to its original position. It has been found that for a given section there is a limiting value of force up to and within which, the deformation entirely disappears on the removal of force.

The value of intensity of stress (or simply stress) corresponding to this limiting force is called elastic limit of the material. Beyond the elastic limit, the material gets into plastic stage and in this stage the deformation does not entirely disappear, on the removal of the force. But as a result of this, there is a residual deformation even after the removal of the force.

## HOOKE'S LAW

It states, "*When a material is loaded, within its elastic limit, the stress is proportional to the strain.*" Mathematically,

Stress / Strain = $E$ = Constant

It may be noted that Hooke's Law equally holds good for tension as well as compression.

**1.** *A steel rod 1 m long and 20 mm × 20 mm in cross-section is subjected to a tensile force of 40 kN. Determine the elongation of the rod, if modulus of elasticity for the rod material is 200 GPa.*

Given Data: Length ($l$) = 1 m = $1 \times 10^3$ mm ; Cross-sectional area ($A$) = 20 × 20 = 400 $mm^2$ ; Tensile force ($P$) = 40 kN = $40 \times 10^3$ N and modulus of elasticity ($E$) = 200 GPa = $200 \times 10^3$ $N/mm^2$.

$$\delta l = \frac{P.l}{A.E} = \frac{(40\times10^3)\times(1\times10^3)}{400\times(20\times10^3)} = 0.5 \text{ mm}$$

**2.** *A hollow cylinder 2 m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa.*

Given Data: Length ($l$) = 2 m = $2 \times 10^3$ mm ; Outside diameter ($D$) = 50 mm ;
Inside diameter ($d$) = 30 mm ; Load ($P$) = 25 kN = $25 \times 10^3$ N
modulus of elasticity ($E$) = 100 GPa = $100 \times 10^3$ $N/mm^2$.

$$A = \frac{\pi}{4}\times(D^2 - d^2) = \frac{\pi}{4}\times[(50)^2 - (30)^2] = 1257 \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{25\times10^3}{1257} = 19.9 \text{ N/mm}^2 = 19.9 \text{ MPa} \qquad \textbf{Ans.}$$

$$\delta l = \frac{P.l}{A.E} = \frac{(25\times10^3)\times(2\times10^3)}{1257\times(100\times10^3)} = 0.4 \text{ mm} \qquad \textbf{Ans.}$$

## STRESSES IN THE BARS OF DIFFERENT SECTIONS

Sometimes a bar is made up of different lengths having different cross-sectional areas as shown in figure 3. In such cases, the stresses, strains and hence changes in lengths for each section is worked out separately as usual. The total changes in length is equal to the sum of the changes of all the individual lengths. It may be noted that each section is subjected to the same external axial pull or push.

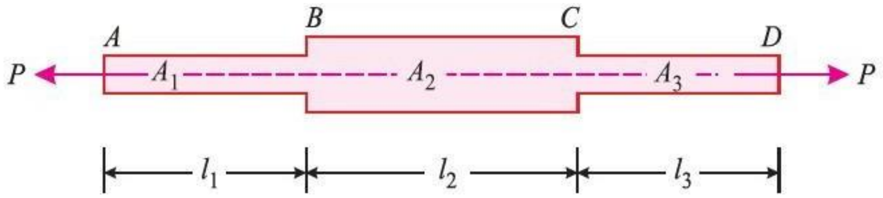



***Fig. 3. Bars of different Sections***

Let $P$ = Force acting on the body,

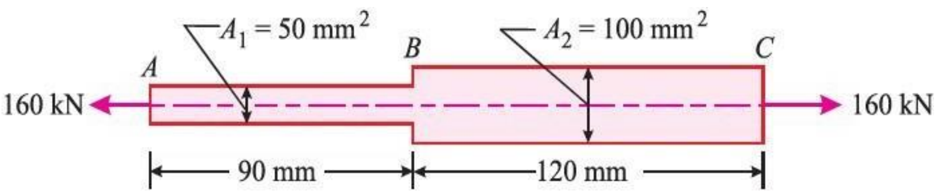
$E$ = Modulus of elasticity for the body,

$l1$ = Length of section 1,

$A1$ = Cross-sectional area of section 1,

$l2$, $A2$ = Corresponding values for section 2 and so on.

**3.** *An automobile component shown in Figure is subjected to a tensile load of 160 kN. Determine the total elongation of the component, if its modules of elasticity is 200 GPa.*

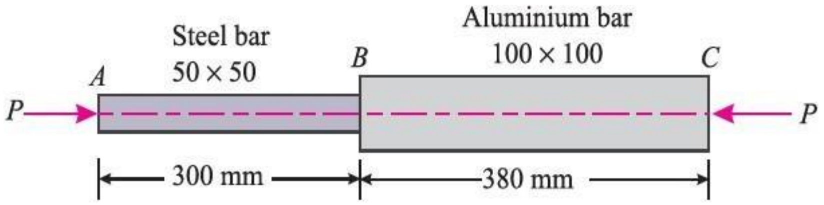



Given Data: Tensile load ($P$) = 160 kN = $160 \times 10^3$ N; Length of section 1 ($l1$) = 90 mm; Length of section 2 ($l2$) = 120 mm; Area of section 1 ($A1$) = 50 mm$^2$; Area of section 2 ($A2$) = 100 mm$^2$ and modulus of elasticity ($E$) = 200 GPa = $200 \times 10^3$ N/mm$^2$.

$$\delta l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2}\right) = \frac{160 \times 10^3}{200 \times 10^3}\left(\frac{90}{50} + \frac{120}{100}\right) \text{mm}$$

$$= 0.8 \times 1.8 + 1.2 = 2.4 \text{ mm} \qquad \textbf{Ans.}$$

***4.** A member formed by connecting a steel bar to an aluminium bar is shown in Figure. Assuming that the bars are prevented from buckling sidewise, calculate the magnitude of force P, that will cause the total length of the member to decrease by 0.25 mm. The values of elastic modulus for steel and aluminium are 210 GPa and 70 GPa respectively.*




Given Data: Decrease in length (€$l$ ) = 0.25 mm ;

Modulus of elasticity for steel ($E_S$) = 210 GPa = 210 × $10^3$ N/mm$^2$;

Modulus of elasticity for aluminium ($EA$) = 70 GPa = 70 × $10^3$ N/mm$^2$;

Area of steel section ($AS$) = 50 × 50 = 2500 mm$^2$;

Area of aluminium section ($A_A$) = 100 × 100 = 10000 mm$^2$;

Length of steel section ($l_S$) = 300 mm and length of aluminium section ($l_A$) = 380 mm.

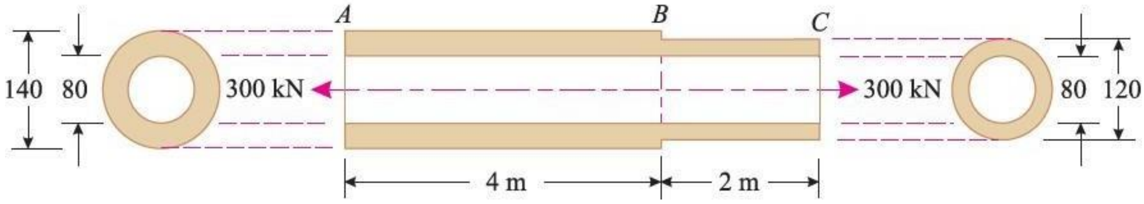
$$0.25 = P\left(\frac{l_S}{A_S E_S} + \frac{l_A}{A_A E_A}\right)$$

$$= P\left(\frac{300}{2500 \times (210 \times 10^3)} + \frac{380}{10000 \times (70 \times 10^3)}\right)$$

$$= \frac{780\,P}{700 \times 10^6}$$

$$P = \frac{0.25 \times (700 \times 10^6)}{780} = 224.4 \times 10^3 \text{ N} = 224.4 \text{ kN} \qquad \textbf{Ans.}$$

***5.** A 6 m long hollow bar of circular section has 140 mm diameter for a length of 4 m, while it has 120 mm diameter for a length of 2 m. The bore diameter is 80 mm throughout as shown in Figure Find the elongation of the bar, when it is subjected to an axial tensile force of 300 kN. Take modulus of elasticity for the bar material as 200 GPa.*



Given Data: Total length ($L$) = 6 m = 6 × $10^3$ mm; Diameter of section 1 ($D_1$) = 140 mm;

Length of section 1 ($l_1$) = 4 m = 4 × $10^3$ mm; Diameter of section 2 ($D_2$) = 120 mm;

Length of section 2 ($l_2$) = 2 m = 2 × $10^3$ mm; Inner diameter ($d_1$) = $d_2$ = 80 mm;

Axial tensile force ($P$) = 300 kN = 300× $10^3$ N and

modulus of elasticity ($E$) = 200 GPa = 200 × $10^3$ N/mm$^2$.

We know that area of portion $AB$,

$$A_1 = \frac{\pi}{4} \times [D_1^2 - d_1^2] = \frac{\pi}{4} \times [(140)^2 - (80)^2] = 3300\,\pi \text{ mm}^2$$

and area of portion $BC$.

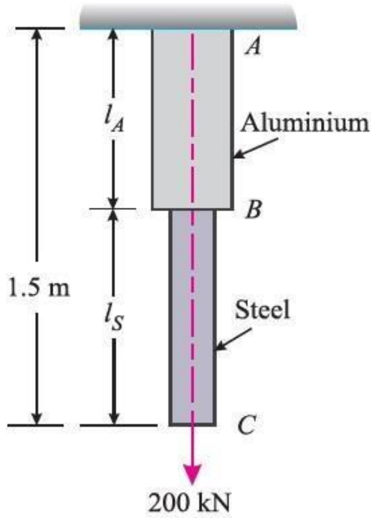
$$A_2 = \frac{\pi}{4} \times [D_2^2 - d_2^2] = \frac{\pi}{4} \times [(120)^2 - (80)^2] = 2000\,\pi \text{ mm}^2$$

∴ Elongation of the bar,

$$\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right] = \frac{300 \times 10^3}{200 \times 10^3} \times \left[\frac{4 \times 10^3}{3300\pi} + \frac{2 \times 10^3}{2000\pi}\right] \text{mm}$$

$$= 1.5 \times (0.385 + 0.318) = 1.054 \text{ mm} \quad \textbf{Ans.}$$

**6.** *A compound bar ABC 1.5 m long is made up of two parts of aluminium and steel and that cross-sectional area of aluminium bar is twice that of the steel bar. The rod is subjected to an axial tensile load of 200 kN. If the elongations of aluminium and steel parts are equal, find the lengths of the two parts of the compound bar. Take E for steel as 200 GPa and E for aluminium as one-third of E for steel.*




Given Data: Total length ($L$) = 1.5 m = 1.5 × $10^3$ mm;

Cross-sectional area of aluminium bar ($A_A$) = 2 $A_S$;

Axial tensile load ($P$) = 200 kN = 200 × $10^3$ N;

Modulus of elasticity of steel ($E_S$) = 200 GPa = 200 × $10^3$ N/mm$^2$ and

Modulus of elasticity of aluminium ($E_A$) = $E_s/3 = 200\ 10^3 / 3$ N/mm$^2$

Let, $l_A$ = Length of the aluminium part,

and $l_S$ = Length of the steel part.

We know that elongation of the aluminium part *AB*,

$$\delta l_A = \frac{P.l_A}{A_A.E_A} = \frac{(200\times10^3)\times l_A}{2A_S\times\left(\frac{200\times10^3}{3}\right)}$$

$$= \frac{1.5\,l_A}{A_S} \qquad ...(i)$$

and elongation of the steel part *BC*,

and elongation of the steel part *BC*,

$$\delta l_S = \frac{P.l_S}{A_S.E_S} = \frac{(200\times10^3)\times l_S}{A_S\times(200\times10^3)} = \frac{l_S}{A_S} \qquad ...(ii)$$

Since elongations of aluminium and steel parts are equal, therefore equating equations (*i*) and (*ii*),

$$\frac{1.5\,l_A}{A_S} = \frac{l_S}{A_S} \quad \text{or} \quad l_S = 1.5\,l_A$$

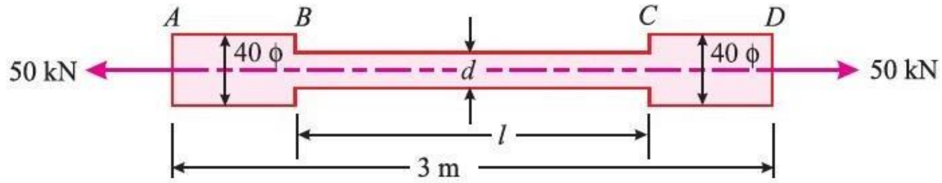
We also know that total length of the bar *ABC* (*L*)

$$1.5\times10^3 = l_A + l_S = l_A + 1.5\,l_A = 2.5\,l_A$$

$$\therefore \qquad l_A = \frac{1.5\times10^3}{2.5} = 600 \text{ mm} \qquad \textbf{Ans.}$$

$$\text{and} \qquad l_S = (1.5\times10^3) - 600 = 900 \text{ mm} \qquad \textbf{Ans.}$$

***7.*** *An alloy circular bar ABCD 3 m long is subjected to a tensile force of 50 kN as shown in Figure. If the stress in the middle portion BC is not to exceed 150 MPa, then what should be its diameter? Also find the length of the middle portion, if the total extension of the bar should not exceed by 3 mm. Take E as 100 GPa.*




Given Data: Total length of circular bar (*L*) = 3m = $3 \times 10^3$ mm = 3000 mm ;

Tensile force (*P*) = 50 kN = $50 \times 10^3$ N;

Maximum stress of portion *BC* ($\sigma_{BC}$) = 150 MPa = 150 N/mm$^2$;

Total extension (€*l*) = 3 mm and modulus of elasticity (*E*) = 100 GPa = $100 \times 10^3$ N/mm$^2$.

*Diameter of the middle portion BC*

Let *d* = Diameter of the middle portion in mm.

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We know that stress in the middle portion $BC$ ($\sigma_{BC}$),

$$150 = \frac{P}{A} = \frac{50 \times 10^3}{\frac{\pi}{4} \times (d)^2} = \frac{63.66 \times 10^3}{d^2}$$

$$\therefore \quad d^2 = \frac{63.66 \times 10^3}{150} = 424.4 \quad \text{or} \quad d = 20.6 \text{ mm} \quad \textbf{Ans.}$$

length of the middle portion $BC$

Let $l_{BC}$ = Length of the middle portion in mm.

We know that area of the end portions $AB$ and $CD$,

$$A_1 = \frac{\pi}{4} \times (40)^2 = 1257 \text{ mm}^2$$

and area of the middle portion $BC$,

$$A_2 = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (20.6)^2 = 333.3 \text{ mm}^2$$

We also know that total extension of bar ($\delta l$),

$$3 = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right] = \frac{50 \times 10^3}{100 \times 10^3} \times \left[\frac{3000 - l}{1257} + \frac{l}{333.3}\right]$$

$$= 0.5\,[2.387 - 0.0008\,l + 0.003\,l] = 0.5\,[2.387 + 0.0022\,l]$$

$$= 1.194 + 0.0011\,l$$

$$\therefore \quad l = \frac{3 - 1.194}{0.0011} = 1.64 \times 10^3 \text{ mm} = 1.64 \text{ m} \quad \textbf{Ans.}$$

NOTE. We have taken total length of the circular bar as $(3000 - l)$ mm.

**8.** *A steel bar ABCD 4 m long is subjected to forces as shown in Figure. Find the elongation of the bar. Take E for the steel as 200 GPa.*

Given Data: Total length of steel bar ($L$) = 4 m = 4 × $10^3$ mm;

Length of first part ($l1$) =

1 m = 1 × $10^3$ mm; Diameter of first part ($d_1$) = 15 mm;

Length of second part ($l_2$) = 2 m = 2 × $10^3$

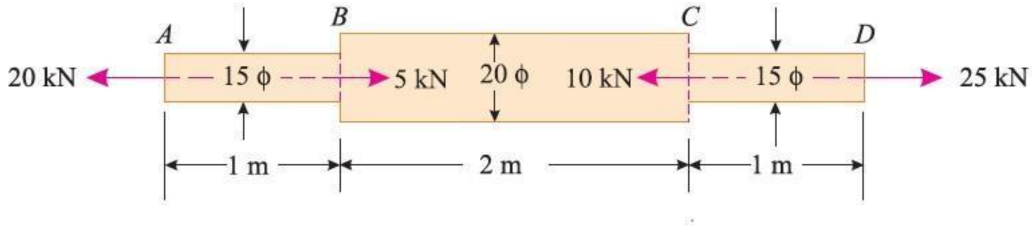
mm; Diameter of second part ($d2$) = 20 mm;

Length of third part ($l_3$) = 1 m = 1 × $10^3$ mm;

Diameter of third part ($d_3$) = 15 mm and

modulus of elasticity ($E$) = 200 GPa = 200 × $10^3$ N/mm$^2$.

We know that area of the first and third parts of the bar,




We know that area of the first and third parts of the bar,

$$A_1 = A_3 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (15)^2 = 177 \text{ mm}^2$$

and area of the middle part of the bar

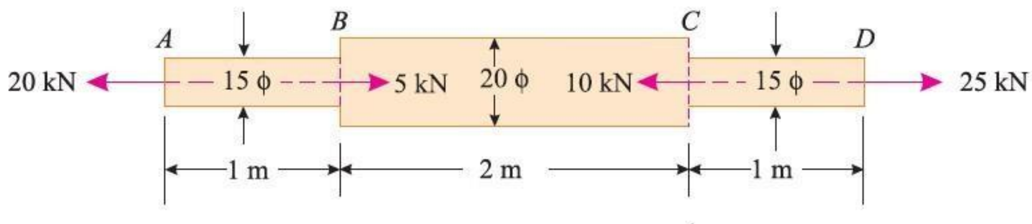
$$A_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (20)^2 = 314 \text{ mm}^2$$

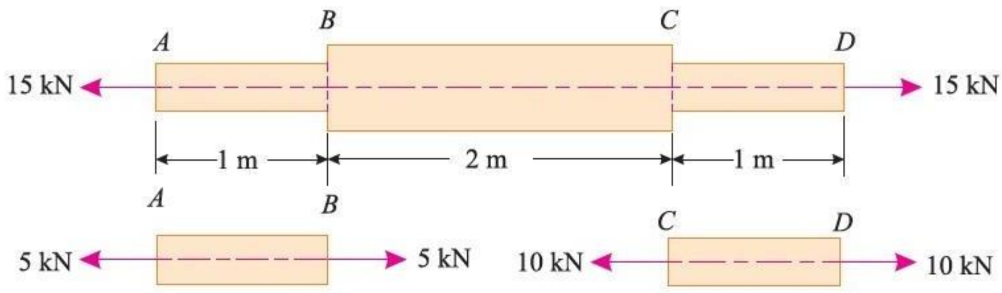
For the sake of simplification, the force of 25 kN acting at *D* may be split up into two forces of 15 kN and 10 kN respectively. Similarly the force of 20 kN acting at *A* may also be split up into two forces of 15 kN and 5 kN respectively.

Now it will be seen that the bar *ABCD* is subjected to a tensile force of 15 kN, part *BC* is subjected to a compressive force of 5 kN and the part *CD* is subjected to a tensile force of 10 kN.

We know that elongation of the bar *ABCD* due to a tensile force of 15 kN,

$$\delta l_1 = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$



$$= \frac{15\times10^3}{200\times10^3}\left[\frac{1\times10^3}{177}+\frac{2\times10^3}{314}+\frac{1\times10^3}{177}\right] \text{ mm} = 1.32 \text{ mm}$$

Similarly elongation of the bar *AB* due to a compression force of 5 kN,

$$\delta l_2 = \frac{P_1 l_1}{A_2 E} = \frac{(5\times10^3)\times(1\times10^3)}{177\times(200\times10^3)} = 0.14 \text{ mm}$$

and elongation of the bar *CD* due to a tensile force of 10 kN,

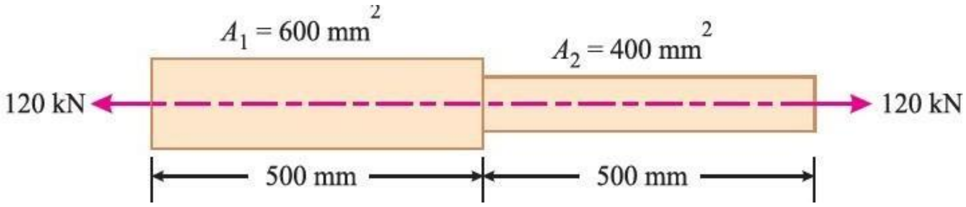
$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{(10\times10^3)\times(1\times10^3)}{177\times(200\times10^3)} = 0.3 \text{ mm}$$

∴ Total elongation of the bar *ABCD*,

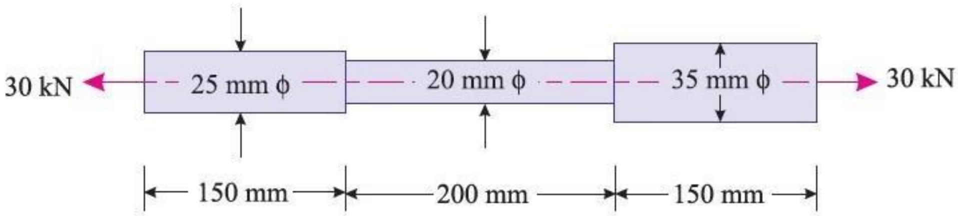
$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 = 1.43 + 0.14 + 0.28 = 1.85 \text{ mm} \qquad \textbf{Ans.}$$

## EXERCISE

1. A steel bar shown in Figure is subjected to a tensile force of 120 kN. Calculate elongation of the bar. Take *E* as 200 GPa. [**Ans.** 1.25 mm]



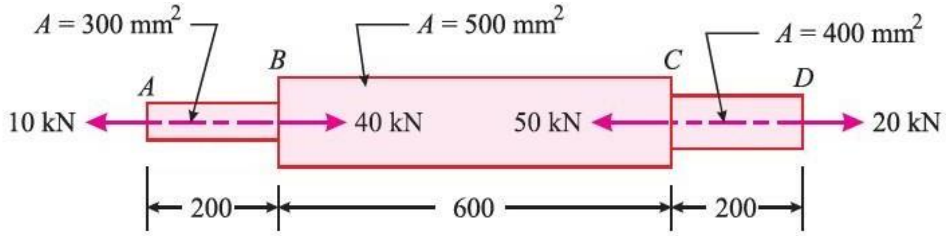

2. A copper bar shown in Figure is subjected to a tensile load of 30 kN. Determine elongation of the bar, if E = 100 GPa. [**Ans.** 0.33 mm]



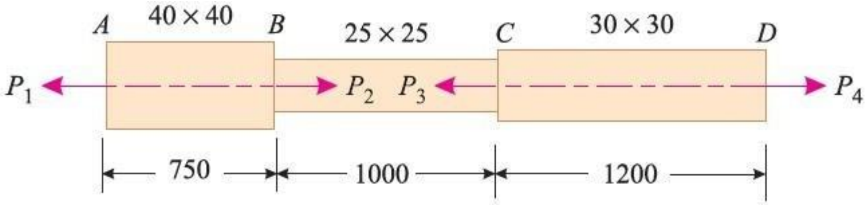

3. A copper bar is 900 mm long and circular in section. It consists of 200 mm long bar of 40 mm diameter, 500 mm long bar of 15 mm diameter and 200 mm long bar of 30 mm diameter. If the bar is subjected to a tensile load of 60 kN, find the total extension of the bar. Take *E* for the bar material as 100 GPa. [**Ans.** 1.963 mm]
4. A stepped bar *ABCD* consists of three parts *AB*, *BC* and *CD* such that *AB* is 300 mm long and 20 mm diameter, BC is 400 mm long and 30 mm diameter and *CD* is 200 mm long and 40 mm diameter. It was observed that the stepped bar undergoes a deformation

of 0.42 mm, when it was subjected to a compressive load $P$. Find the value of $P$, if $E$ = 200 GPa. [**Ans.** 50 kN]

5. A member *ABCD* is subjected to point load as shown in Figure. Determine the total change in length of the member. Take $E$ = 200 GPa. [**Ans.** 0.096 mm (decrease)]




6. A steel bar ABCD is subjected to point loads of P1, P2, P3 and P4 as shown in Figure.




Determine the magnitude of the force P3 necessary for the equilibrium, if P1 = 120 kN, P2 = 220 and P4 = 160 kN. Also determine the net change in the length of the steel bar. Take E = 200 GPa. [**Ans.** 260 kN; 0.55 mm]

**[Hint. AB will be subjected to 120 kN (tension). BC will be subjected to 100 kN (compression) and CD will be subjected to 160 kN (tension).**