Inverse of a Matrix _ Gauss Jordan Method

Let us consider a 3×3 non singular matrix A.
If the matrix x is the inverse of A, then $Ax = I$

(ie) $$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To find the inverse of A, we first consider the augmented matrix $[A, I] = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{31} & 0 & 0 & 1 \end{pmatrix}$

Here our aim is reduce the matrix A in $[A, I]$ into the unit matrix I by means of elementary row transformations, so that, A is reduced to I, the other matrix represents A^{-1}

(ie) $[A, I] \rightarrow [I, A^{-1}]$.

1) Find the inverse of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ using Gauss Jordan method.

Soln:

Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Now $[A, I] = \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0.5 & 1.5 & -1.5 & 1 & 0 \\ 0 & 3.5 & 8.5 & -0.5 & 0 & 1 \end{pmatrix} \begin{matrix} \\ R_2 \leftrightarrow R_2 - \frac{3}{2}R_1 \\ R_3 \leftrightarrow R_3 - \frac{1}{2}R_1 \end{matrix}$$

$$= \begin{pmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0.5 & 1.5 & -1.5 & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} R_3 \leftrightarrow R_3 - \frac{3.5}{0.5} R_2$$

$$= \begin{pmatrix} 2 & 1 & 0 & 6 & -3.5 & 0.5 \\ 0 & 0.5 & 0 & 6 & -4.25 & 0.75 \\ 0 & 0 & -2 & 10 & -7 & +1 \end{pmatrix} \begin{matrix} R_1 \leftrightarrow R_1 + \frac{1}{2} R_3 \\ R_2 \leftrightarrow R_2 + \frac{1.5}{2} R_3 \end{matrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 & -6 & 5 & -1 \\ 0 & 0.5 & 0 & 6 & -4.25 & 0.75 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{pmatrix} R_1 \leftrightarrow R_1 - \frac{1}{0.5} R_2$$

$$= \begin{pmatrix} 1 & 0 & 0 & -3 & 5/2 & -1/2 \\ 0 & 1 & 0 & 6/0.5 & -4.25/0.5 & 0.75/0.5 \\ 0 & 0 & 1 & 10/-2 & -7/-2 & 1/-2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & -3 & 2.5 & -0.5 \\ 0 & 1 & 0 & 12 & -8.5 & 1.5 \\ 0 & 0 & 1 & -5 & 3.5 & -0.5 \end{pmatrix}$$

Hence Inverse of A, $A^{-1} = \begin{pmatrix} -3 & 2.5 & -0.5 \\ 12 & -8.5 & 1.5 \\ -5 & 3.5 & -0.5 \end{pmatrix}$

<u>To check:</u> $AA^{-1} = I$