



DEPARTMENT OF MATHEMATICS

UNIT - III - SOLUTIONS OF EQUATIONS AND EIGEN VALUE PROBLEMS

ITERATIVE METHODS (or) INDIRECT METHODS :

GAUSS JACOBI (or) JACOBI'S METHOD :

Let the system of simultaneous equations be

$$\left. \begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned} \right\} \text{--- (1)}$$

Assume :
 $|a_1| > |b_1| + |c_1|$
 $|b_2| > |a_2| + |c_2|$
 $|c_3| > |a_3| + |b_3|$

(The diagonal elts. should be dominant, so that the iteration process can be applied.)
 This system of equations can also be written as:

$$\left. \begin{aligned} x &= \frac{1}{a_1} (d_1 - b_1y - c_1z) \\ y &= \frac{1}{b_2} (d_2 - a_2x - c_2z) \\ z &= \frac{1}{c_3} (d_3 - a_3x - b_3y) \end{aligned} \right\} \text{--- (2)}$$

Let the first approximation be x_0, y_0 and z_0 .
 Sub. x_0, y_0 , and z_0 in (2), we get,

$$\begin{aligned} x_1 &= \frac{1}{a_1} (d_1 - b_1y_0 - c_1z_0) \\ y_1 &= \frac{1}{b_2} (d_2 - a_2x_0 - c_2z_0) \\ z_1 &= \frac{1}{c_3} (d_3 - a_3x_0 - b_3y_0) \end{aligned}$$

Sub. the values of x_1, y_1, z_1 in (2), we get

$$\begin{aligned} x_2 &= \frac{1}{a_1} (d_1 - b_1y_1 - c_1z_1) \\ y_2 &= \frac{1}{b_2} (d_2 - a_2x_1 - c_2z_1) \\ z_2 &= \frac{1}{c_3} (d_3 - a_3x_1 - b_3y_1) \end{aligned}$$



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This process is repeated till the difference btwn. two consecutive approximations is negligible.

Solve the following system by Gauss-Jacobi method

$$10x - 5y - 2z = 3$$

$$4x - 10y + 3z = -3$$

$$x + 6y + 10z = -3$$

[Form the table]

Soln:

$$a_1: |10| > |-5| + |-2|$$

$$b_2: |-10| > |4| + |3|$$

$$c_3: |10| > |1| + |6|$$

Since the diagonal elements are dominant, the iteration process is applied here.

The gn. system can be written as,

$$x = \frac{1}{10} (3 + 5y + 2z)$$

$$y = \frac{1}{-10} (3 + 4x + 3z)$$

$$z = \frac{1}{10} (-3 - x - 6y)$$

1st iteration: Let the initial values be

Let the initial values be $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{10} [3 + 5(0) + 2(0)] = 0.3$$

$$y_1 = \frac{1}{-10} [3 + 4(0) + 3(0)] = -0.3$$

$$z_1 = \frac{1}{10} [-3 - 0 - 6(0)] = -0.3$$



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ii iteration:

$$x_2 = \frac{1}{10} [3 + 5(0.3) + 2(-0.3)] = 0.39$$

$$y_2 = \frac{1}{10} [3 + 4(0.3) + 3(-0.3)] = 0.33$$

$$z_2 = \frac{1}{10} [-3 - (0.3) - 6(0.3)] = -0.51$$

iii iteration:

$$x_3 = 0.363$$

$$y_3 = 0.303$$

$$z_3 = -0.537$$

iv iteration:

$$x_4 = 0.3441$$

$$y_4 = 0.2841$$

$$z_4 = -0.5181$$

v iteration:

$$x_5 = 0.3384$$

$$y_5 = 0.2822$$

$$z_5 = -0.5048$$

vi iteration:

$$x_6 = 0.3401$$

$$y_6 = 0.2839$$

$$z_6 = -0.5031$$

vii iteration:

$$x_7 = 0.3413$$

$$y_7 = 0.2851$$

$$z_7 = -0.5043$$

viii iteration:

$$x_8 = 0.3416$$

$$y_8 = 0.2852$$

$$z_8 = -0.50519$$

ix iteration:

$$x_9 = 0.3415$$

$$y_9 = 0.28511$$

$$z_9 = -0.5052$$

x iteration:

$$x_{10} = 0.34148$$

$$y_{10} = 0.28504$$

$$z_{10} = -0.50522$$

$$x \approx 0.3415$$

$$y \approx 0.2850$$

$$z = -0.5052$$



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Q. Solve the following equations using Jacobi's iteration method:

$$30x - 2y + 3z = 75$$

$$x + 17y - 2z = 48$$

$$x + y + 9z = 15$$

Soln:

$$a_1 \cdot |30| > |1 \cdot 2 + 1 \cdot 3|$$

$$b_1 \cdot |17| > |1 \cdot 1 + 1 \cdot 2|$$

$$c_1 \cdot |9| > |1 \cdot 1 + 1 \cdot 1|$$

∵ Since the diagonal elements are dominant, the iteration process is applied here.

The gn. system can be written as,

$$x = \frac{1}{30} (75 + 2y - 3z)$$

$$y = \frac{1}{17} (48 - x + 2z)$$

$$z = \frac{1}{9} (15 - x - y)$$

1st iteration:

$$x_1 = \frac{1}{30} (75 + 2y_0 - 3z_0)$$

$$y_1 = \frac{1}{17} (48 - x_0 + 2z_0)$$

$$z_1 = \frac{1}{9} (15 - x_0 - y_0)$$

Let the initial values be $x_0 = y_0 = z_0 = 0$

$$x_1 = 2.5$$

$$y_1 = 2.8235$$

$$z_1 = 1.6666$$



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ii iteration:

$$x_2 = \frac{1}{30} (75 + 2y_1 - 3z_1)$$

$$y_2 = \frac{1}{17} (48 - x_1 + 2z_1)$$

$$z_2 = \frac{1}{9} (15 - x_1 - y_1)$$

$$\Rightarrow x_2 = \frac{1}{30} (75 + 2(2.8235) - 3(1.6666)) = 2.5217$$

$$y_2 = \frac{1}{17} (48 - 2.5 + 2(1.6666)) = 2.8725$$

$$z_2 = \frac{1}{9} (15 - 2.5 - 2.8235) = 1.0757$$

iii iteration:

$$x_3 = 2.5839$$

$$y_3 = 2.8016$$

$$z_3 = 1.0673$$

iv iteration:

$$x_4 = 2.5800$$

$$y_4 = 2.7971$$

$$z_4 = 1.0682$$

v iteration:

$$x_5 = 2.5796$$

$$y_5 = 2.7974$$

$$z_5 = 1.0692$$

vi iteration:

$$x_6 = 2.5795$$

$$y_6 = 2.7975$$

$$z_6 = 1.0692$$

vii iteration:

$$x_7 = 2.5795$$

$$y_7 = 2.7975$$

$$z_7 = 1.0692$$

From vi & vii iterations, approx. we get,

$$x = 2.5795$$

$$y = 2.7975$$

$$z = 1.0692$$