CHI - SQUARE TEST

$$\chi^2 = \frac{\Sigma (O_i^2 - E_i)^2}{E_i}$$

where O_i = Observed frequency

E_i = Experimental frequency or Expected frequency

$$= \frac{\Sigma O_i}{n}$$

Degrees of freedom, $\nu = n-1$

Properties :-

i) The mean of χ^2 distribution is equal to the number of degrees of freedom.

ii) The variance of χ^2 distribution is twice the degrees of freedom.

iii) If χ^2 is a chi-square variate with ν degrees of freedom, then $\chi^2/2$ is a gamma variate with parameter $\nu/2$

iv) Standard χ^2 variate tends to standard normal variate as $n \to \infty$.

Applications :

i) To test if the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$

ii) To test the goodness of fit.

iii) To test the independence of attributes.

iv) To test the homogenity of independant estimates of the population variance.

Degrees of freedom:

No. of values in a set which may be assigned arbitrarily

1. The table below gives the number of aircraft accidents that occured during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tue	Wed	Thurs	Fri	Sat
No. of accidents	14	18	12	11	15	14

Given:- Total number of accidents $= 84$

No of days $= 6$

$\therefore$ Expected freq of the accident ie) $E_i = \frac{84}{6} = 14$

Null Hypothesis : H_0: the accidents are uniformly distributed.

Alternative Hypothesis : H_1: the accidents are not uniformly distributed.

Level of Significance:- LOS is fixed at 5%.

O_i	E_i	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
14	14	0	0
18	14	16	1.14
12	14	4	0.285
11	14	9	0.642
15	14	1	0.071
14	14	0	0

$$\Sigma \frac{(O_i - E_i)^2}{E_i} = 2.14285$$

Test statistic! $\chi^2 = \frac{\Sigma(O_i - E_i)^2}{E_i} = 2.1428$

Degrees of freedom :- $\nu = n-1$
$= 6 - 1 = 5$

$$\chi^2_\alpha = 11.04$$

Conclusion :- $\chi^2_{cal} = 2.1428 < 11.07 = \chi^2_\alpha$

$\therefore$ Ho is accepted at 5% LOS

ie) the accident are uniformly distributed.

2. A dice was thrown 498 times. Denoting x to be the number appearing on the top face of it. The observed frequency of x is given below.

x!	1	2	3	4	5	6
f:	69	78	85	82	86	98

What option you could form for the accuracy of the die?

Given : Expected frequency, $E_i = \frac{\text{Total Frequency}}{6}$

$$= \frac{498}{6} = 83$$

$O_i x$	O_i	$(O_i - E_i)^2$	E_i	$(O_i - E_i)^2 / E_i$
1	69	196	83	2.3614
2	78	25	83	0.3012
3	85	4	83	0.0481
4	82	1	83	0.0120
5	86	9	83	0.1084
6	98	225	83	2.7108

$$\frac{\Sigma (O_i - E_i)^2}{E_i} = 5.5419$$

Null Hypothesis :-

H_0 : A die is unbiased

Alternative Hypothesis :-

H_1 : A die is not unbiased ie) biased

Level of significance:

Level of significance is fixed at 5%. $\alpha = 5$

Test statistic:

$$\chi^2 = \frac{\Sigma (O_i - E_i)^2}{E_i} = 5.542$$

Critical value: Degrees of freedom, $\nu = n - 1$
$= 6 - 1 = 5$

$$\chi^2_\alpha = 11.07$$

Conclusion :-

$$\chi^2_{cal} = 5.542 < 11.07 = \chi^2_\alpha$$

$\therefore$ H_0 is accepted at 5% LOS ie) A die is unbiased.

Chi Square test for Independence of Attributes.

$$\chi^2 = \Sigma \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

where $O_i \to$ observed frequency

$E_i \to$ Expected frequency

$$E_i = \frac{(\text{Row total } B_i)(\text{Column total } A_j)}{\text{Whole total}}$$ $i = 1$ to s, $j = 1$ to t

Degrees of freedom $\nu = (s-1 * t-1)$

1) On the basis of information noted below, find out whether the new ~~techniques~~ treatment is comparatively superior to the conventional one.

	Favourable	Notfavourable	Total
New	60	30	90
Conventional	40	70	110
Total	100	100	200.

Soln!

To find E_i

$\frac{90 \times 100}{200} = 45$ $\frac{90 \times 100}{200} = 45$

$\frac{110 \times 100}{200} = 55$ $\frac{110 \times 100}{200} = 55$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2/E_i$
60	45	15	5
30	45	-15	5
40	55	-15	4.09
70	55	15	4.09

$$\Sigma \frac{(O_i - E_i)^2}{E_i} = 18.18$$

Null Hypothesis:-

H_0: There is no difference between new & conventional treatment

Alternative Hypothesis:

H_1: There is a difference between new & conventional treatment

Level of significance:-

LOS is fixed at 5%. i.e) $\alpha = 5\%$

Test statistics:

$$\chi^2 = \frac{\Sigma (O_i - E_i)^2}{E_i} = 18.18$$

Degrees of freedom:-

$$\nu = ((s-1)*(t-1)) = (2-1 * 2-1)$$
$$= 1*1$$
$$= 1$$

$$\chi^2_{tab} = 3.841$$

Conclusion:

$$\chi^2 = 18.18 > 3.841 = \chi^2_\alpha$$

H_0 is rejected at 5% LOS.

i.e) there is difference between new & conventional treatment.