Test of significance of small samples:

Student's t-test

Test for single mean:

Null Hypothesis : $H_0 : \mu = \mu_0$

Test statistic : $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$, if SD is given.

$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$, if SD is not given.

To find s: $s^2 = \dfrac{\Sigma(x-\bar{x})^2}{n-1}$

Degrees of freedom : $\nu = n-1$

Note: Confidence limit : $\bar{x} \pm t_\alpha \dfrac{s}{\sqrt{n-1}}$

1) A random sample of 10 boys had the following IQ's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ's of 100? Find a reasonable range in which most of the mean IQ's value of sample 10 boys.

Given:- $n = 10$, $\mu = 100$.

$$\bar{x} = \frac{70+120+110+101+88+83+95+98+107+100}{10}$$

$= 97.2$.

To find s: $s^2 = \dfrac{\Sigma(x-\bar{x})^2}{n-1}$

x :	70	120	110	101	88	83	95	98	107	100
$x-\bar{x}$:	-27.2	22.8	12.8	3.8	-9.2	-14.2	-2.2	0.8	9.8	2.8
$(x-\bar{x})^2$	739.84	519.84	163.84	14.44	84.64	201.64	4.84	0.64	96.04	7.84

$$\Sigma(x-\bar{x})^2 = 1833.6$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1833.6}{10-1} = 203.73$$

$$\Rightarrow s = 14.27$$

Null Hypothesis :- $H_0 : \mu = 100$

Alternative Hypothesis : $H_1 : \mu \neq 100$. [two tailed test]

Level of Significance :-

Level of significance is fixed at 5%. $\alpha = 0.05$

Test statistic :- $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{97.2-100}{14.27/\sqrt{10}} = \dfrac{-2.8}{4.5125}$

$$|t| = 0.62$$

Critical value :-

t_{tab} for degree of freedom, $\nu = n-1$
$= 10-1 = 9$

$$t_{tab} = 2.262\ (t_\alpha)$$

Conclusion :- $t = 0.62 < 2.262 = t_\alpha$

$\therefore$ H_0 is accepted at 5% LOS

i.e) the population mean IQ's is 100.

Confidence limit :-

$$\mu = \bar{x} \pm t_\alpha \frac{s}{\sqrt{n-1}} = 97.2 \pm (2.262) \times \frac{14.27}{\sqrt{10-1}}$$

$$= 97.2 \pm 10.759$$

$$= 107.95, 86.45$$

2. The weight of 10 peoples of a locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 kg. it is resonable to believe that the average weight of people locality greater than 64kg. Test at 5% LOS.

Given :- $n = 10$, $\mu = 64$

$$\bar{x} = \frac{70 + 67 + 62 + 68 + 61 + 68 + 70 + 64 + 64 + 66}{10}$$

$$\bar{x} = 66.$$

To find S :- $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

x:	70	67	62	68	61	68	70	64	64	66
$x-\bar{x}$:	4	1	-4	2	-5	2	4	-2	-2	0
$(x-\bar{x})^2$:										

$$\Sigma(x-\bar{x})^2 = 90, \quad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{90}{10-1} = 10.$$

Null Hypothesis : $H_0 : \mu = 64$

Alternative Hypothesis :- $H_1 = \mu > 64$ (one tail right)

Level of Significance :- Level of significance is fixed at 5%

ie $\alpha = 5$

Test statistic:- $t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{66-64}{3.16/\sqrt{10}}$

$= 2.02$

t_{tab} for degree of freedom $\nu = n-1$
$= 10-1 = 9$

$t_{tab} = t_{\alpha} = 1.833$ (at twotailed at 10%)

Conclusion: $t_{tab} = 2.02 > 1.833 = t_{tab}$

Ho is rejected at 5% LOS

i.e) the avg weight of people locality is greater than 64kg.

Test for difference of mean:-

Null Hypothesis : $H_0: \mu_1 = \mu_2$

Test statistic : $t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

Where $S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}$ (or) $s^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$

Degrees of freedom ; $\nu = n_1+n_2-2$

1. In a test examination given to two groups of students the marks obtained were as follows.

Group I : 18 20 36 50 49 36 34 49 41

Group II : 29 28 26 35 30 44 46.

Examine whether the significance of difference between the average marks secured by the students of the above two groups.

Given:- Group I : $n_1 = 9$

Group II : $n_2 = 7$

$$\bar{x}_1 = \frac{18+20+36+50+49+36+34+49+41}{9} = 37$$

$$\bar{x}_2 = \frac{29+28+26+35+30+44+46}{7} = 34$$

x_1	$x_1-\bar{x}_1$	$(x_1-\bar{x}_1)^2$	x_2	$x_2-\bar{x}_2$	$(x_2-\bar{x}_2)^2$
18	-19	361	29	-5	25
20	-17	289	28	-6	36
36	-1	1	26	-8	64
50	13	169	35	1	1
49	12	144	30	-4	16
36	-1	1	44	10	100
34	-3	9	46	12	144
49	12	144			
41	4	16			

$\Sigma(x_1-\bar{x}_1)^2 = 1134$ $\Sigma(x_2-\bar{x}_2)^2 = 386$

$$s^2 = \frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}$$

$$= \frac{1134 + 386}{9+7-2} = 108.57$$

$$S = 10.42$$

Null Hypothesis :- $H_0 : \mu_1 = \mu_2$

Alternative Hypothesis : $H_1 : \mu_1 \neq \mu_2$

Level of Significance : Level of significance is fixed at 5%

ie) $\alpha = 5\%$

Test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{37-34}{10.42\sqrt{\frac{1}{9}+\frac{1}{7}}} = 0.5713$$

Critical value :

t_{tab} for degrees of freedom, $\nu = n_1+n_2-2$

$= 9+7-2$

$= 14$

ie) $t_{tab}(t_\alpha) = 2.145$

Conclusion :

$t_{cal} < t_{tab}$ ie) $0.5713 < 2.145$

$\therefore$ Null Hypothesis is accepted

$\therefore$ There is no significant difference in the average marks of the two groups of students

d) A sample of two types of electric bulbs were tested for length of life and the following data were obtained

Samples	size	mean	SD
I	8	1134	35
II	7	1024	40

. Test at 5%

Given:- $n_1 = 8$, $\bar{x}_1 = 1134$, $s_1 = 35$

$n_2 = 7$, $\bar{x}_2 = 1024$, $s_2 = 40$

Null Hypothesis: $H_0: \mu_1 = \mu_2$

Alternative Hypothesis: $H_1: \mu_1 \neq \mu_2$ (two tailed)

Level of significance: Level of significance is fixed at 5% $\alpha = 5\%$

Test statistic: $t = \dfrac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$s = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{8(35)^2 + 7(40)^2}{8+7-2}$$

$$= 1615.38$$

$$S = 40.19$$

$$t = \frac{1134 - 1024}{40.19\sqrt{\frac{1}{8} + \frac{1}{7}}} = \frac{110}{20.8} = 5.288$$

Critical value:- t_{tab} for degrees of freedom, $\nu = n_1 + n_2 - 2$
$= 8 + 7 - 2$
$= 13$

$$t_{tab} = t_\alpha = 2.160.$$

Conclusion:- $t_{cal} = 5.288 > 2.160 = t_{tab}$

$\therefore$ Ho is rejected at 5% LOS.