F- Test for equality of variance (or) Variance Ratio Test

Null Hypothesis :- $H_0 : \sigma_1^2 = \sigma_2^2$

Test statistic : $F = \frac{S_1^2}{S_2^2}$ where $S_1^2 > S_2^2$

where $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ (or) $S_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1}$

$S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$ (or) $S_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}$

Degrees of freedom : (ν_1, ν_2) where $\nu_1 = (n_1 - 1)$ & $\nu_2 = n_2 - 1$

Note 1: F Greater than one always

Note 2: Suppose S_2^2 greater than S_1^2 then $F = \frac{S_2^2}{S_1^2}$

with degrees of freedom $\nu_1 = n_2 - 1$, $\nu_2 = n_1 - 1$.

Application :

F Test is used to test the two samples have come from the same population.

1. Two random sample of 11 and 9 items show that the sample standard deviation of their weight as 0.8 & 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the true variances are equal, against the alternative hypothesis that they are not.

Given : $n_1 = 11$, $s_1 = 0.8$

$n_2 = 9$, $s_2 = 0.5$

$$s_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{11(0.8)^2}{11-1} = 0.704$$

$$s_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{9(0.5)^2}{9-1} = 0.2812$$

$$s_1^2 > s_2^2$$

Null Hypothesis:- $H_0 : \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis:- $H_1 : \sigma_1^2 \neq \sigma_2^2$

Level of significance: Level of significance is fixed at 5%

Test statistic: $F = \frac{s_1^2}{s_2^2} = \frac{0.704}{0.2814} = 2.5$

Critical value: Degrees of freedom $(\nu_1, \nu_2) = (n_1 - 1, n_2 - 1)$

$= (10, 8)$

Critical value, $F_{tab} = F_\alpha = 3.35$

Conclusion:-

$F_{cal} = 2.5 < 3.5 = F_\alpha$

$\therefore H_0$ is accepted at $\alpha = 5\%$

2. Two random samples gave the following results

Sample	size	sample mean	sum of squares of deviations from the means.
1	12	14	108
2	10	15	90

Test whether the samples came from the same population.

Given:-

$n_1 = 12$, $\bar{x}_1 = 14$, $\Sigma(x_1 - \bar{x}_1)^2 = 108$

$n_2 = 10$, $\bar{x}_2 = 15$, $\Sigma(x_2 - \bar{x}_2)^2 = 90$

$$s_1^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2}{n_1 - 1} = \frac{108}{12-1} = 9.818$$

$$s_2^2 = \frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1} = \frac{90}{10-1} = 10.$$

$$s_1^2 < s_2^2$$

Null Hypothesis: $H_0: \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis: $H_1: \sigma_1^2 \neq \sigma_2^2$

Level of significance: Level of significance is fixed at 5% ie) $\alpha = 5\%$.

Test statistic: $F = \frac{s_2^2}{s_1^2} = \frac{10}{9.818}$

$$F = 1.018$$

Critical value:-

Degrees of freedom $(\nu_1, \nu_2) = (n_2 - 1, n_1 - 1)$

$= (10-1, 12-1) = (9, 11)$

Critical value, $F_\alpha = 2.90$.

Conclusion:

$F_{cal} = 1.018 < 2.90 = F_\alpha$

$\therefore$ H_0 is accepted at 5% Level of significance.

3) Test whether the population variances are identical

Sample I: 10 11 16 12 10 11 12 16

Sample II: 7 9 3 7 9 3 15 at 1% LOS

Given: $n_1 = 8$, $n_2 = 7$ $\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{98}{8} = 12.25$ $\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{53}{7} = 7.57$

x_1	$(x_1-\bar{x}_1)^2$	x_2	$(x_2-\bar{x}_2)^2$
10	5.0625	7	0.3249
11	1.5625	9	2.0449
16	14.0625	3	20.8849
12	0.0625	7	0.3249
10	5.0625	9	2.0449
11	1.5625	3	20.8849
12	0.0625	15	55.2049
16	14.0625		
98	$\Sigma(x_1-\bar{x}_1)^2 = 41.5$	53	$\Sigma(x_2-\bar{x}_2)^2 = 101.7143$

$$s_1^2 = \frac{\Sigma(x_1-\bar{x}_1)^2}{n_1 - 1} = \frac{41.5}{7} = 5.9285$$

$$s_2^2 = \frac{\Sigma(x_1-\bar{x}_2)^2}{6} = \frac{101.7143}{6} = 16.9523$$

$s_1^2 < s_2^2$

Null Hypothesis:- $H_0: \sigma_1^2 = \sigma_2^2$

Alternative Hypothesis: $H_1: \sigma_1^2 \neq \sigma_2^2$

Level of significance: Level of significance is fixed at 1% ie $\alpha = 1\%$

Test Statistic :- $F = \frac{s_2^2}{s_1^2} = \frac{16.9524}{5.9286} = 2.86$

Critical Value :- Degrees of freedom $= (\nu_1, \nu_2)$

$= (n_2-1, n_1-1)$

$= (6, 7)$

$F_\alpha = 7.19$

Conclusion: $F = 2.86 < 7.19 = F_\alpha$

$\therefore H_0$ is accepted at 1% level of significance.

CHI - SQUARE TEST

$\chi^2 = \frac{\Sigma (O_i^2 - E_i)^2}{E_i}$ where O_i = Observed frequency

E_i = Experimental frequency or Expected frequency

$= \frac{\Sigma O_i}{n}$

Degrees of freedom, $\nu = n-1$

Properties :-

i) The mean of χ^2 distribution is equal to the number of degrees of freedom.

ii) The variance of χ^2 distribution is twice the degrees of freedom.

iii) If χ^2 is a chi-square variate with ν degrees of freedom, then $\chi^2/2$ is a gamma variate with parameter $\nu/2$

iv) Standard χ^2 variate tends to standard normal variate as $n \to \infty$.