Test of significance for Difference of Means.

Null Hypothesis $H_0: \mu_1 = \mu_2$.

Test statistic:

i) If $\sigma_1 = \sigma_2 = \sigma \Rightarrow$ $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

ii) If $\sigma_1 \neq \sigma_2 \Rightarrow z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

1. The means of two large samples of size 2000 and 1000 are 68.0 and 67.5 gm respectively. Can the sample be regarded as drawn from the sample population of Standard deviation 2.25 gm? Test at 5% Level of Significance

Given:-
$n_1 = 2000$, $n_2 = 1000$; $\bar{x}_1 = 68$, $\bar{x}_2 = 67.5$; $\sigma = 2.25$

Null Hypothesis :-
Two samples have been drawn from the same population of S.D = 2.25 ie $H_0: \mu_1 = \mu_2$; $\sigma = 2.25$

Alternative Hypothesis :-
$H_1: \mu_1 \neq \mu_2$. (Two tailed Test)

Level of Significance :- Level of significance $\alpha = 5\% = 0.05$

Test Statistic :-

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{68.0 - 67.5}{2.25\sqrt{\frac{1}{2000} + \frac{1}{1000}}} = \frac{0.5 \times 10\sqrt{10}}{2.25\sqrt{1.5}}$$

$$= 5.7377$$

Critical value:

Critical value at 5% level of significance is $Z_\alpha = 1.96$.

Conclusion:-

Since $|Z| = 5.7377 > 1.96 = Z_\alpha$

Null Hypothesis $H_0 : \mu_1 = \mu_2$ is not drawn from the same population of $SD = 2.25$

$\therefore$ Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$ is accepted

2. A simple sample of height of 6400 ~~sailors~~ soldiers has a mean of 67.85 inches and S.D of 2.56 inches while a simple sample of heights of 1600 sailors has a mean of 68.55 inches and S.D of 2.52 inches. Do the data indicate that the sailors are on the average taller than soldiers?

Given : $n_1 = 6400$, $\bar{x}_1 = 67.85$, $s_1 = 2.56$

$n_2 = 1600$, $\bar{x}_2 = 68.55$, $s_2 = 2.52$

Null Hypothesis:-

The sailors are on the average taller than soldiers

ie) $H_0 : \mu_1 = \mu_2$

Alternative Hypothesis:-

$H_1 : \mu_1 < \mu_2$.

Level of significance :-

Level of significance is fixed at $\alpha = 5\%$.

Test statistic :-

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{67.85 - 68.55}{\sqrt{\frac{(2.56)^2}{6400} + \frac{(2.52)^2}{1600}}}$$

$$= \frac{-0.7}{4.993 \times 10^{-3}} = \frac{-0.7}{0.070}$$

$$= -9.9$$

$$|z| = 9.9$$

Critical Value :-

Critical value at 5% level of significance is $z_\alpha = 1.645$

Conclusion :-

Since $|z| = 9.9 > 1.645 = z_\alpha$

Null Hypothesis $H_0 : \mu_1 = \mu_2$ is rejected and

Alternative Hypothesis $H_1 : \mu_1 < \mu_2$ is accepted.

3. The average hourly wages of a sample of 150 workers in plant A was ₹ 2.56 with S.D 1.08. The average wages of a sample of 200 workers in plant B was ₹ 2.87 with S.D 1.28 an average applicant. Assume that hourly wages paid by plant B of their then this paid by their plant A.

$n_1 = 150$, $s_1 = 1.08$, $\bar{x}_1 = 2.56$

$n_2 = 200$, $s_2 = 1.28$, $\bar{x}_2 = 2.87$.

Null Hypothesis :- $H_0 : \mu_0 = \mu_1$

Alternate Hypothesis :- $H_1 : \mu_0 < \mu_2$ [left tailed alternative hypothesis]

Level of Significance : Level of significance is fixed at $\alpha = 5\% \; (-1.645)$

Test Statistic: $$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{2.56 - 2.87}{\sqrt{\frac{(1.08)^2}{150} + \frac{(1.28)^2}{200}}} = -2.45$$

$$|z| = 2.45$$

Critical value :-

Critical value at 5% level of significance $Z_\alpha = -1.645$

Conclusion :-

since $|z| = 2.45 > -1.645 = Z_\alpha$

$\therefore$ H_0 is rejected at 5% level of significance.