Test of significance for Single Proportion:

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

1. A manufacturer claimed that atleast 95% of the equipments which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance.

Given: Sample size $n = 200$,

Number of pieces conforming to specification,

$$= 200 - 18 = 182$$

p = proportion of pieces confirming to specifications

$= \frac{182}{200} = 0.91$

P = Population Proportion $= \frac{95}{100} = 0.95$

$Q = 1 - P = 1 - 0.95 = 0.05$

Null hypothesis H_0:

The proportion of pieces confirming to specifications i.e) $P = 95\%$

Alternative Hypothesis H_1: $P < 0.95$ (Left tail test)

Level of significance:- LOS is fixed at 5%.

Test Statistic :

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{200}}} = \frac{-0.04}{0.0154}$$

$$= -2.59$$

Critical value:-

Since alternative hypothesis is left tailed, the tabulated value of z at 5% level of significance is 1.645.

Conclusion:-

Since calculated value $|z| = 2.6$ is greater than 1.645. we reject the null hypothesis H_0 at 5% level of significance. Hence the manufacturer claim is rejected.

In a sample of 1000 people in Karnataka 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance?

Given:- $n = 1000$.

p = sample proportion of rice eaters

$= \frac{540}{1000} = 0.54$

P = population proportion of rice eaters

$= 1/2 = 0.5$

$Q = 1 - P = 0.5$

Null Hypothesis:-

Both rice and wheat are equally popular in the state.

Alternative Hypothesis: $P \neq 0.5$ (two-tailed test)

Level of Significance:-

Level of significance is fixed at 1%.

Test statistic:-

$$Z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1000}}} = 2.532.$$

The calculated value of $Z = 2.532$.

Critical value :-

Critical value at 1% level of significance is 2.58

Conclusion:-

Since calculated value $2.532 = |Z| < |Z_\alpha| = 2.58$
table value

we accept the Null Hypothesis Ho.

ie) Both rice and wheat are equally popular in the state.

In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in the city are smokers?

Given:- $n=600$ Number of smokers $=325$

$p =$ sample proportion of smokers

$= \frac{325}{600} = 0.5417$

$P =$ Population proportion of smokers in the city

$= \frac{1}{2} = 0.5$

$Q = 1-P = 0.5$

Null Hypothesis Ho: The number of smokers and non-smokers are equal in the city.

Alternative Hypothesis H_1: $P > 0.5$ (Right tailed)

Level of significance :- Level of significance is fixed at 5%

Test Statistic :-

$$Z = \frac{p - P}{\sqrt{PQ/n}}$$

$$= \frac{0.5417 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{600}}} = 2.04$$

Calculated value = 2.04

Critical value :-

Critical value at 5% level of significance ~~1.645~~ for right tail is 1.645.

Conclusion :-

As the Calculated value > table value ie 2.04 > 1.645. we reject the null hypothesis

∴ The majority of men in the city are smokers.

Difference of Proportions :-

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \text{ and } q = 1 - p$$

1. Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that the proportions of men and women in favour of the proposal are same, at 5% level.

Given sample size $n_1 = 400$, $n_2 = 600$.

proportion of men $= p_1 = \frac{200}{400} = \frac{1}{2} = 0.5$

proportion of women $= p_2 = \frac{325}{600} = 0.541$.

Null Hypothesis Ho :

assume that there is no significant difference between the option of men and women are same.

$H_0 : p_1 = p_2 = p$

Alternative Hypothesis :- $H_1 : p_1 \neq p_2$. [two tailed]

Level of Significance :-

The level of significance is fixed at 5%.

Test Statistic :-

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \Rightarrow p = \frac{n_1p_1+n_2p_2}{n_1+n_2}$$

$$= \frac{400(0.5)+600(0.541)}{400+600} = \frac{525}{1000} = 0.525$$

$$q = 1-p = 1-0.525 = 0.475$$

$$\therefore Z = \frac{0.5-0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400}+\frac{1}{600}\right)}} = \frac{-0.041}{0.032} = -1.34$$

$$|Z| = 1.34$$

Critical value :-

Critical value at 5% level of significance is 1.96

Conclusion :-

Since the ~~table~~ calculated value is greater than table value Null Hypothesis is accepted.

$\therefore$ Proportions of the men and women and in favour of the proposal are same.

Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion state whether there is a significant decrease in the consumption of tea after the increase in excise duty.

Given:- $n_1 = 1000$, $n_2 = 1200$

$p_1 = \frac{800}{1000} = 0.8$, $p_2 = \frac{800}{1200} = 0.667$

Now,

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{1000(0.8) + 1200(0.667)}{1000 + 1200} = \frac{1600}{2200} = 0.727$$

$$q = 1 - 0.727 = 0.272$$

Null hypothesis H_0: Assume that there is no significant difference in the consumption of tea before and after the increase in excise ~~sty~~ duty.

$$H_0 : p_1 = p_2.$$

Alternative Hypothesis:- $H_1 : p_1 > p_2$ (right tailed test)

Level of significance:- Level of significance is fixed at 5%.

Test statistic:-

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.8 - 0.667}{\sqrt{(0.727)(0.273)\left(\frac{1}{1000} + \frac{1}{1200}\right)}}$$

$$= \frac{0.8 - 0.667}{0.019} = 7$$

Critical value:-

critical value at 5% level of significance is 1.645

Conclusion:-

Since the calculated value > table value ..

Null hypothesis is rejected.

i.e) There is a difference in the consumption of tea before and after the increase in excise duty.

In a large city A, 20% of a random sample of 900 School boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 School boys had the same defect. Is the difference between the proportions significant.

Given:- $n_1 = 900$; $n_2 = 1600$

$$p_1 = \frac{20}{100} = 0.2 \quad ; \quad p_2 = \frac{18.5}{100} = 0.185$$

Null Hypothesis:-

The difference between the two proportions are not significant.

Alternate Hypothesis:-

The difference between the two proportions are significant

Level of Significance:-

Level of significance is fixed at 5%

Test statistic:

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{900(0.2) + 1600(0.185)}{900 + 1600}$$

$$= \frac{180 + 296}{2500} = \frac{476}{2500} = 0.1904$$

$q = 1-p = 1-0.1904 = 0.8096.$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.2 - 0.185}{\sqrt{(0.1904)(0.8096)\left(\frac{1}{900}+\frac{1}{1600}\right)}}$$

$$= \frac{0.015}{0.016} = 0.9375$$

Critical value :-

Critical value at 5% level of significance is 1.96

Conclusion :-

Since the table value is greater than calculated value ie) $1.96 > 0.9375$. Null Hypothesis is accepted

ie., $P_1 = P_2$

ie) the difference between the two proportions are not significant.