



Random Variable:

Random variable is a real valued function that assigns a numerical value to each possible outcome of an experiment.

Eg:

consider an experiment, tossing a coin twice.
The sample space is

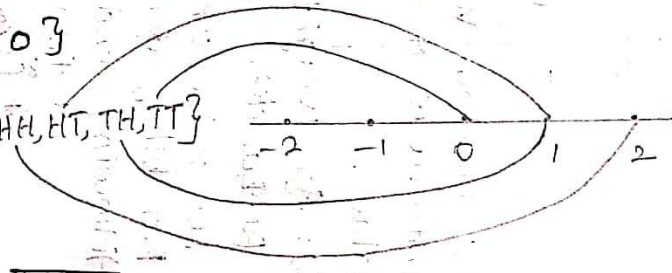
$$S = \{HH, HT, TH, TT\}$$

let x be the random variable such that

$$x(s) = x(\text{No. of heads})$$

$$\therefore x(s) = \{2, 1, 1, 0\}$$

$$S = \{HH, HT, TH, TT\}$$



Types of Random variable:

- * Discrete Random variable
- * Continuous Random variable

Note:

$$* P(X \geq x) = 1 - P(X < x)$$

$$* P(X \leq x) = 1 - P(X > x)$$



Discrete

* A Random variable X is said to be discrete if it assumes either finite or countably infinite.

Eg: No. of students present

* Probability Mass Function $P(x)$

* Let x be discrete random variable then probability mass function satisfies the following conditions.

$$* P(x_i) \geq 0$$

$$* \sum_{i=1}^n P(x_i) = 1$$

* To find constant, $\sum_{i=1}^n P(x_i) = 1$

* To find cumulative distribution function,

$$F(x) = P(X \leq x)$$

* cumulative distribution is given, then find PMF

$$P(x) = F(x_i) - F(x_{i-1})$$

* To find mean: $E(x) = \sum_{i=1}^n x_i P(x_i)$; $E(x^2) = \sum_{i=1}^n x_i^2 P(x_i)$

$$\text{variance } V(x) = E(x^2) - (E(x))^2$$

Continuous

* A Random Variable X is said to be continuous if it takes only an interval values.

Eg: Age, Weight

* Probability density function $f(x)$

* Let x be continuous random variable. The probability density function satisfies the following conditions

$$* f(x) \geq 0$$

$$* \int_{-\infty}^{\infty} f(x) dx = 1$$

* To find constant, $\int_{-\infty}^{\infty} f(x) dx = 1$

* To find cumulative distribution function

$$F(x) = \int_{-\infty}^x f(x) dx$$

* cumulative distribution is given, then find

$$f(x) = \frac{d}{dx} F(x)$$

* To find mean: $E(x) = \int_{-\infty}^{\infty} x f(x) dx$; $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$

$$\text{variance: } V(x) = E(x^2) - (E(x))^2$$



11. A random variable X has the following probability function

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

- Find k .
- $P(X < 6)$, $P(X \geq 6)$, $P(0 < X < 5)$
- Distributive or cumulative function
- $P(\frac{1}{2} < X < \frac{5}{2} / X > 1)$
- Find the smallest value of x such that $P(X \leq x) > \frac{1}{2}$.

Soln.

i). Find k :

$$\sum_{i=0}^7 P(x_i) = 1$$

$$\sum_{i=0}^7 P(x_i) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} a &\rightarrow 10 \\ b &\rightarrow 9 \\ c &\rightarrow -1 \end{aligned}$$

$$= \frac{-9 \pm \sqrt{81 - 4(10)(-1)}}{2(10)}$$

$$= \frac{-9 \pm \sqrt{81 + 40}}{20} = \frac{-9 \pm \sqrt{121}}{20}$$

$$= \frac{-9 \pm 11}{20} = -1, \frac{1}{10}$$

$$k = -1, \frac{1}{10}$$

Here $k = 0.1$ ($\because k = -1$ is not possible)



SNS COLLEGE OF TECHNOLOGY

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DEPARTMENT OF MATHEMATICS

UNIT-1 (PROBABILITY AND RANDOM VARIABLES)



x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

$$ii). P(x < 6) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= 0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$$

$$P(x < 6) = 0.81$$

$$P(x \geq 6) = 1 - P(x < 6)$$

$$= 1 - 0.81$$

$$= 0.19$$

$$P(0 < x < 5) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= 0.1 + 0.2 + 0.2 + 0.3$$

$$= 0.8$$

iii). Distributive or cumulative function:

$$F(x) = P(x \leq x)$$

x	0	1	2	3	4	5	6	7
$P(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17
$F(x)$	0	0.1	0.3	0.5	0.8	0.81	0.83	1

$$iv). P(\frac{1}{2} < x < \frac{5}{2} / x > 1)$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore P(\frac{1}{2} < x < \frac{5}{2} / x > 1) = \frac{P(\frac{1}{2} < x < \frac{5}{2} \cap x > 1)}{P(x > 1)}$$

$$= \frac{P(0.5 < x < 2.5 \cap x > 1)}{P(x > 1)}$$

The diagram shows a number line with points 0.5, 1, and 2.5. A double-headed arrow above the line spans from 0.5 to 2.5. Another double-headed arrow below the line spans from 1 to 2.5, representing the intersection of the two sets.

$$= \frac{P[1 < x < 2.5]}{1 - P(x \leq 1)}$$

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$$\begin{aligned}
 &= \frac{P(X=2)}{1 - [P(X=0) + P(X=1)]} \\
 &= \frac{0.2}{1 - [0 + 0.1]} \\
 &= \frac{0.2}{1 - 0.1} \\
 &= \frac{0.2}{0.9} \\
 &= 0.222
 \end{aligned}$$

v). $P(X \leq x) > \frac{1}{2}$

$\Rightarrow F(x) > \frac{1}{2}$

The values are 4, 5, 6, 7

The smallest value is 4.

21. A random variable X has the probability function

x	0	1	2	3	4	5	6	7	8
$P(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

- i). Find a ii). $P(X < 3)$, $P(X \geq 3)$, $P(0 < X < 6)$
 iii). distribution function.

Soln.

i). Find a :

$$\sum_{i=0}^8 P(x_i) = 1$$

$$\Rightarrow \sum_{i=0}^8 P(x_i) = 1$$

$$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$81a = 1 \Rightarrow a = \frac{1}{81} = 0.012$$



$$\therefore a = 0.012$$

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.012	0.036	0.060	0.084	0.108	0.132	0.156	0.180	0.204

i) $P(x < 3)$

$$P(x < 3) = P(x=0) + P(x=1) + P(x=2)$$

$$= 0.012 + 0.036 + 0.060 = 0.108$$

$$P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - 0.108$$

$$= 0.892$$

$$P(0 < x < 6) = P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$$

$$= 0.036 + 0.060 + 0.084 + 0.108 + 0.132$$

$$= 0.420$$

iii) Distribution function:

$$F(x) = P(x \leq x)$$

x	0	1	2	3	4	5	6	7	8
$P(x)$	0.012	0.036	0.060	0.084	0.108	0.132	0.156	0.180	0.204
$F(x)$	0.012	0.048	0.108	0.192	0.300	0.432	0.588	0.768	0.972