RANDOM VARIABLES

Definition : A real variable 'X' whose value is determined by the outcome of a random experiment is called a random variable.

Example : A random experiment consists of two tosses of a coin. Consider the random variable which is the number of heads (0, 1 or 2).

Outcome :	HH	HT	TH	TT
Value of X :	2	1	1	0

Types of random Variables :

There are two types of random variables :

1. Discrete random variable
2. Continuous random Variable.

Discrete random variable :

A random variable which can assume only a countable number of real values is called a discrete random variable.

Examples :

1. Number of telephone calls per unit time.
2. Marks obtained in a test.

Distribution function (or) Cumulative Distribution Function of the random variable X :

The C.D.F of a random variable X is defined as,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i)$$

in $(-\infty, \infty)$

PROBLEMS :

① Find the constant 'k' from the following probability distribution of a discrete random variable X.

Values of $X = x$	1	2	3	4	5	Total
$p(x)$	0.1	0.2	k	2k	0.1	1

Solution : We know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\therefore \sum_{i=1}^{5} p(x_i) = 1$$

$$p(1) + p(2) + p(3) + p(4) + p(5) = 1$$

$$0.1 + 0.2 + k + 2k + 0.1 = 1$$

$$3k + 0.4 = 1$$

$$3k = 1 - 0.4 = 0.6$$

$$3k = 0.6$$

$$k = \frac{0.6}{3} = 0.2$$

$$\boxed{k = 0.2}$$

② For the following probability distribution,

(i) Find the distribution function of X,

(ii) What is the smallest value of 'x' for which $P(X \leq x) > 0.5$

Solution :

(i) The distribution function of 'X' is given by,

$X = x$	$F(x) = P(X \le x)$
0	$F(0) = P(X \le 0) = P(X = 0) = \frac{1}{4} = 0.25$
1	$F(1) = P(X \le 1) = P(X = 0) + P(X = 1) = \frac{1}{4} + \frac{2}{4} = \frac{3}{4} = 0.75$
2	$F(2) = P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$

(ii) The smallest value of 'x' for which $P(X \le x) > 0.5$ is 1.

(3) A random variable 'X' has the following probability function

Value of $X = x$	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) Find k (ii) Evaluate $P(X < 6)$, $P(X \ge 6)$ and $P(0 < X < 5)$,
(iii) If $P(X \le k) > \frac{1}{2}$, find the minimum value of k and determine the distribution function of X.

Solution :

(i) We know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\sum_{i=0}^{7} p(x_i) = 1$$

$$p(0) + p(1) + p(2) + \ldots + p(7) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$10K^2 + 9K - 1 = 0$

$\therefore K = \frac{-9 \pm \sqrt{81+40}}{20}$ $\quad K = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ $\quad a=10, b=9, c=-1.$

$= \frac{-9 \pm \sqrt{121}}{20} = \frac{-9 \pm 11}{20} = \frac{-9+11}{20}$ (or) $\frac{-9-11}{20}$

$\therefore K = \frac{2}{20}, \frac{-20}{20}$

$K = \frac{1}{10}$ or $K = -1$

$K = -1$ is not possible, Since probability cannot be a negative value.

$\therefore \boxed{K = \frac{1}{10}}$

Subs $K = \frac{1}{10}$ in the given table,

$X = x$	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10} = \frac{17}{100}$

(ii) $P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$

$= \frac{10+20+20+30+1}{100}$

$\boxed{P(X<6) = \frac{81}{100}}$

$$P(X \geq 6) = 1 - P(X < 6)$$

$$= 1 - \frac{81}{100} = \frac{100 - 81}{100}$$

$$\boxed{P(X \geq 6) = \frac{19}{100}}$$

$$P(0 < X < 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10} = \frac{4}{5}$$

$$\boxed{P(0 < X < 5) = \frac{4}{5}}$$

(iii) Given: $P(X \leq k) > \frac{1}{2}$

$P(X \leq 3) = P(X=0) + \cdots + P(X=3)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10}$

$= 0.5$

By trial, $\boxed{k = 4}$

$P(X \leq 4) = P(X=0) + \cdots + P(X=4)$

$= 0.8$

Distribution function of X:

X = x	F(x) = P(X ≤ x)
0	$F(0) = P(X \leq 0) = P(X=0) = 0$
1	$F(1) = P(X \leq 1) = P(X=0) + P(X=1) = 0 + \frac{1}{10} = \frac{1}{10}$
2	$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$
3	$F(3) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10}$

4 $F(4) = P(X \le 4) = P(X=0) + \cdots + P(X=4)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$

$= \frac{8}{10}$

5 $F(5) = P(X \le 5) = P(X=0) + \cdots + P(X=5)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$

$= \frac{10 + 20 + 20 + 30 + 1}{100}$

$= \frac{81}{100}$

6 $F(6) = P(X \le 6) = P(X=0) + \cdots + P(X=6)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$

$= \frac{10 + 20 + 20 + 30 + 1 + 2}{100}$

$= \frac{83}{100}$

7 $F(7) = P(X \le 7) = P(X=0) + \cdots + P(X=7)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100} + \frac{17}{100}$

$= \frac{83}{100} + \frac{17}{100} = 1$