Total probability:

The probability of A in the sample space in P(A) can be expressed interms of conditional probability. Suppose 'n' mutually exclusive event B_n, we can prove that

$$P(A) = \sum_{i=1}^{N} P(A/B_n) \cdot P(B_n)$$

Baye's Theorem:

Let $A_1, A_2, A_3, \ldots, A_n$ be 'n' mutually exclusive and exhaustive event, with $P(A_i) \neq 0$ and B be an independent event $B \subset \bigcup_{i=1}^{n} A_i$ with $P(B) \neq 0$ such that

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)}$$

1. In a Bolt Factory machine A, B, C manufacture 25%, 35%, 40% of the total of their output. 5%, 4% and 2% are defective bolts. A bolt is drawn at a random from the product and is found is to be defective. What are the probability that it was manufactured by machine A, B, C.

Soln.:

Let A_i be the probability of manufacturing bolt.

let B be the probability of defective

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Now

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)}$$

$P(A_i)$	$P(B/A_i)$	$P(A_i) \cdot P(B/A_i)$
$P(A_1) = 25\% = 0.25$	0.05	0.0125
$P(A_2) = 0.35$	0.04	0.014
$P(A_3) = 0.40$	0.02	0.008
	$\sum P(A_i) \cdot P(B/A_i) =$	0.0345

$\therefore$ $P(A_1/B) = \frac{0.0125}{0.0345} = 0.362$

$P(A_2/B) = \frac{0.014}{0.0345} = 0.405$

$P(A_3/B) = \frac{0.008}{0.0345} = 0.231$

2). A, B, C manufacture 25, 35, 40 and defective 5, 4, 2 (Similar to 1st problem)

Soln:

Let A_i be the probability of manufacturing bolt.

Let B be the defective bolt.

Now,

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(B/A_i)}$$

$P(A_i)$	$P(B/A_i)$	$P(A_i)\cdot P(B/A_i)$
25	5	125
35	4	140
40	2	80
	$\sum P(A_i)\cdot P(B/A_i) = 345$	

$$P(A_1/B) = \frac{125}{345} = 0.362$$

$$P(A_2/B) = \frac{140}{345} = 0.406$$

$$P(A_3/B) = \frac{80}{345} = 0.232$$

3\] The first bag contains 3w balls, 2R, 4B and second bag contains 4w, 3R, 5B, third bag contains 3w, 4R, 2B. One bag is choosen at random and from it 3 balls are drawn. Out of these 2 balls are w, one is R ball. what are the probability that they have taken from 1^{st}, 2^{nd}, 3^{rd} bag.

Soln.

Let A_i be the probability of selecting bags.

Let B be the probability of taking balls that are 2w, 1R.

$P(A_i)$	$P(B/A_i)$	$P(A_i)\cdot P(B/A_i)$
$P(A_1) = 1/3$	$\frac{3C_2 \cdot 2C_1}{9C_3} = 0.071$	0.024
$P(A_2) = 1/3$	$\frac{4C_2 \cdot 3C_1}{12C_3} = 0.082$	0.027
$P(A_3) = 1/3$	$\frac{3C_2 \cdot 4C_1}{9C_3} = 0.143$	0.048

$\sum P(A_i)\cdot P(B/A_i) = 0.099$

$$> \frac{3C_2 \cdot 2C_1}{9C_3} = \frac{3\times2\times1\times2\times3\times2\times1}{2\times1\times9\times8\times7} = \frac{1}{14} = 0.071$$

$$> \frac{4C_2 \cdot 3C_1}{12C_3} = \frac{4\times3\times3\times3\times2\times1}{2\times1\times12\times11\times10} = \frac{9}{110} = 0.082$$

$$> \frac{3C_2 \cdot 4C_1}{9C_3} = \frac{3\times2\times4\times3\times2\times1}{2\times1\times9\times8\times7} = \frac{1}{7} = 0.143$$

Now,

$$P(A_1/B) = \frac{0.024}{0.099} = 0.247$$

$$P(A_2/B) = \frac{0.027}{0.099} = 0.272$$

$$P(A_3/B) = \frac{0.048}{0.099} = 0.484$$

4]. The chances of 3 candidates A, B, C becoming a manager of company are in the ratio 3:5:4. The probability that special bonus that will be introduce by them. If selected are 0.6, 0.5, 0.4 respectively and the bonus schemes introduced. What is the probability that B has become one manager?

Soln.

Let A_i be the probability of selecting a manager.

Let B be the probability that the special bonus introduce by 3 candidates.

$$P(A_i/B) = \frac{P(A_i)\cdot P(B/A_i)}{\sum_{i=1}^{n} P(A_i)\cdot P(B/A_i)}$$

$P(A_i)$	$P(B/A_i)$	$P(A_i) \cdot P(B/A_i)$
$P(A_1) = \frac{3}{12} = 0.25$	0.6	0.150
$P(A_2) = \frac{5}{12} = 0.41$	0.5	0.205
$P(A_3) = \frac{4}{12} = 0.3$	0.4	0.120
		$\Sigma P(A_i) \cdot P(B/A_i) = 0.475$

$$\therefore P(A_2/B) = \frac{0.205}{0.475} = 0.432$$

5] Let 5 men out of 100 and 25 women out of 100 are colour blind. A colour blind person is choosen at random. what is the probability of his being male. (Assume that male and female are in equal proportion)

Soln.

Let M be the probability of male.

Let F be the probability of female.

Let C be the probability of colour blind person.

$P(M) = \frac{1}{2} = 0.500$

$P(F) = \frac{1}{2} = 0.500$

$P(C/M) = \frac{5}{100} = 0.050$

$P(C/F) = \frac{25}{100} = 0.250$

$$P(M/C) = \frac{P(M) \cdot P(C/M)}{\sum_{i=1}^{n} P(M) \cdot P(C/M)}$$

$$= \frac{\frac{1}{2} \times 0.05}{0.025 + 0.125}$$

$$= \frac{0.025}{0.150}$$

$$P(M/C) = 0.167$$

(OR)

Let $P(A_i)$ be male or female.
Let $P(B)$ be selecting a colour blind person at random.

$P(A_i)$	$P(B/A_i)$	$P(A_i) \cdot P(B/A_i)$
$M = \frac{1}{2} = 0.5$	$5/100 = 0.05$	0.025
$F = \frac{1}{2} = 0.5$	$25/100 = 0.250$	0.125
	$\sum P(A_i) \cdot P(B/A_i) =$	0.150

$$\therefore P(M/B) = \frac{0.025}{0.150}$$

$$= 0.167$$