



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



UNIT - III

Two Dimensional Random Variable

- * Joint Distribution
- * Marginal Distribution
- * Conditional Distribution
- * Covariance Distribution
- * Correlation Distribution
- * Regression Distribution
- * Functions of Random variable.



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



Discrete

Continuous

I. Joint probability mass function

i). $P(x_i, y_j) \geq 0$

ii). $\sum_{i=1}^n \sum_{j=1}^n P(x_i, y_j) = 1$

2]. To find constant:

$$\sum_{i=1}^n \sum_{j=1}^n P(x_i, y_j) = 1$$

3]. Marginal distribution function of x :

$$P(x) = \sum_{j=1}^n P(x_i, y_j)$$

marginal distribution function of y :

$$P(y) = \sum_{i=1}^n P(x_i, y_j)$$

4]. Cumulative distribution:

$$F(x, y) = P(x \leq x, y \leq y)$$

J. Joint Probability Density function

i). $f(x, y) \geq 0$

ii). $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dy dx = 1$

2]. To find constant

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dy dx = 1$$

3]. Marginal distribution function of x :

$$f(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

Marginal distribution function of y :

$$f(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

4]. Cumulative distribution:

$$F(x, y) = \int_{-\infty}^x \int_{-\infty}^y f(x, y) dy dx$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



Discrete

5]. To check x & y are independent :

$$P(i, j) = P(x=i) \cdot P(y=j)$$

6]. Conditional distribution

$$P(x=x_i / y=y_j) = \frac{P(x=x_i, y=y_j)}{P(y=y_j)}$$

$$P(y=y_j / x=x_i) = \frac{P(x=x_i, y=y_j)}{P(x=x_i)}$$

Continuous

5]. To check x & y are independent

$$F(x, y) = F(x) \cdot F(y)$$

6]. Conditional Distributions

$$P(x/y) = \frac{F(x, y)}{F(y)}$$

$$F(y/x) = \frac{F(x, y)}{F(x)}$$

7]. Joint cumulative function is given then to find PDF

$$f(x, y) = \frac{\partial^2}{\partial x \partial y} F(x, y)$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



J. From the following table. for bivariate distribution of (x, y) . Find

i). $P(x \leq 1)$ ii). $P(y \leq 3)$ iii). $P(x \leq 1, y \leq 3)$

iv). $P(x \leq 1 | y \leq 3)$ v). $P(y \leq 3 | x \leq 1)$

vi). Marginal distribution function of x & y .

vii). Conditional distribution of x given $y=2$

viii). Estimate x & y are independent

ix). $P(x+y \leq 4)$

Soln. \downarrow $x \backslash y \rightarrow$	1	2	3	4	5	6	$P(x)$
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\rightarrow \frac{8}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{10}{16}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{8}{64}$
$P(y)$	$\downarrow \frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	1

i). $P(x \leq 1)$

$$\begin{aligned}
 P(x \leq 1) &= P(x=0) + P(x=1) \\
 &= \frac{8}{32} + \frac{10}{16} \\
 &= \frac{28}{32} = \frac{7}{8}
 \end{aligned}$$

ii). $P(y \leq 3)$

$$\begin{aligned}
 P(y \leq 3) &= P(y=1) + P(y=2) + P(y=3) \\
 &= \frac{3}{32} + \frac{3}{32} + \frac{11}{64} \\
 &= \frac{23}{64}
 \end{aligned}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



iii). $P(X \leq 1, Y \leq 3)$

$$X = 0, 1$$

$$Y = 1, 2, 3$$

$$P(X \leq 1, Y \leq 3) = P(0, 1) + P(0, 2) + P(0, 3) + P(1, 1) + P(1, 2) + P(1, 3)$$

$$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}$$

$$= \frac{1+2+2+4}{32}$$

$$= \frac{9}{32}$$

iv). $P(X \leq 1 | Y \leq 3) = \frac{P(X \leq 1, Y \leq 3)}{P(Y \leq 3)}$

$$= \frac{P(0, 1) + P(0, 2) + P(0, 3) + P(1, 1) + P(1, 2) + P(1, 3)}{P(Y=1) + P(Y=2) + P(Y=3)}$$

$$= \frac{0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}}{\frac{3}{32} + \frac{3}{32} + \frac{11}{64}}$$

$$= \frac{9/32}{23/64}$$

$$= \frac{18}{23}$$

v). $P(Y \leq 3 | X \leq 1) = \frac{P(X \leq 1, Y \leq 3)}{P(X \leq 1)}$

$$= \frac{P(0, 1) + P(0, 2) + P(0, 3) + P(1, 1) + P(1, 2) + P(1, 3)}{P(X=0) + P(X=1)}$$

$$= \frac{0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}}{28/32}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



$$= \frac{9/32}{28/32}$$

$$= \frac{9}{28}$$

vi) marginal distribution function of x

x	0	1	2
$P(x)$	$8/32$	$10/32$	$8/32$

marginal distribution function of y :

y	1	2	3	4	5	6
$P(y)$	$3/32$	$3/32$	$11/64$	$13/64$	$6/32$	$16/64$

vii) Conditional distribution function of x on $y=2$.

$$P(x/y=2) =$$

$$P(x=0/y=2) = \frac{P(x=0, y=2)}{P(y=2)} = 0$$

$$P(x=1/y=2) = \frac{P(x=1, y=2)}{P(y=2)} = \frac{1/16}{3/32} = \frac{2}{3}$$

$$P(x=2/y=2) = \frac{P(x=2, y=2)}{P(y=2)} = \frac{1/32}{3/32} = \frac{1}{3}$$

viii) x & y are independent.

$$\Rightarrow P(x=i, y=j) = P(x=i) \cdot P(y=j)$$

Consider $P(2, 3)$

$$P(2, 3) = P(x=2) \cdot P(y=3)$$

$$\frac{1}{32} \neq \frac{8}{64} \cdot \frac{11}{64}$$

$\therefore x$ & y are not independent.



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



$$ix). P(x+y \leq 4)$$

$$P(x+y \leq 4) = P(0, 1) + P(0, 2) + P(0, 3) + P(0, 4) + P(1, 1) \\ + P(1, 2) + P(1, 3) + P(2, 1) + P(2, 2)$$

$$= 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32}$$

$$= \frac{1+2+2+2+4+1+1}{32}$$

$$= \frac{13}{32}$$

2j. If the joint PDF of (x, y) is given by

$P(x, y) = k(2x + 3y)$, $x=0, 1, 2$; $y=1, 2, 3$. Find all the marginal probability distributions. Also find the prob. distribution of $(x+y)$ and $P(x+y > 3)$.

Soln.

Given $P(x, y) = k(2x + 3y)$

$y \backslash x$	0	1	2	$P(y)$
1	$3k$	$5k$	$7k$	$15k$
2	$6k$	$8k$	$10k$	$24k$
3	$9k$	$11k$	$13k$	$33k$
$P(x)$	$18k$	$24k$	$30k$	$72k$

$$\text{Total probability} = 1$$

$$72k = 1$$

$$k = \frac{1}{72}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



$y \backslash x$	0	1	2	$P(y)$
1	$\frac{3}{72}$	$\frac{5}{72}$	$\frac{7}{72}$	$\frac{15}{72}$
2	$\frac{6}{72}$	$\frac{8}{72}$	$\frac{10}{72}$	$\frac{24}{72}$
3	$\frac{9}{72}$	$\frac{11}{72}$	$\frac{13}{72}$	$\frac{33}{72}$
$P(x)$	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$	1

Marginal probability function of x

$$P(x=0) = \frac{18}{72}$$

$$P(x=1) = \frac{24}{72}$$

$$P(x=2) = \frac{30}{72}$$

marginal probability function of y

$$P(y=1) = \frac{15}{72}$$

$$P(y=2) = \frac{24}{72}$$

$$P(y=3) = \frac{33}{72}$$

Probability distribution of $x+y$:

Probability

$$P(0, 1) = \frac{3}{72}$$

$$P(x+y=1)$$

$$P(x+y=2) \quad P(1, 1) + P(0, 2) = \frac{5}{72} + \frac{6}{72} = \frac{11}{72}$$

$$P(x+y=3) \quad P(2, 1) + P(1, 2) + P(0, 3) = \frac{7}{72} + \frac{8}{72} + \frac{9}{72} = \frac{24}{72}$$

$$P(x+y=4) \quad P(1, 3) + P(2, 2) = \frac{11}{72} + \frac{10}{72} = \frac{21}{72}$$

$$P(x+y=5) \quad P(2, 3) = \frac{13}{72}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



$$\begin{aligned}P(x+y > 3) &= P(x+y=4) + P(x+y=5) \\&= \frac{21}{72} + \frac{13}{72} \\&= \frac{34}{72}\end{aligned}$$

3]. The two dimensional random variable (x, y) has joint probability mass function

$$f(x, y) = \frac{x+2y}{27}, \quad x=0, 1, 2; \quad y=0, 1, 2.$$

Find the conditional distribution of y for $x=x$.

Also find conditional distribution of y given $x=1$.

Soln.

$$\text{Given } f(x, y) = \frac{x+2y}{27}$$

$x \backslash y$	0	1	2	$P(x)$
0	0	$2/27$	$4/27$	$6/27$
1	$1/27$	$3/27$	$5/27$	$9/27$
2	$2/27$	$4/27$	$6/27$	$12/27$
$P(y)$	$3/27$	$9/27$	$15/27$	1

i). $P(y/x=x) = P$

When $x=0$,

$$P(y=0/x=0) = \frac{P(x=0, y=0)}{P(x=0)} = \frac{0}{6/27} = 0$$

$$P(y=1/x=0) = \frac{P(x=0, y=1)}{P(x=0)} = \frac{2/27}{6/27} = \frac{2}{6}$$

$$P(y=2/x=0) = \frac{P(x=0, y=2)}{P(x=0)} = \frac{4/27}{6/27} = \frac{4}{6}$$



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

Joint distribution, Marginal, Conditional distribution



when $x=1$,

$$P(y=0/x=1) = \frac{P(x=1, y=0)}{P(x=1)} = \frac{1/27}{9/27} = \frac{1}{9}$$

$$P(y=1/x=1) = \frac{P(x=1, y=1)}{P(x=1)} = \frac{3/27}{9/27} = \frac{3}{9}$$

$$P(y=2/x=1) = \frac{P(x=1, y=2)}{P(x=1)} = \frac{5/27}{9/27} = \frac{5}{9}$$

when $x=2$,

$$P(y=0/x=2) = \frac{P(x=2, y=0)}{P(x=2)} = \frac{2/27}{12/27} = \frac{2}{12}$$

$$P(y=1/x=2) = \frac{P(x=2, y=1)}{P(x=2)} = \frac{4/27}{12/27} = \frac{4}{12}$$

$$P(y=2/x=2) = \frac{P(x=2, y=2)}{P(x=2)} = \frac{6/27}{12/27} = \frac{6}{12}$$

ii). $P(y/x=1)$

$$P(y=0/x=1) = \frac{1}{9}$$

$$P(y=1/x=1) = \frac{3}{9}$$

$$P(y=2/x=1) = \frac{5}{9}$$