Continuous Distributions:

Uniform Distribution:

If x is a continuous random variable which follows uniform distribution, then

$$f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{otherwise} \end{cases}$$

MGF, mean & variance:

MGF:

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$= \int_a^b e^{tx} \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\int_a^b e^{tx}\, dx$$

$$= \frac{1}{b-a}\left[\frac{e^{tx}}{t}\right]_a^b$$

$$= \frac{1}{t(b-a)}\left(e^{tb} - e^{ta}\right)$$

$$M_x(t) = \frac{e^{bt}-e^{at}}{t(b-a)}$$

Mean:

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_a^b x \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{2(b-a)}(b^2-a^2)$$

$$= \frac{(b+a)(b-a)}{2(b-a)}$$

$$E(x) = \frac{b+a}{2}$$

Variance :

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_a^b x^2 \frac{1}{b-a}\,dx$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$= \frac{1}{3(b-a)}(b^3-a^3)$$

$$= \frac{(b-a)(b^2+a^2+ab)}{3(b-a)}$$

$$E(X^2) = \frac{b^2+a^2+ab}{3}$$

$$\therefore\ Var(X) = \frac{b^2+a^2+ab}{3} - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{b^2+a^2+ab}{3} - \frac{b^2+a^2+2ba}{4}$$

$$= \frac{4(b^2+ab+a^2) - 3(b^2+a^2+2ba)}{12}$$

$$= \frac{4b^2+4ab+4a^2-3b^2-3a^2-6ba}{12}$$

$$= \frac{b^2+a^2-2ab}{12}$$

$$= \frac{(b-a)^2}{12}$$

$$M_x(t) = \frac{e^{bt}-e^{at}}{t(b-a)}$$

$$\text{mean} = \frac{b+a}{2}$$

$$\text{Variance} = \frac{(b-a)^2}{12}$$

1. If x is a uniform distribution with mean 1 and variance 4/3. Find $P(X<0)$

Soln.

mean : $\frac{b+a}{2} = 1$

$b+a = 2 \longrightarrow (1)$

Variance: $\frac{(b-a)^2}{12} = \frac{4}{3}$

$(b-a)^2 = 16$

$b-a = 4 \rightarrow (2)$

Solve:

$b+a = 2$

$b-a = 4$

$2b = 6 \Rightarrow b = 3$

$(1) \Rightarrow \quad a = 2 - 3$

$a = -1$

$$\therefore \quad f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{4}, & -1 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Now, $$P(X<0) = \int_{-1}^{0} f(x)\,dx$$

$$= \int_{-1}^{0} \frac{1}{4}\,dx$$

$$= \frac{1}{4}(x)_{-1}^{0}$$

$$= \frac{1}{4}(0+1)$$

$$P(X<0) = \frac{1}{4}$$

3. The Number of personal computer PC sold daily at a computer world is uniform distribution with min 2000 PC max 5000 PC.

i). The prob. that daily sales will fall between 2500 & 3000 PC.

ii). what is the prob. that computer world sell atleast 4000 PC.

iii). what is the prob. that computer world sell exactly 2500 PC.

Soln.

WKT, $$f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{otherwise} \end{cases}$$

Here $a = 2000$ and $b = 5000$

$$f(x) = \begin{cases} \frac{1}{3000}, & 2000 < x < 5000 \\ 0, & \text{otherwise} \end{cases}$$

i). $P(2500 < x < 3000) = \int_{2500}^{3000} \frac{1}{3000}\, dx$

$$= \frac{1}{3000}\left[x\right]_{2500}^{3000}$$

$$= \frac{1}{3000}\left[3000 - 2500\right]$$

$$= \frac{500}{3000}$$

$$= \frac{1}{6}$$

ii). $P(X \geq 4000) = \int_{4000}^{5000} \frac{1}{3000}\, dx$

$$= \frac{1}{3000}\left[x\right]_{4000}^{5000}$$

$$= \frac{1}{3000}\left[5000 - 4000\right]$$

$$= \frac{1000}{3000}$$

$$= \frac{1}{3}$$

iii). Exactly 2500

$P(X = 2500) = 0$

3) A random variable has an uniform distribution over an $(-3, 3)$. Compute $P(X=2)$, $P(X<2)$, $P(|X|<2)$, $P(|X-2|<2)$. Find K if $P(X>K)=\frac{1}{3}$.

Soln.

WKT $f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{otherwise} \end{cases}$

$$f(x) = \begin{cases} \frac{1}{6}, & -3<x<3 \\ 0, & \text{otherwise} \end{cases}$$

i). $P(X=2) = 0$

ii). $P(X<2) = \int_{-3}^{2} \frac{1}{6}\, dx$

$$= \frac{1}{6}\int_{-3}^{2} dx$$

$$= \frac{1}{6}[x]_{-3}^{2}$$

$$= \frac{1}{6}[2+3]$$

$$P(X<2) = \frac{5}{6}$$

iii). $P(|X|<2) = P(-2<x<2)$

$$= \int_{-2}^{2} \frac{1}{6}\, dx$$

$$= \frac{1}{6}[x]_{-2}^{2}$$

$$= \frac{1}{6}[2+2]$$

$$= \frac{4}{6} = \frac{2}{3}$$

iv). $P(|x-2|<2) = P(-2 < x-2 < 2)$

$= P(0 < x < 4)$

Now,

$$P(0<x<4) = \int_0^3 \frac{1}{6}\,dx$$

$$= \frac{1}{6}[x]_0^3$$

$$= \frac{1}{6}[3-0]$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

v). $P(x>k) = \frac{1}{3}$

$$\int_k^3 f(x)\,dx = \frac{1}{3}$$

$$\int_k^3 \frac{1}{6}\,dx = \frac{1}{3}$$

$$\frac{1}{6}[x]_k^3 = \frac{1}{3} \Rightarrow \frac{1}{6}[3-k] = \frac{1}{3}$$

$$3-k = \frac{6}{3}$$

$$3-k = 2$$

$$-k = 2-3 = -1$$

$$k = 1$$

4). Bus arrives at a specified stop at 15 minutes intervals starting at 7am ie. they arrived at 7, 7.15, 7.30, ... If a passenger arrive at a stop at a random time, ie., is uniform distribution between 7 and 7.30.
Find the probability,

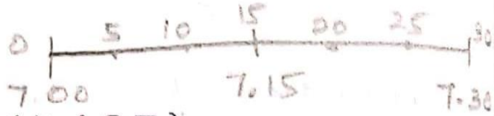
i). less than 5 mins.
ii). more than 10 mins.
iii). atleast 12 mins. for a bus.

Soln.

WKT
$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{30}, & 0 < x < 30 \\ 0, & \text{otherwise} \end{cases}$$

i). less than 5 mins.



$$= P(0 < X < 10) + P(15 < X < 25)$$

$$= \int_0^{10} \frac{1}{30}\,dx + \int_{15}^{25} \frac{1}{30}\,dx$$

$$= \frac{1}{30}(10-0) + \frac{1}{30}(25-15)$$

$$= \frac{10}{30} + \frac{10}{30}$$

$$= \frac{20}{30}$$

$$= \frac{2}{3}$$

ii). more than 10 mins:

$$= P(0 < X < 5) + P(15 < X < 20)$$

$$= \int_0^5 \frac{1}{30} dx + \int_{15}^{20} \frac{1}{30} dx$$

$$= \frac{1}{30}(5-0) + \frac{1}{30}(20-15)$$

$$= \frac{5}{30} + \frac{5}{30}$$

$$= \frac{10}{30}$$

$$= \frac{1}{3}$$

. atleast 12 mins

$$= P(0 < X < 3) + P(15 < X < 18)$$

$$= \int_0^3 \frac{1}{30} dx + \int_{15}^{18} \frac{1}{30} dx$$

$$= \frac{1}{30}(x)_0^3 + \frac{1}{30}(18-15)$$

$$= \frac{3}{30} + \frac{3}{30}$$

$$= \frac{6}{30}$$

$$= \frac{1}{5}$$