



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore - 641 035

DEPARTMENT OF MATHEMATICS

BINOMIAL DISTRIBUTION



Binomial Distribution:

Bernoulli trial:

Each trial has two possible outcomes generally called Success (p) and failure (q). Such a trial is known as Bernoulli trial.

Binomial Distribution:

A random variable x is said to follow Binomial distribution, if it assume only non-negative values of its probability mass function is given by,

$$P[X=x] = {}^n C_x p^x q^{n-x}, \quad x=0,1,\dots,n \text{ and } p+q=1, q=1-p$$

Properties:

- i). There must be a fixed number of trials.
- ii). All trials must have identical probabilities of Success (p)
- iii). The trials must be independent of each other.

MGF, mean and variance of Binomial Distribution:

$$\begin{aligned}
 M_x(t) &= \sum_{x=0}^n e^{tx} P(x) \\
 &= \sum_{x=0}^n e^{tx} {}^n C_x p^x q^{n-x} \\
 &= \sum_{x=0}^n (pe^t)^x {}^n C_x q^{n-x} \\
 &= {}^n C_0 q^n + {}^n C_1 (pe^t)^1 q^{n-1} + {}^n C_2 (pe^t)^2 q^{n-2} + \dots + {}^n C_n (pe^t)^n
 \end{aligned}$$

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$$= q^n + nC_1 (pe^t) q^{n-1} + nC_2 (pe^t)^2 q^{n-2} + \dots + (pe^t)^n$$

$$\text{MGF} = (q + pe^t)^n \quad \because (q+p)^n = q^n + nC_1 q^{n-1} p + \dots + p^n$$

mean:

$$E[X] = \left[\frac{d}{dt} m_x(t) \right]_{t=0}$$

$$= \left[n(q+pe^t)^{n-1} pe^t \right]_{t=0}$$

$$= np(q+p)^{n-1}$$

$$= np \cdot 1^{n-1}$$

$$\because p+q=1$$

$$\text{mean} = np$$

variance:

$$\text{var}(X) = E[X^2] - (E[X])^2$$

$$E[X^2] = \left[\frac{d^2}{dt^2} m_x(t) \right]_{t=0}$$

$$= \frac{d}{dt} \left[\frac{d}{dt} m_x(t) \right]_{t=0}$$

$$= \left[\frac{d}{dt} \left[np(q+pe^t)^{n-1} e^t \right] \right]_{t=0}$$

$$= np \left[(q+pe^t)^{n-1} e^t + e^t (n-1)(q+pe^t)^{n-2} pe^t \right]_{t=0}$$

$$\because d(uv) = uv' + vu'$$

$$= np \left[(q+p)^{n-1} + (n-1)p(q+p)^{n-2} \right]$$

$$= np \left[1^{n-1} + (n-1)p \cdot 1^{n-2} \right] \because p+q=1$$

$$= np \left[1 + (n-1)p \right]$$

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$$\begin{aligned} &= np [1 + np - p] \\ E[x^2] &= np + n^2 p^2 - np^2 \\ \text{Var}(x) &= E(x^2) - [E(x)]^2 \\ &= np + n^2 p^2 - np^2 - n^2 p^2 \\ &= np - np^2 \\ &= np(1-p) \\ \text{Var}(x) &= npq \end{aligned}$$

$$\begin{aligned} \therefore m_x(t) &= (q + pe^t)^n \\ \text{mean} &= np \\ \text{variance} &= npq \end{aligned}$$

7. For a Binomial variate, mean is 36, variance is 12. Find p, q, n .

Soln.

$$\text{Given mean: } np = 36 \rightarrow (1)$$

$$\text{variance: } npq = 12 \rightarrow (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{12}{36}$$

$$q = \frac{1}{3}$$

$$\text{wkt } p + q = 1$$

$$p + \frac{1}{3} = 1$$

$$p = 1 - \frac{1}{3} = \frac{3-1}{3}$$

$$p = \frac{2}{3}$$

$$(1) \Rightarrow n \left(\frac{2}{3}\right) = 36 \Rightarrow n = \frac{36 \left(\frac{3}{2}\right)}{1}$$

$$n = 54$$

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Q]. Find the Binomial distribution for which mean is 4, variance is 3.

Soln.

$$\text{Given mean: } np = 4 \rightarrow (1)$$

$$\text{variance: } npq = 3 \rightarrow (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{3}{4}$$

$$q = \frac{3}{4}$$

$$\text{WKT } p+q = 1$$

$$p = 1 - \frac{3}{4}$$

$$p = \frac{1}{4}$$

$$(1) \Rightarrow n\left(\frac{1}{4}\right) = 4$$

$$n = 16$$

Binomial distribution is

$$P[X=x] = {}^nC_x p^x q^{n-x}$$

$$P[X=x] = {}^{16}C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{16-x}$$

Q]. The mean of BD is 20 and Standard deviation is 4. determine the parameter of distribution. (n, p, q)

Soln.:

$$\text{Given mean: } np = 20 \rightarrow (1)$$

$$\text{S.D. : } \sqrt{\text{variance}} \Rightarrow npq = 4^2 = 16 \rightarrow (2)$$

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$$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = \frac{16}{20}$$

$$q = \frac{4}{5}$$

$$\therefore p + q = 1$$

$$p = 1 - \frac{4}{5}$$

$$p = \frac{1}{5}$$

$$(1) \Rightarrow n\left(\frac{1}{5}\right) = 20$$

$$n = 100$$

4]. IF x is a Binomial variate with $n=6$ and $9P[X=4] = P[X=2]$. Find Binomial distribution.

Soln.

$$\text{Given } 9P(X=4) = P(X=2)$$

$$\text{BD: } P[X=x] = {}^n C_x p^x q^{n-x}$$

$$9[{}^6 C_4 p^4 q^{6-4}] = {}^6 C_2 p^2 q^{6-2}$$

$$9[{}^6 C_2 p^4 q^2] = {}^6 C_2 p^2 q^4$$

$$9 \frac{p^4}{p^2} = \frac{q^4}{q^2}$$

$$9p^2 = q^2$$

$$= (1-p)^2$$

$$9p^2 = 1 + p^2 - 2p$$

$$9p^2 - p^2 + 2p - 1 = 0$$

$$8p^2 + 2p - 1 = 0$$

$$8p^2 + 4p - 2p - 1 = 0$$

$$\begin{array}{r} -8 \\ 4 \quad -2 = 2 \end{array}$$

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$$4P(2P+1) - 1(2P+1) = 0$$

$$(2P+1)(4P-1) = 0$$

$$\begin{array}{l} 2P+1 = 0 \\ P = -\frac{1}{2} \end{array} \quad \left| \quad \begin{array}{l} 4P-1 = 0 \\ P = \frac{1}{4} \end{array} \right.$$

$\therefore P = \frac{1}{4}$ (\because negative value is not possible)

$$\therefore P+q = 1$$

$$q = 1 - \frac{1}{4}$$

$$q = \frac{3}{4}$$

BD:

$$P[X=x] = {}^6C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{6-x}$$

5]. Four coins are tossed simultaneously.
What is the probability of getting
i). 2 heads ii). at least 2 heads
iii). at most 2 heads.

Soln.

Given: 4 coins tossed. So $n=4$

Probability of getting head $p = \frac{1}{2}$

$$\therefore q = \frac{1}{2}$$

BD:

$$\begin{aligned} P[X=x] &= {}^n C_x p^x q^{n-x} \\ &= {}^4 C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x} \end{aligned}$$

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i). 2 heads

$$\begin{aligned}P[X=2] &= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} \\&= \frac{4 \times 3}{2 \times 1} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \\&= 6 \left(\frac{1}{2}\right)^4 = \frac{6}{16} \\&= 0.375\end{aligned}$$

ii). at least 2 heads

$$\begin{aligned}P[X \geq 2] &= P(X=2) + P(X=3) + P(X=4) \\&= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} + {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} + \\&\quad {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4} \\&= \frac{4 \times 3}{2 \times 1} \left(\frac{1}{2}\right)^4 + 4C_1 \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 \\&= \left(\frac{1}{2}\right)^4 [6 + 4 + 1] \\&= \frac{11}{16} \\&= 0.688\end{aligned}$$

iii). at most 2 heads

$$\begin{aligned}P[X \leq 2] &= P(X=0) + P(X=1) + P(X=2) \\&= {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} \\&= \left(\frac{1}{2}\right)^4 + 4 \left(\frac{1}{2}\right)^4 + \frac{4 \times 3}{2 \times 1} \left(\frac{1}{2}\right)^4\end{aligned}$$

$$\begin{array}{|l} \because nC_0 = 1 \\ nC_1 = n \end{array}$$

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$$\begin{aligned} &= \left(\frac{1}{2}\right)^4 [1+4+6] \\ &= \frac{11}{16} \\ P[X \leq 2] &= 0.688 \end{aligned}$$
$$\begin{aligned} P[X \leq 2] &= 1 - P[X > 2] \\ &= 1 - [P(X=3) + P(X=4) + P(X=5) + P(X=6)] \end{aligned}$$

6]. 6 dice are thrown 729 times. How many times do you expect at least 3 dice to show 5 or 6.

Soln.

$$\text{Give } n = 6$$

$$N = 729$$

$$\text{BD: } P[X=x] = {}^n C_x p^x q^{n-x}$$

probability of getting 5 or 6 in the dice.

$$= P(5) + P(6)$$

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$p = \frac{1}{3}$$

$$\therefore p + q = 1$$

$$q = 1 - \frac{1}{3}$$

$$q = \frac{2}{3}$$

at least 3 dice

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$$P[X \geq 3] = P(X=3) + P(X=4) + P(X=5) + P(X=6)$$

$$= {}^6C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{6-3} + {}^6C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{6-4} +$$

$${}^6C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{6-5} + {}^6C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{6-6}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 + {}^6C_2 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + {}^6C_1 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)$$

$$+ \left(\frac{1}{3}\right)^6 (1)$$

$$= 20 \frac{1}{27} \left(\frac{8}{27}\right) + \frac{6 \times 5}{2 \times 1} \left(\frac{1}{81}\right) \left(\frac{4}{9}\right) + 6 \left(\frac{1}{243}\right) \left(\frac{2}{3}\right) + \frac{1}{729}$$

$$= 0.219 + 0.082 + 0.016 + 0.001$$

$$= 0.318$$

For single time, possibility is 0.318

For 729 times, 729×0.318

$$= 231.82$$

$$= 232$$

7). In a large consignment of electric bulbs 20% are defective. A random sample of 20 is taken for inspection. Find the probability that

i). All are good

ii). Atmost there are 3 defectives.

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Soln.:

$$n = 20$$

$$p = 10\% \text{ defective} = \frac{10}{100}$$

$$p = 0.1$$

$$\therefore p + q = 1$$

$$q = 1 - p = 1 - 0.1$$

$$q = 0.9$$

$$\begin{aligned} \text{BD: } P[X=x] &= {}^n C_x p^x q^{n-x} \\ &= {}^{20} C_x (0.1)^x (0.9)^{20-x} \end{aligned}$$

i). All are good (no one is defective)

$$P[X=0] = {}^{20} C_0 (0.1)^0 (0.9)^{20-0}$$

$$= 1(0.9)^{20}$$

$$= 0.122$$

ii). Atmost there are 3 defectives.

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= 0.122 + {}^{20} C_1 (0.1) (0.9)^{20-1} + {}^{20} C_2 (0.1)^2 (0.9)^{20-2} + {}^{20} C_3 (0.1)^3 (0.9)^{20-3}$$

$$= 0.122 + 20 (0.1) (0.9)^{19} + \frac{20 \times 19}{2 \times 1} (0.1)^2 (0.9)^{18}$$

$$+ \frac{20 \times 19 \times 18}{3 \times 2 \times 1} (0.1)^3 (0.9)^{17}$$

$$= 0.122 + 0.270 + 0.285 + 0.190$$

$$P(X \leq 3) = 0.867$$

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