RANDOM VARIABLES

Definition : A real variable 'X' whose value is determined by the outcome of a random experiment is called a random variable.

Example : A random experiment consists of two tosses of a coin. Consider the random variable which is the number of heads (0, 1 or 2).

Outcome :	HH	HT	TH	TT
Value of X :	2	1	1	0

Types of random variables :

There are two types of random variables :

1. Discrete random variable

2. Continuous random variable.

Discrete random variable :

A random variable which can assume only a countable number of real values is called a discrete random variable.

Examples :

1. Number of telephone calls per unit time.

2. Marks obtained in a test.

Distribution function (or) Cumulative Distribution function of the random variable X :

The C.D.F of a random variable X is defined as,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i)$$

in $(-\infty, \infty)$

PROBLEMS :

① Find the Constant 'k' from the following probability distribution of a discrete random variable X.

Values of X = x	1	2	3	4	5	Total
P(x)	0.1	0.2	k	2k	0.1	1

Solution : We Know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\therefore \quad \sum_{i=1}^{5} p(x_i) = 1$$

$$p(1) + p(2) + p(3) + p(4) + p(5) = 1$$

$$0.1 + 0.2 + k + 2k + 0.1 = 1$$

$$3k + 0.4 = 1$$

$$3k = 1 - 0.4 = 0.6$$

$$3k = 0.6$$

$$k = \frac{0.6}{3} = 0.2$$

$$\boxed{k = 0.2}$$

② For the following probability distribution,

(i) Find the distribution function of X,

(ii) What is the smallest value of 'x' for which $P(X \leq x) > 0.5$

Solution :

(i) The distribution function of 'x' is given by,

$X = x$	$F(x) = P(X \leq x)$
0	$F(0) = P(X \leq 0) = P(X=0) = \dfrac{1}{4} = 0.25$
1	$F(1) = P(X \leq 1) = P(X=0) + P(X=1)$ $= \dfrac{1}{4} + \dfrac{2}{4} = \dfrac{3}{4} = 0.75$
2	$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$ $= \dfrac{1}{4} + \dfrac{2}{4} + \dfrac{1}{4} = 1$

(ii) The smallest value of 'x' for which $P(X \leq x) > 0.5$ is 1.

③ A random variable 'x' has the following probability function

Value of $X = x$	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

(i) Find k (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$,
(iii) If $P(X \leq k) > \dfrac{1}{2}$, find the minimum value of k and determine the distribution function of X.

Solution :

(i) We know that,

$$\sum_{i=1}^{\infty} P(x_i) = 1$$

$$\sum_{i=0}^{7} P(x_i) = 1$$

$$P(0) + P(1) + P(2) + \ldots + P(7) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$\therefore k = \frac{-9 \pm \sqrt{81+40}}{20} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{array}{l} a = 10 \\ b = 9 \\ c = -1 \end{array}$$

$$= \frac{-9 \pm \sqrt{121}}{20} = \frac{-9 \pm 11}{20} = \frac{-9+11}{20} \text{ (or) } \frac{-9-11}{20}$$

$$\therefore k = \frac{2}{20} , \frac{-20}{20}$$

$$k = \frac{1}{10} \quad \text{or} \quad k = -1$$

$k = -1$ is not possible, Since probability cannot be a negative value.

$$\therefore \boxed{k = \frac{1}{10}}$$

Subs $k = \frac{1}{10}$ in the given table.

$X = x$	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10}$ = $\frac{17}{100}$

(ii) $P(X < 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$$

$$= \frac{10 + 20 + 20 + 30 + 1}{100}$$

$$\boxed{P(X < 6) = \frac{81}{100}}$$

$$P(X \geq 6) = 1 - P(X < 6)$$

$$= 1 - \frac{81}{100} = \frac{100 - 81}{100}$$

$$\boxed{P(X \geq 6) = \frac{19}{100}}$$

$$P(0 < X < 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10} = \frac{4}{5}$$

$$\boxed{P(0 < X < 5) = \frac{4}{5}}$$

(iii) Given: $P(X \leq k) > \frac{1}{2}$

By trial, $\boxed{k = 4}$

Distribution function of X:

$$P(X \leq 3) = P(X=0) + \cdots + P(X=3)$$
$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10}$$
$$= 0.5$$

$$P(X \leq 4) = P(X=0) + \cdots + P(X=4)$$
$$= 0.8$$

$X = x$	$F(x) = P(X \leq x)$
0	$F(0) = P(X \leq 0) = P(X=0) = 0$
1	$F(1) = P(X \leq 1) = P(X=0) + P(X=1)$ $= 0 + \frac{1}{10} = \frac{1}{10}$
2	$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$ $= 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$
3	$F(3) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2)$ $+ P(X=3)$ $= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10}$

4. $F(4) = P(x \leq 4) = P(x=0) + \cdots + P(x=4)$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10}$$

5. $F(5) = P(x \leq 5) = P(x=0) + \cdots + P(x=5)$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$$

$$= \frac{10 + 20 + 20 + 30 + 1}{100}$$

$$= \frac{81}{100}$$

6. $F(6) = P(x \leq 6) = P(x=0) + \cdots + P(x=6)$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$$

$$= \frac{10 + 20 + 20 + 30 + 1 + 2}{100}$$

$$= \frac{83}{100}$$

7. $F(7) = P(x \leq 7) = P(x=0) + \cdots + P(x=7)$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$$

$$+ \frac{2}{100} + \frac{17}{100}$$

$$= \frac{83}{100} + \frac{17}{100} = 1$$