

(An Autonomous Institution)



### **DEPARTMENT OF MATHEMATICS**

$$f_{v}(u) = \int_{-\infty}^{\infty} f_{vv}(u,v) \, dv$$

$$= \int_{0}^{\infty} e^{u-av} \, dv$$

$$= e^{u} \int_{0}^{\infty} e^{-av} \, dv$$

$$= e^{u} \left[ \underbrace{e^{-av}}_{2} \right]_{0}^{\infty}$$

$$= \underbrace{e^{u}}_{2} \left[ o + e^{-2u} \right]$$

$$f_{v}(u) = \underbrace{e^{-u}}_{2},$$

Step 6:  

$$\int_{0}^{u/2} (u) = \begin{cases}
e^{u/2}, & u > 0
\end{cases}$$

(4) If X and Y are independent random variables with Pdf's  $e^{-\chi}$ ,  $\chi > 0$  and  $e^{-y}$ , y > 0 respectively, find the density function of  $U = \frac{\chi}{\chi + \gamma}$  and  $V = \chi + \gamma$ . Are U & V independent.

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$$= e^{-(\chi + y)}$$



Solution:

Step 1:  
Given: 
$$f(x) = e^{-x}$$
,  $x > 0$   
 $f(y) = e^{-y}$ ,  $y > 0$ 

Since X and Y are independent,

$$f(x,y) = f(x).f(y)$$

$$= e^{-x}.e^{-y}$$

$$f(x,y) = e^{-(x+y)}, x>0,y>0$$

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Let 
$$U = \frac{X}{X+Y}$$

$$V = X + Y$$

i.e., 
$$u = \frac{x}{x+y}$$
,  $v = x+y$   
 $y = v-x$ 

$$\begin{array}{c|c} y = v - uv \\ \hline x + y = x \\ uv = x \\ x = uv \end{array}$$

$$uv = x$$
  $\frac{\partial y}{\partial v} = 1 - u$ 

$$\frac{\partial x}{\partial u} = v ; \frac{\partial x}{\partial v} = u \qquad \frac{\partial y}{\partial u} = -v$$

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{2} - v & 1 - u \end{vmatrix} = |v(1 - u) + vu|$$

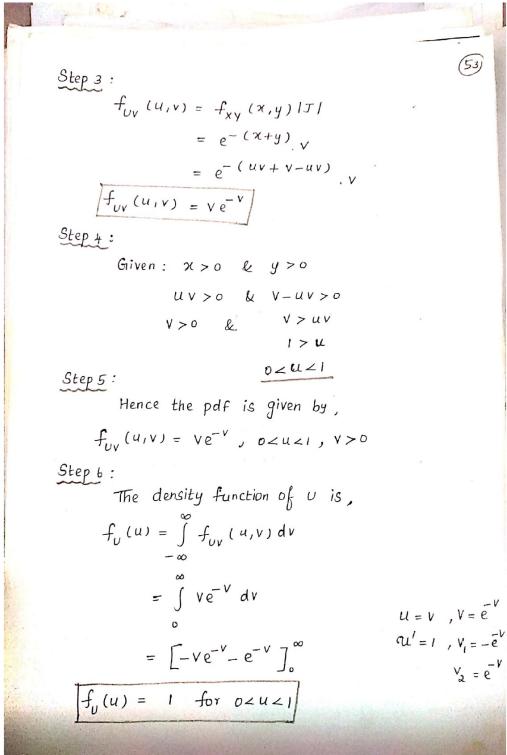
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