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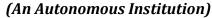


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A continuous random variable 'X' is said to follow an exponential distribution with parameter x >0 if its probability density function (p.d.f) is given by, $f(\mathbf{x}) = \begin{cases} \alpha e^{-\alpha \mathbf{x}} , \ \alpha \ge 0 \\ 0 , \text{ otherwise} \end{cases}$ Moment generating function : $M_{x}(t) = E(e^{tx})$ $= \int e^{tx} f(x) dx$ = $\int_{0}^{\infty} e^{tx} de^{-\alpha x} dx$ [: x = 0] $= \alpha \int_{e}^{\infty} \overline{e}^{(t+\alpha)x} dx$ $= \alpha \left[\frac{-(t+\alpha)x}{-(t+\alpha)x} \right]^{\infty}$ $= \frac{\alpha}{t \neq \alpha} \left[\frac{e^{2}}{e^{2}} - e^{2} \right]$ $= \frac{\alpha}{4t \, \overline{\bullet} \, \alpha} \left[0 - 1 \right]$ $= + \alpha$ $+ + \alpha$

P.GOMATHI/AP/MATHEMATICS







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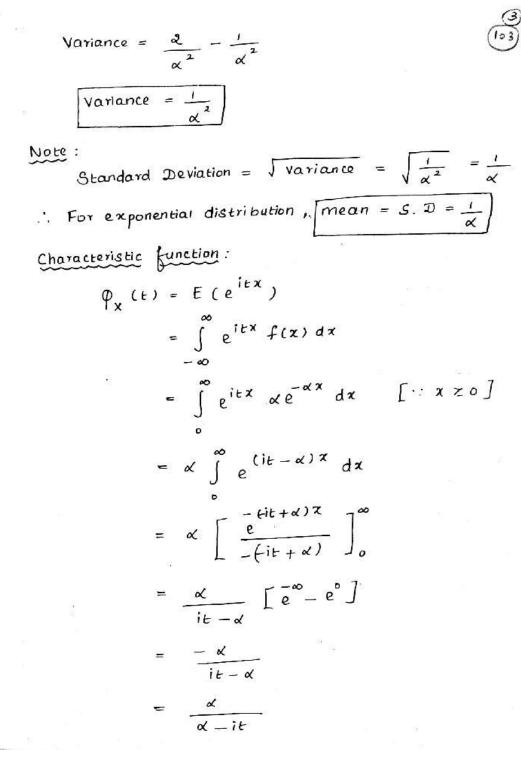
 $M_{\chi}(t) = \frac{\alpha}{\alpha - t}$

Dividing numerator and denominator by 'a', we get, $M_{\chi}(t) = \frac{1}{1 - \frac{t}{\alpha}} \qquad M_{\chi}(t) = \alpha(\alpha - t)^{-1}$ $M_{\chi}(\alpha) = \alpha(-1)(\alpha - t)^{-2}(-1)(\alpha - t)^{-2}(-1)(-1)(\alpha - t)^{-2}(-1)(\alpha - t)^{-2}(-1)(-1)(\alpha - t)^{-2}(-1)(-1)(\alpha - t)^{-2}(-1)(\alpha - t)^{-2}(-1)(-1)(\alpha - t)^{-2}(-1)(-1)(\alpha - t)^{-2}(= 1 + \frac{t}{\alpha} + \frac{t^{2}}{\alpha^{2}} + \dots + \frac{t^{\gamma}}{\alpha^{\gamma}} + \dots$ $M_{\chi}(t) = \frac{\infty}{\sum_{Y=0}^{\infty}} \left(\frac{t}{\alpha}\right)^{\gamma} \qquad M_{\chi}^{"}(t) = \omega \chi (\alpha - t)^{-3}$ $= 2\alpha (\alpha - t)^{-3}$ an and variance: $M_{\chi}^{"}(0) = \frac{2\alpha}{\alpha^{3}} = \frac{2}{\lambda^{1}}$ Mean and variance : $\mu'_{r} = E(x^{r}) \qquad \text{Var} = \left(\frac{2}{\pi^{2}}\right) - \left(\frac{1}{\pi}\right)^{2} \qquad \frac{2}{\pi^{2}}$ = Coefficient of $\frac{t^{\gamma}}{\gamma I}$ in $M_{\chi}(t) = \frac{1}{\chi^2}$ $= \frac{\gamma!}{\sqrt{\gamma}}, \gamma = 1, 2, \cdots$ $Mean = \mu'_1 = \frac{1}{\alpha}$ $\mu_{2}^{\prime} = \frac{2!}{\sqrt{2}}$ Variance = $\mu_2' - (\mu_1')^2$



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ан В са If 0 < 0(21, Variance > Mean $\alpha = 1$, Variance = Mean ≪>1 , Variance ∠ Mean Hence for an exponential distribution, Variance > = , or ~ Mean for different values of the parameter. Exponential distribution lacks memory:) If x is exponentially distributed with parameter X, then for any two positive integens 's' and 't', P[x>s+t/x>s] = P[x>t] proof : The p.d.f of x is, $f(x) = \begin{cases} x e^{-\alpha x}, & x \ge 0\\ 0, & \text{otherwise} \end{cases}$ Consider $P[x > t] = \int_{0}^{\infty} f(x) dx$ $= \int_{E}^{\infty} de^{-dx} dx$ $= \alpha \left[\frac{e^{-\alpha x}}{-\alpha} \right]_{L}^{\infty}$ $= - \left[e^{-\omega} - e^{-\omega t} \right]$



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$$P(x > t) = e^{-\alpha t} \longrightarrow 0$$

Consider,

$$P[x > \delta + t | x > \delta] = P[x > \delta + t | x > \delta]$$

$$= \frac{P(x > \delta + t)}{P(x > \delta)}$$

$$= \frac{e^{-\alpha (\delta + t)}}{e^{-\delta \alpha}} (using 0)$$

$$= e^{-\delta d} e^{-\alpha t} e^{-\delta d}$$

$$= e^{-\alpha t} \longrightarrow 0$$

From () and (2),

$$P[x > \delta + t | x > \delta] = P[x > t]$$
Thus exponential distribution lacks memory.

 The mileage which car owners get with certain kind of radial type is a random variable having an exponential distribution with mean 4000 km. Find the probabilities that one of these tires will last
 (i) atleast 2000 km (ii) atmost 3000 km. Solution:

Problems : [Memoryless Property of Exponential Distribution

Given: $x_{1} = \frac{1}{400}$ Mean = $\frac{1}{4} = \frac{1}{4000}$ Then $f(x) = de^{-dx}$ $f(x) = \frac{1}{4000}e^{-x/4000}$, x > 0

(i) Atleast 2000 km

$$P[x > 2000] = \int f(x) dx$$

2000

$$\int_{-\frac{1}{4000}}^{\infty} \frac{-x/4000}{dx} dx$$

$$= \frac{1}{4000} \left[\frac{\frac{-x}{4000}}{\frac{-1}{4000}} \right]_{2000}^{\infty}$$
$$= - \left[e^{\infty} - e^{-\frac{1}{2}} \right].$$

$$P [x > 2000] = e^{-0.5}$$

$$P[x > 2000] = 0.6665$$
(ii) Atmost 3000 km:

$$P(x \le 3000) = \int f(x) dx$$

$$= \int_{0}^{3000} e^{-x/4000} dx$$

$$= \int_{-1/4000}^{-1} e^{-x/4000} \int_{0}^{3000}$$

$$= - [-e^{0} + e^{-3/4}]$$

$$= 1 - e^{-0.45}$$

$$P(x \le 3000) = 0.5270$$
(2) For an exponential distribution with mean 120 days,
find the probability that such a watch will
(i) have to be set in less than 24 days and
(ii) not have to be reset in atleast 180 days
Solution:
Given : Mean = $\frac{1}{x} = \frac{120}{120}$

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