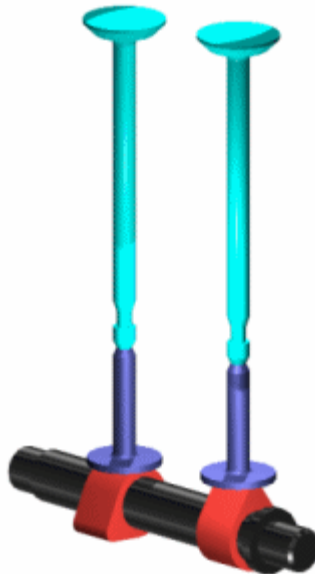


# CAM MECHANISMS

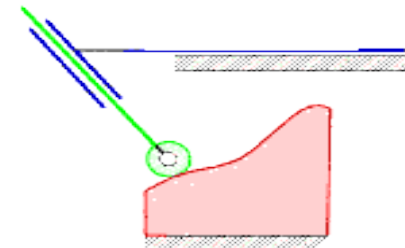
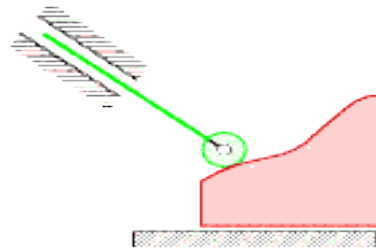
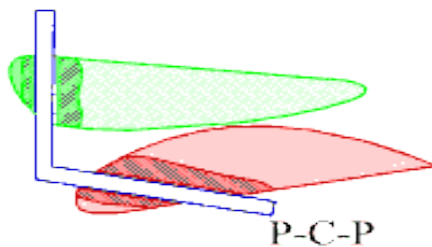
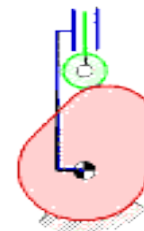
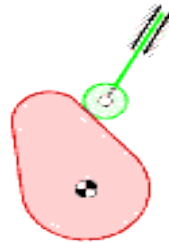
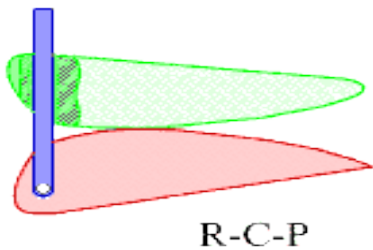
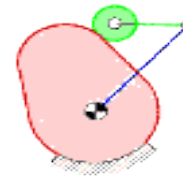
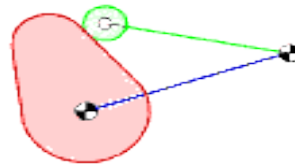
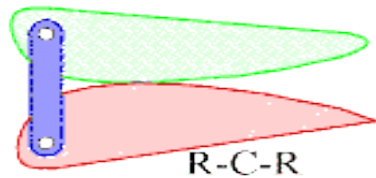


Cam makes a higher kinematic pair with follower. Cam mechanisms are widely used because with them, different types of motion can be possible. Cams can provide unusual and irregular motions that may be impossible with the other types of mechanisms.

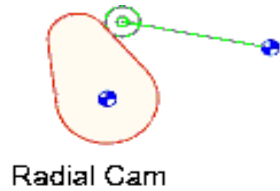
However, the manufacturing of cams is expensive and the wear effect due to the contact stresses is a disadvantage. On the other hand, cams are not proper for the systems with high speeds and heavy loads.

# Classification of Cams

According to **kinematic structure**:



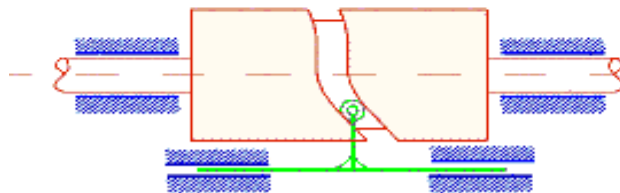
# According to **shape of the cam:**



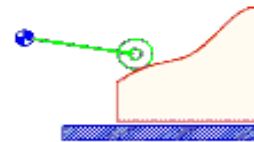
Radial Cam



Face Cam



Cylindrical Cam

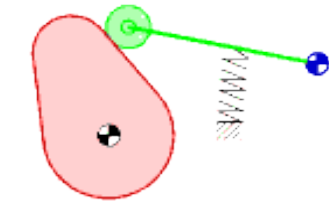


Wedge Cam

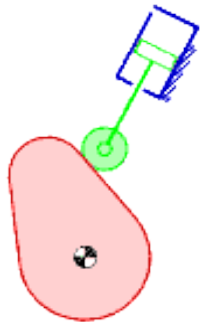
Spherical,  
Helical,  
Conical,  
3 dimensional

According to the **type of contact:**

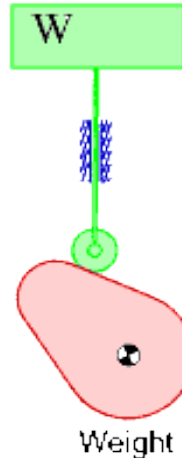
Force closed



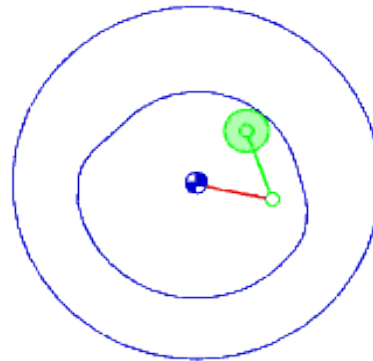
Spring Force



Hydraulic or  
Pneumatic Pressure

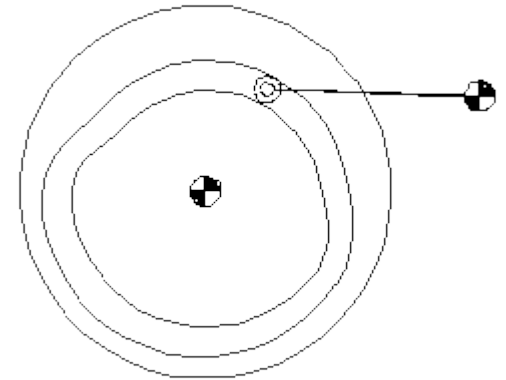


Weight



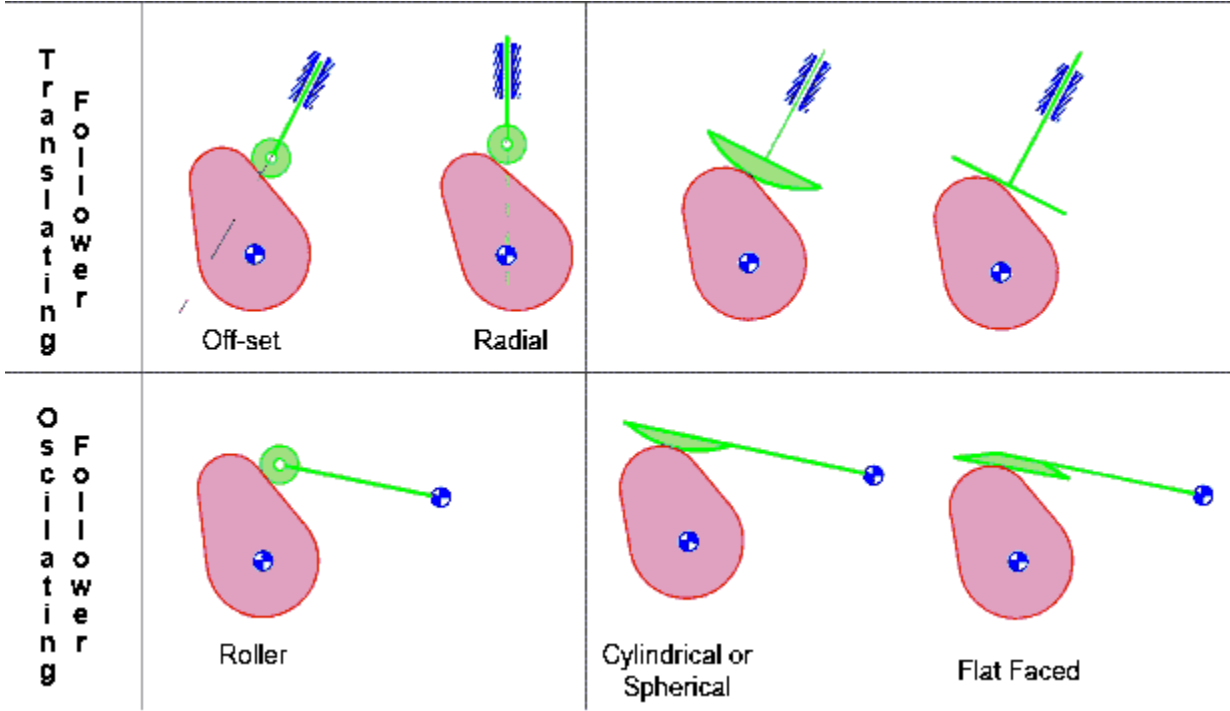
Centrifugal Force

Form closed

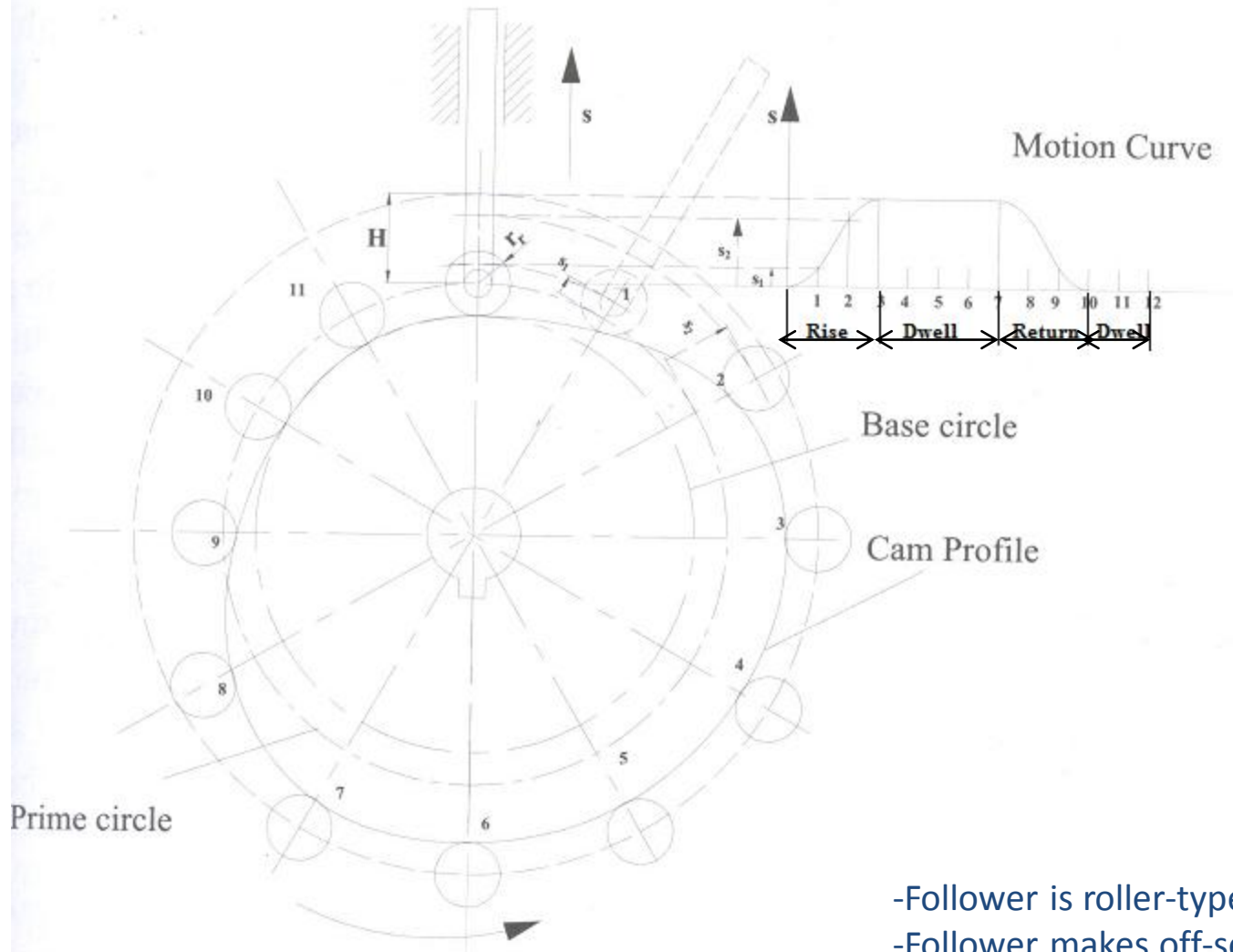


# Classification of Followers

- According to **the construction of the surface in contact**(roller, flat faced, cylindrical, spherical)
- According to **the type of movement** (translating or oscillating)
- The translating follower is also classified as **radial or off-set** according to the location of movement with respect to the cam rotation axis.



## Drawing Cam Profile for a Given Motion Curve of the Follower



- Follower is roller-type
- Follower makes off-set translation
- Follower radius is given
- Base circle radius is given



# BASIC CAM MOTION CURVES

- In this section some well known motion curves will be explained for the rise portion of the motion curve only.

- 1. Linear Motion :**

- Equation describing a linear motion with respect to time is:

$$s = a_1 t + a_0$$

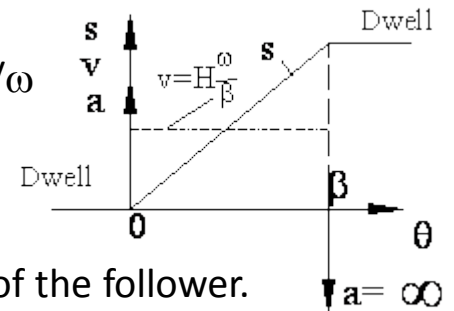
Assuming constant angular velocity for the input cam ( $\omega$ ), since  $t = \theta/\omega$

$$s = a_1 \theta/\omega + a_0$$

Let  $H =$  Total follower rise (Stroke)

$\beta =$  angular rotation of the cam corresponding to the total rise of the follower.

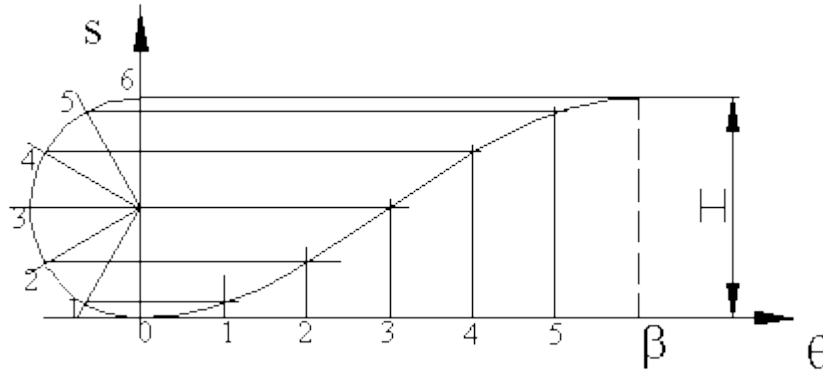
$$s = H \frac{\theta}{\beta} \quad v = H \frac{\omega}{\beta} \quad a = 0$$



- The motion curve and velocity and acceleration curves are as shown. Note that the acceleration is zero for the entire motion ( $a=0$ ) but is infinite at the ends. Due to infinite accelerations, high inertia forces will be created at the start and at the end even at moderate speeds. The cam profile will be discontinuous.

**One basic rule in cam design is that this motion curve must be continuous and the first and second derivatives (corresponding to the velocity and acceleration of the follower) must be finite even at the transition points.**

## 2. Simple Harmonic Motion



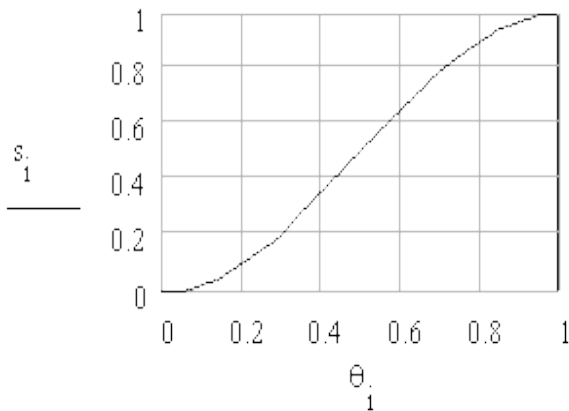
- Simple harmonic motion curve is widely used since it is simple to design. The curve is the projection of a circle about the cam rotation axis as shown in the figure. The equations relating the follower displacement velocity and acceleration to the cam rotation angle are:

$$s = \frac{1}{2} H \left( 1 - \cos\left(\frac{\pi\theta}{\beta}\right) \right)$$

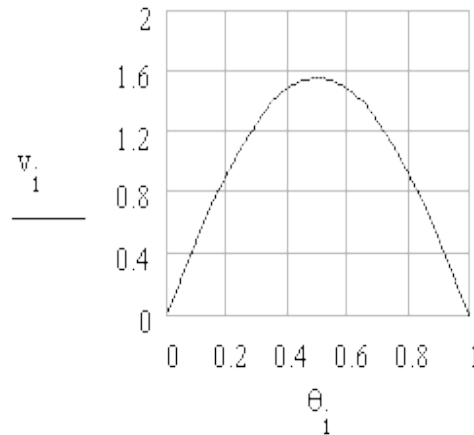
$$v = \frac{1}{2} \frac{H\pi\omega}{\beta} \sin\left(\frac{\pi\theta}{\beta}\right)$$

$$a = \frac{1}{2} H \left( \frac{\pi\omega}{\beta} \right)^2 \cos\left(\frac{\pi\theta}{\beta}\right)$$

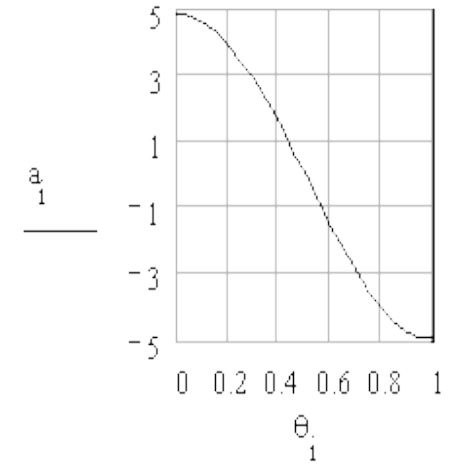
## 2. Simple Harmonic Motion(cont'd)



(a)



(b)

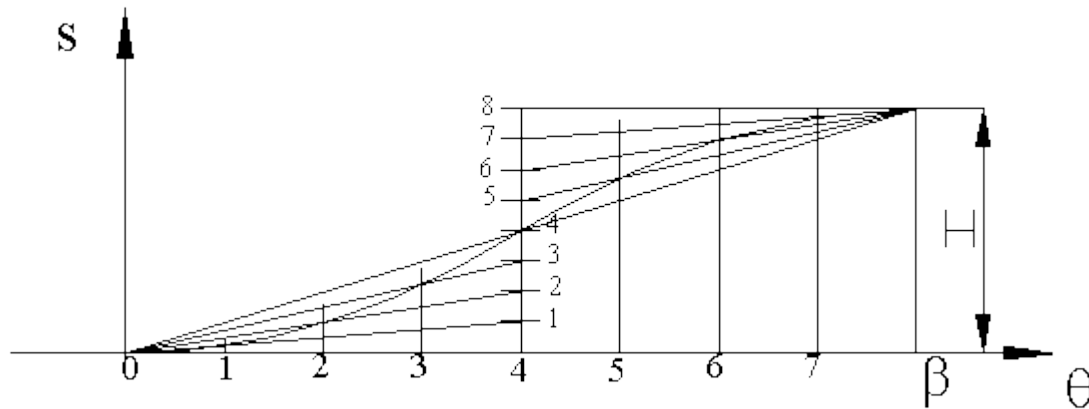


(c)

- $V_{\max} = \frac{1}{2} H \frac{\pi \omega}{\beta}$
- $a_{\max} = \frac{1}{2} H \left( \frac{\pi \omega}{\beta} \right)^2$

Note that even though the velocity and acceleration is finite, the maximum acceleration is discontinuous at the start and end of the rise period. Hence the third derivative, jerk, will be infinite at the start and end of the rise portion. This curve will not be suitable for high or moderate speeds.

# 3. *Parabolic or Constant Acceleration Motion Curve*



- For the range  $0 < \theta < \beta/2$

$$s = 2H \left( \frac{\theta}{\beta} \right)^2$$

$$v = 4H\omega \left( \frac{\theta}{\beta^2} \right)$$

$$a = 4H \left( \frac{\omega}{\beta} \right)^2$$

- For the range  $\beta/2 < \theta < \beta$

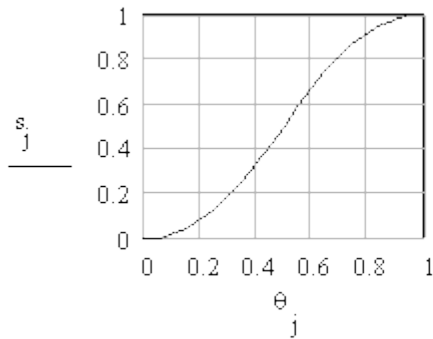
$$s = H \left[ 1 - 2 \left( 1 - \frac{\theta}{\beta} \right)^2 \right]$$

$$v = 4H \frac{\omega}{\beta} \left( 1 - \frac{\theta}{\beta} \right)$$

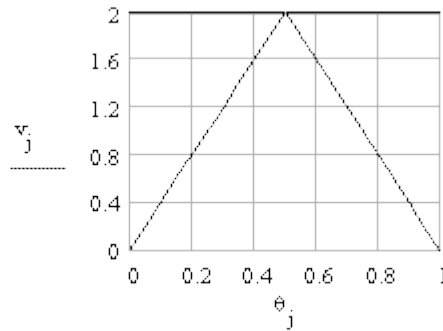
$$a = -4H \left( \frac{\omega}{\beta} \right)^2$$

# 3. *Parabolic or Constant Acceleration Motion Curve (cont'd)*

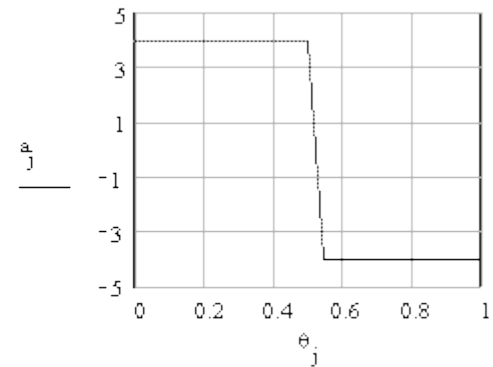
- In this case the velocity and accelerations will be finite. There is a constant acceleration for the first half and a constant deceleration in the second half of the cycle. However the third derivative, jerk, will be infinite at the two ends as in the case of simple harmonic motion. Displacement, velocity and acceleration curves are as shown. This motion curve has the lowest possible acceleration.



(a)



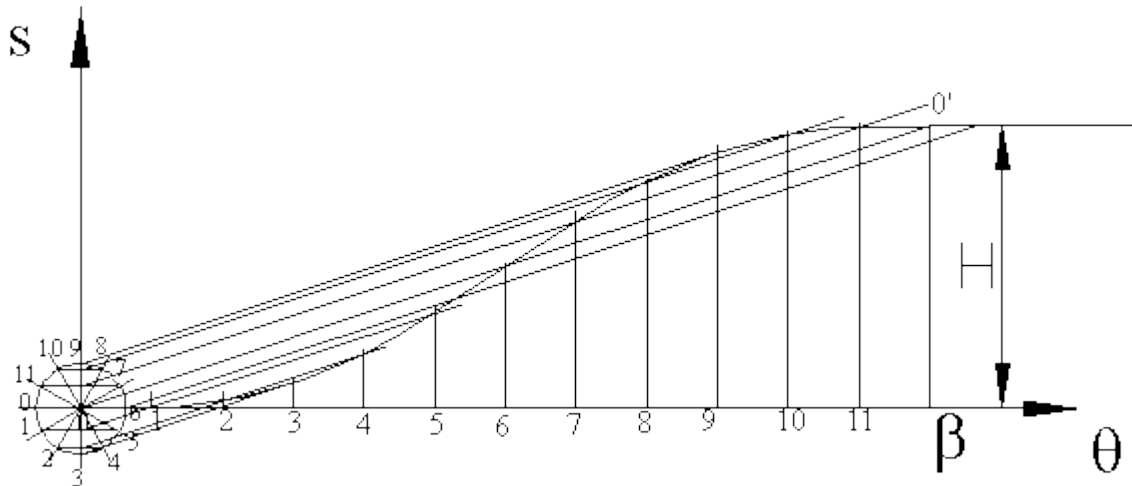
(b)



(c)

# 4. Cycloidal Motion Curve

- If a circle rolls along a straight line without slipping, a point on the circumference traces a curve that is called a *cycloid*. This curve can be drawn by drawing a circle with center C on the line OO'. The circumference of the circle is equal to the total rise; or the diameter is  $H/\pi$ . The circumference is divided into a number of equal parts corresponding to the divisions along the horizontal axis. The points around the circle are first projected to the vertical centerline of the circle and then parallel to OO' to the corresponding vertical line on the diagram.



# 4. Cycloidal Motion Curve (cont'd)

- The equations relating the follower displacement, velocity and acceleration to the cam rotation angle are:

$$s = \frac{H}{\pi} \left[ \frac{\pi\theta}{\beta} - \frac{1}{2} \sin\left(\frac{2\pi\theta}{\beta}\right) \right]$$

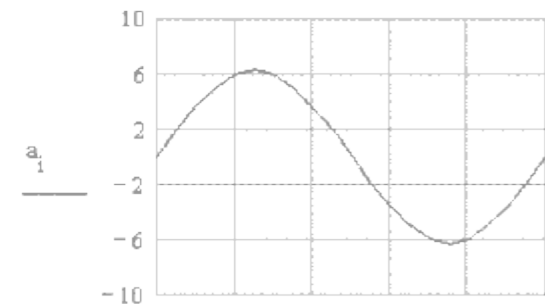
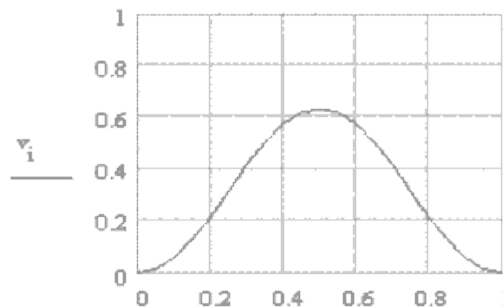
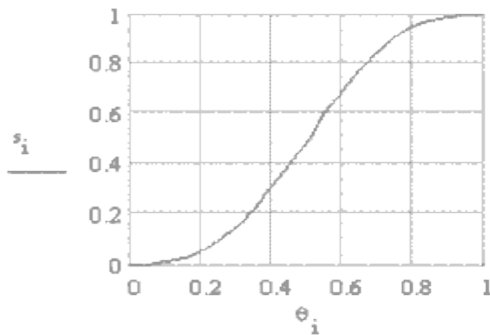
$$V = \frac{H\omega}{\beta} \left[ 1 - \cos\left(\frac{2\pi\theta}{\beta}\right) \right]$$

$$a = \frac{2H\pi\omega^2}{\beta^2} \sin\left(\frac{2\pi\theta}{\beta}\right)$$

$$v_{\max} = \frac{2H}{\beta} \omega$$

$$a_{\max} = \frac{2H\pi}{\beta^2} \omega^2$$

Within the curves we have thus far seen, cycloidal motion curve has the best dynamic characteristics. The acceleration is finite at all times and the starting and ending acceleration is zero. It will yield a cam mechanism with the lowest vibration, stress, noise and shock characteristics. Hence for high speed applications this motion curve is recommended.



# Normalization of the motion curves

- In order to compare the motion curves that were discussed we take,
- $\omega=1$  rad/s  
H= 1 unit  
 $\beta= 1$  radian
- This procedure is known as “***normalization***”. Using this procedure one can then easily compare all these curves with respect to each other. This comparison is shown in the Figure.  $C_v$ ,  $C_a$  and  $C_j$ , are the maximum velocity, acceleration and jerk values for the normalized curves. One can determine the maximum velocity, acceleration and jerk for any H,  $\omega$  and  $\beta$  as:



$$v_{\max} = C_v \frac{H}{\beta} \omega; \quad a_{\max} = C_a \frac{H}{\beta^2} \omega^2; \quad j_{\max} = C_j \frac{H}{\beta^3} \omega^3$$

Also the equations given for the normalized motion curves can be converted for any rise H, angular velocity  $\omega$  and crank rotation  $\beta$  by multiplying the equation by H and substituting  $\theta/\beta$  instead of  $\theta$ .

For simple harmonic motion:  
 $s = 1/2(1 - \cos(\pi\theta))$  (from Table)

multiply this equation by H and substitute  $\theta/\beta$  instead of  $\theta$ ,

$$s = H/2(1 - \cos(\pi\theta/\beta)) \text{ (the real eq'n)}$$

$$V_{\max} = (\pi/2) H\omega/\beta = 1,57 H\omega/\beta = C_v H\omega/\beta$$

$$a_{\max} = \frac{1}{2} H \left( \frac{\pi\omega}{\beta} \right)^2 = C_a H\omega^2/\beta^2 = 4,93 H\omega^2/\beta^2$$

Motion Curve	Velocity	Acceleration	Jerk	Note
Parabolic $0 < \theta < 1/2$ $s = 2\theta^2$ $1/2 < \theta < 1$ $s = 1 - 2(1-\theta)^2$	 $C_v = 2$	 $C_a = 4$	 $C_j = \infty$	Has Backlash problems. Lowest Max. Acc.
Cubic #1 $0 < \theta < 1/2$ $s = 4\theta^3$ $1/2 < \theta < 1$ $s = 1 - 4(1-\theta)^3$	 $C_v = 3$	 $C_a = 12.5$	 $C_j = \infty$	Not recommended
Cubic #2 $s = 3\theta^2 - 2\theta^3$	 $C_v = 1.5$	 $C_a = 6$	 $C_j = \infty$	Not suitable for high speeds
Simple harmonic $s = 1/2(1 - \cos(\pi\theta))$	 $C_v = 1.57$	 $C_a = 4.93$	 $C_j = \infty$	Good for low or medium speeds.
Cycloid $s = \frac{1}{\pi}(\pi\theta - \frac{1}{2}\sin(2\pi\theta))$	 $C_v = 2$	 $C_a = 4.93$	 $C_j = 39.5$	Good for high speeds. Requires precision machining.
Double harmonic $s = 1/4 - 1/2\cos(\pi\theta) + 1/4\cos(2\pi\theta)$	 $C_v = 2$	 $C_a = 9.9$	 $C_j = \infty$	Good for dwell-rise-return motion
Trapezoidal Acceleration	 $C_v = 2$	 $C_a = 5.3$	 $C_j = 44$	Good for high speeds. Requires precision machining

# Cam Size Determination

- Cam size determination is related to the determination of the base circle. In almost all applications it is required to minimize the size of the cam. Large cams are not desired due to the following reasons:
  1. More space is required.
  2. Unbalanced mass increases
  3. Follower has a longer path for each cycle. Therefore, angular velocity of the follower and the surface velocity increases.

However as we decrease the cam size, the following factors arise:

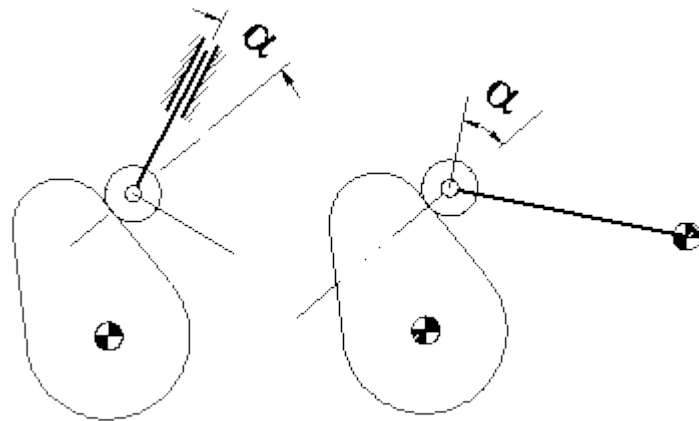
1. The force transmission characteristics deteriorate. The cam profile becomes more steep.
  2. The curvature of the cam profile decreases (sharp curves)
  3. Strength requirements due to the forces and moments acting on the cam.
- In practice the cam size is determined by considering two factors:
    - a) The pressure angle
    - b) The minimum radius of curvature

# Pressure Angle

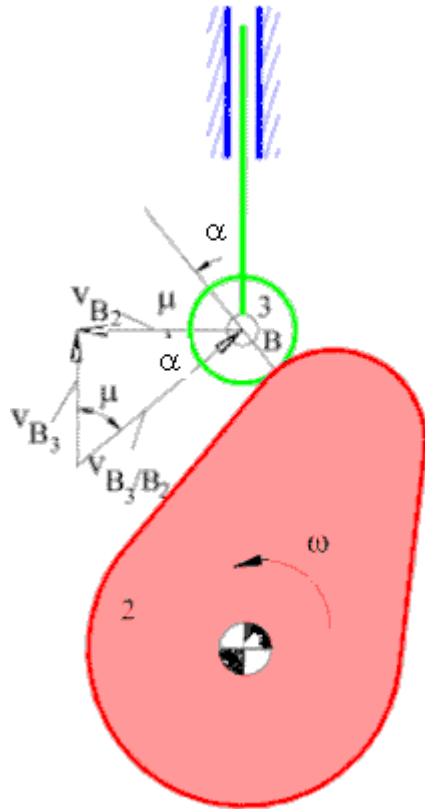
- The **pressure angle**, which is the reciprocal of the transmission angle  $\mu$  (i.e.  $\alpha = \pi/2 - \mu$ ) is defined as:

- $$\tan \alpha = \frac{\text{Force component tending to apply pressure on the follower bearings}}{\text{Force component tending to move the follower.}}$$

In cams there is point contact and the force is transmitted along the common normal of the two contacting curves. Cams with roller followers have pressure angles whereas with flat faced followers don't have.



# Pressure Angle (cont'd)



The pressure angle is a function of cam rotation angle. The force ratio for the pressure angle may be expressed by using the kinematics of the mechanism.

**For an inline translating roller follower radial cam**

$$\mathbf{v}_{B_3} = \mathbf{v}_{B_2} + \mathbf{v}_{B_3/B_2}$$

$$v_{B_3} = \text{Follower velocity} = ds/dt = \dot{s}(\theta)$$

$$v_{B_2} = \text{Velocity of point B on the cam} = r\omega$$

( $r = OB = s + r_b + r_r$ .  $r_b$  is the base circle radius,  $r_r$  is the roller radius)

The relative velocity direction is along the path tangent.

$$\tan \alpha = \frac{v_{B_3}}{v_{B_2}} = \frac{ds/dt}{r\omega} = \frac{ds/d\theta}{r} = \frac{s'(\theta)}{s(\theta) + r_b + r_r}$$

# Pressure Angle (cont'd)

$$\tan \alpha_{\max} = \frac{\left(\frac{ds}{d\theta}\right)_{\max}}{\frac{H}{2} + r_b + r_f}$$

$$\left(\frac{ds}{d\theta}\right)_{\max} = C_v \frac{H}{\beta}$$

This value can easily be determined from the motion curve.  $r_r$  may be determined from strength considerations

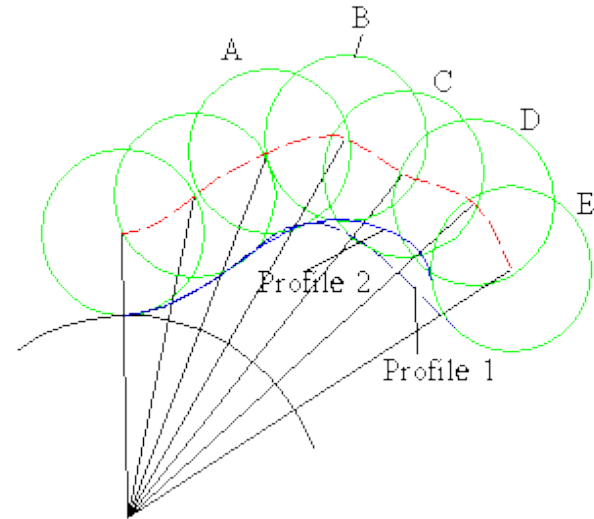
- The designer selects an acceptable maximum pressure angle and finds the base circle radius.
- Maximum pressure angle usually depends on the speed, load and place of application of the cam mechanism. In general,
- - If the cam speed is less than 30 rpm:  $\alpha_{\max} \leq 45^\circ$
  - If the cam speed is greater than 30 rpm:  $\alpha_{\max} \leq 60^\circ$

# Cam Curvature

- In practice for roller followers it is common to determine the cam size using the maximum pressure angle criteria and then check that the cam curvature is satisfactory. In case of flat faced followers, the cam curvature is the determining criteria for the cam size. Graphically when laying out the cam profile, first the successive positions of the follower according to the cam motion curve is drawn while keeping the cam fixed.

# Cam Curvature (cont'd)

For example, for a cam with the selected roller radius and base circle radius, the positions of the roller are A,B,C,D and E. However, it is not possible to draw a smooth curve that is tangent to all the roller circles. With Profile#1, the cam profile is tangent to the circles at positions A,C and E, and in Profile#2 the cam profile is tangent to positions A,B and E. There is no



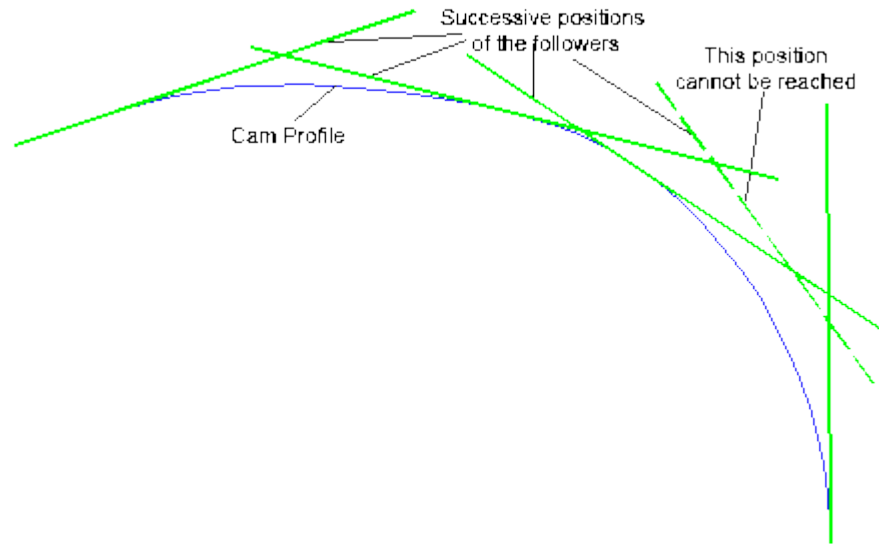
cam profile that will be tangent to all the positions of the roller. This is known as **“undercutting”** and occurs whenever the radius of curvature of the cam profile is less than the radius of the roller ( $\rho_p < r_r$ ). In order to eliminate this condition a larger base circle radius is selected or, if strength conditions permit, the radius of the roller is reduced.

# Cam Curvature (cont'd)

- For roller followers the following recommendations may be made to avoid undercutting:
- Use smaller roller diameter (this is limited by the contact stress at the surface)
- Utilize a larger cam size (this is usually not desired. It must be applied if necessary)
- Employ an internal cam (the curvature is less critical but they are more expensive to manufacture)



# Cam Curvature (cont'd)



A similar case is shown in case of flat faced followers. The cam profile is not tangent to all the successive positions of the follower.