Chapter 1 Static Force Analysis

When the inertia forces are neglected in comparison to the externally applied load, one may go for static force analysis. If the body is under equilibrium condition, then this equilibrium is known as static equilibrium and this condition is applicable in many machines where the movement is relatively slow. These include clamps, latches, support linkages, and many hand operated tools, such as pliers and cutters. In case of lifting cranes also, the bucket load and the static weight loads may be quite high relative to any dynamic loads due to accelerating masses and hence one may go for static force analysis.

When the inertia effect due to the mass of the components is also considered, it is called dynamic force analysis.

Applied and Constraint forces:

- When two or more bodies are connected together to form a group or system, the pair of action and reaction forces between any two of the connecting bodies is called constrained forces.
- These forces constrain the connected bodies to behave in a specific manner defined by the nature of the connection.
- Forces acting on this system of bodies from outside the system are called applied forces.

Electric, Magnetic and gravitational forces are example of forces that may be applied without actual physical contact. But most of the forces we are concerned in mechanical equipment occur through direct physical or mechanical contact.

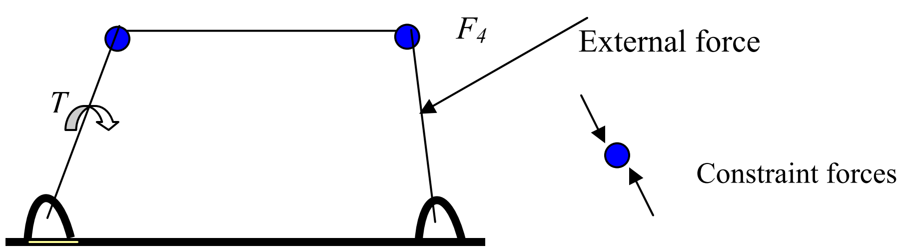


Figure 1: Four bar mechanism showing external and constraint forces

Constraint forces of action and reaction at a mechanical contact occur in pairs and thus have no net force effect on the system of bodies being considered.

When a part of the body is considered in isolation the effect of such force is considered by using the freebody diagram.

Characteristics of a force are its magnitude, its direction and its point of application

Two equal and opposite forces along two parallel but noncollinear straight lines in a body cannot be combined to constitute a single force and they constitute a couple. The arm of the couple is the perpendicular distance between their lines of action and the plane of the couple is the plane containing the two lines of action.

The moment of the couple $\boldsymbol{M}$ is a vector directed normal to the plane of the couple and the sense of $\boldsymbol{M}$ is in accordance to the right-hand rule for rotation.

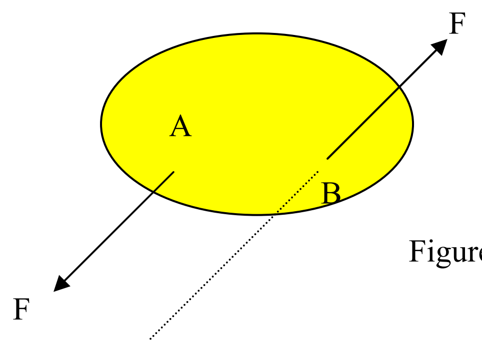


Figure 2

The moment of couple

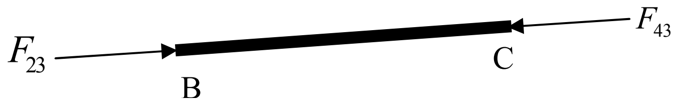
$$\mathbf{M} = \mathbf{R_{BA}} \times \mathbf{F}$$

The value of M is independent of the choice of the reference point about which the moments are taken, because the vector R_{BA} is the same for all positions of the origin.

As the moment vector M is independent of any particular origin or line of application, hence it is a free vector.

Free-body diagram

A free body diagram is a sketch or drawing of the body, isolated from the rest of the machine and its surroundings, upon which the forces and moments are shown in action. In case of the four bar mechanism shown in figure 1 the free body diagram of link 3 is as shown below.



Free body diagram of link 3

When a link or body is subjected to only two forces it is called a **two-force member** and when it is subjected to 3 forces it is called a **three-force member**. Similarly one may consider multi-force member also.

Static equilibrium: A body is in static equilibrium if

- the vector sum of the forces acting on the body is zero i.e., $\sum F = 0$
- the vector sum of all the moments about any arbitrary point is zero i.e., $\sum M = 0$

Hence a two force member as shown in figure 3(a) will be in equilibrium if (i) both forces are equal and opposite and (b) their line of action coincide. If the forces are equal and opposite but not collinear as shown in Figure 3(b) they will form a couple and body will start to rotate. Hence these two forces should be equal, opposite and collinear.

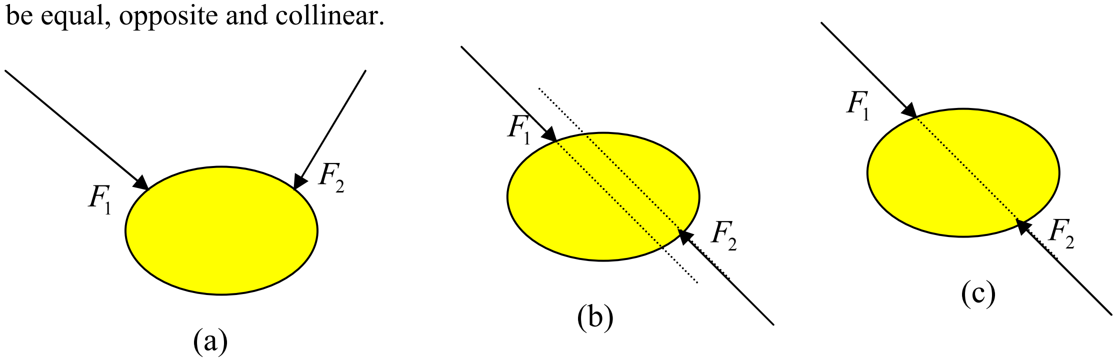


Figure 3. Equilibrium of a two force member

Similarly a three force member will be in equilibrium if the vector sum of all these forces equal to zero and to satisfy the vector sum of all the moments about any arbitrary point equal to zero, their line of action should meet at a point.

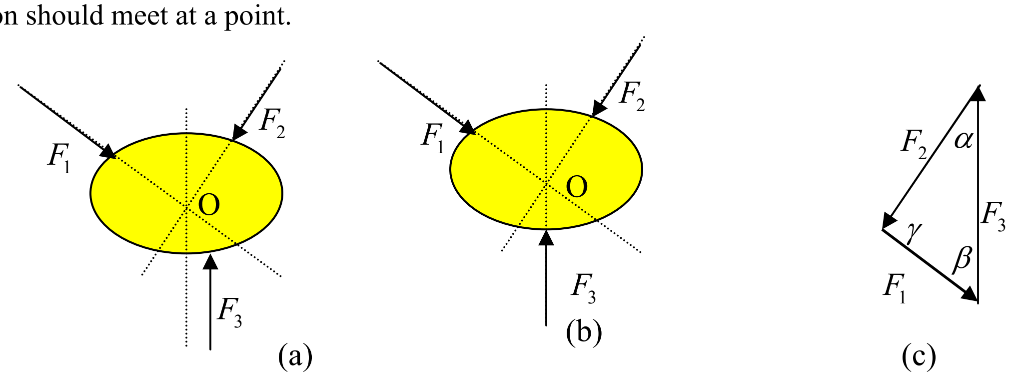


Figure 4: Equilibrium of three-force member

Figure 4(a) shows a body subjected to three forces F_1, F_2 and F_3. Also the line of action of F_1 and F_2 coincide at point O. Hence the resultant of F_1 and F_2 must pass through point O and it should be equal and opposite to force F_3. Hence for equilibrium, line of action of F_3 should pass through point O as shown in Figure 4(b). In figure 4(c) the forces are shown to form a close polygon (triangle) and one may use Lami's theorem (sine rule of tringle) to find the unknown forces if atleast one force is known both in magnitude and direction and the line of action of one more force is known. According to this theorem

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

where α, β and γ are angle as shown in figure 4(c).

For more than three forces one may draw force vector polygon or resolve the forces and moments to get the required force components.

To find the constraint forces in a mechanism one may either go for analytical or graphical method of solution if the maximum number of forces in a member is limited to three and if the system has more than three force members one should go for analytical methods.

Example 1: Find the bearing forces and the torque required for static equilibrium of the four bar mechanism shown in fig 1.

Solution:

Analytical: For Planar mechanism $\sum F_X = 0$, $\sum F_Y = 0$ and $\sum M_Z = 0$.

Step1: Let us first write all the quantities in vector form

$$\vec{R}_{AB} = AB\cos\theta_2\hat{i} + AB\sin\theta_2\hat{j}$$

$$\vec{R}_{CB} = BC\cos\theta_3\hat{i} + Bc\sin\theta_3\hat{j}$$

$$\vec{R}_{DC} = CD\cos\theta_4\hat{i} + CD\sin\theta_4\hat{j}$$

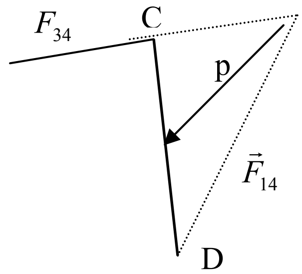
$$\vec{R}_{QD} = DQ\cos\theta_4\hat{i} + DQ\sin\theta_4\hat{j}$$

Free body diagram of link 3

Similarly

$$\vec{P} = P\cos\alpha\hat{i} + P\sin\alpha\hat{j}$$

Here link 3 is a two-force member and at this stage we know only the line of action of the forces F_{23} and F_{43} which should be along the line BC.



Free body diagram of
Link 4

Link 4 is a three-force member in whih force P is completely known and the line of action of force F_{34} which is equal and opposite to F_{43} is known. Only the point of application of force F_{14}, which is at point D, is known. As link 4 is a three force member, taking moment about D,

$$\sum M_Z = 0 \Rightarrow \vec{R}_{CD} \times \vec{F}_{34} + \vec{R}_{QD} \times \vec{P} = 0$$

As P is completely known one may obtain $\vec{F}_{34}$

One may note that link 3 is a two-force member, so $\vec{F}_{23} = -\vec{F}_{43} = \vec{F}_{34}$

Link 2 which is acted upon by two forces i.e., $\vec{F}_{12}$ and $\vec{F}_{32}$, and the external applied torque, will be in equilibrium only if $\vec{F}_{12}$= - $\vec{F}_{32}$, i.e., these forces are equal and opposite and the resulting moment of the couple is equal to the applied torque.

Also one may find the torque by taking moment about point A.

Graphical method

As link 4 is a three force member, the line of action of $\vec{F}_{14}$ should pass through the intersection of the line of action of P and $\vec{F}_{34}$.

Taking proper scale and by drawing the force polygon one may obtain the magnitude of $\vec{F}_{34}$ and $\vec{F}_{14}$.

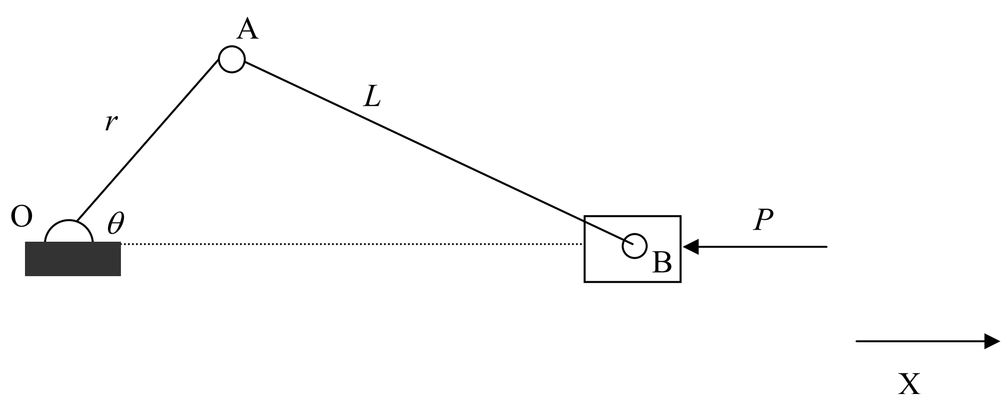
Then considering equilibrium of link 3, force $\vec{F}_{23}$ can be determined.

Then determine the torque taking moment about A.

When multiple forces act on a mechanism, one may use **superposition theory,** which states that in a linear system, the net effect (e.g., bearing forces or torque) due to all the forces taken simultaneously will be equal to the summation of the effects due to individual forces taken one at a time.

If one wishes to find only the torque acting on the mechanism, the method of virtual work may be used. It states **the work performed during a virtual displacement from equilibrium is equal to zero.** The virtual displacement is defined as an imaginary infinitesimal displacement of the system that is consistent with the constraints on the system. For example, the constraints on the slider-crank mechanism are that all members including the frame are rigid and all joints maintain contact

Example 2. Calculate the torque required (assuming no friction in the bearing) for static equilibrium of an in-line reciprocating engine in the position when crank angle θ = 45 deg (from inner dead center). The dimensions are crank length *r* =30 mm, connecting rod length *L* = 70mm, and the piston force is *P* = 40 N.



Solution

Here OB is link 1, crank OA is the 2nd link, connecting rod AB is the 3rd link and the piston is the 4th link.
Crank radius r =30 mm, Length of connecting rod =70 mm

Letting <ABO = β

$$r \sin \beta = L \sin \theta$$

Hence, $\beta = \sin^{-1}\left(\frac{30 \sin 45}{70}\right) = 17.64^0$

Taking the positive X axis as shown in the figure

$$R_{AO} = 30\angle 45 = 30\cos 45\hat{i} + 30\sin 45\hat{j} = 21.213\hat{i} + 21.213\hat{j}$$

$$R_{BA} = 70\angle 342.35 = 70\cos 342.35\hat{i} + 70\sin 342.35\hat{j} = 66.70\hat{i} - 21.213\hat{j}$$

It may be observed that link 3 is a two force member and subjected to forces F_{23}

The free-body diagram of link 4, i.e., that of piston is shown below. For the present case, it is a three-force member subjected to a force P due to gas pressure, vertical reaction force F_{14} and force of connecting rod on piston (F_{34}) at the gudgeon pin. Force P is known completely both in magnitude and direction and the line of action and point of application of force F_{34} is known. Now drawing the force polygon as shown in Figure (b) one will be able to find the unknown forces F_{14} and F_{34}.

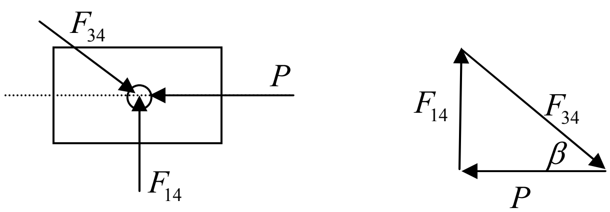


Figure 5 (a) Free-body diagram of link 4 (b) force polygon

Now one may use a vector method or use simple algebraic calculation using Lami's theorem to find the forces. Also one may use graphical method to fid the same. All these methods are described briefly below

Vector method

As $\sum F = 0,$ hence, $P + F_{14} + F_{34} = 0$,

So, $(0.953\hat{i} - 0.303\hat{j})F_{34} - P\hat{i} + F_{14}\hat{j} = 0$

Equating the ith and jth compoment of the forces one may obtain

$$F_{34} = \frac{40}{0.953} = 41.973\text{N}$$
$$F_{14} = 0.304 F_{34} = 12.72\text{N}$$

Hence $F_{34} = 41.973\angle 342.35$ N and $F_{14} = 12.72\angle 90$ N.

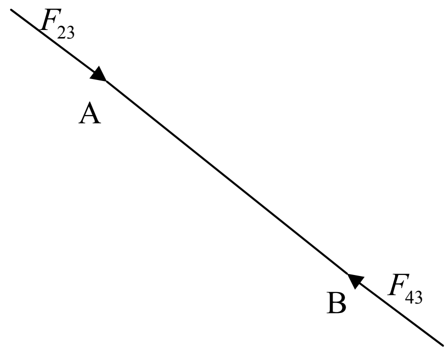
Using **Lami's formula** from the force diagram shown in Figure (b)

$$\frac{F_{34}}{\sin 90} = \frac{F_{14}}{\sin\beta} = \frac{P}{\sin(90-\beta)}$$

Hence

$$F_{34} = \frac{40}{\sin(90-17.64)} = 41.974 \text{ N} \text{ and}$$

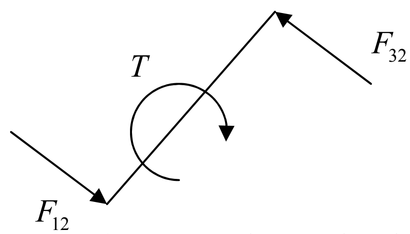
$$F_{14} = \frac{40\sin(17.64)}{\sin(90-17.64)} = 12.72 \text{ N}.$$



Now considering free-body diagram of link 3 $F_{23} = -F_{43}$

But, $F_{43} = -F_{34} = -41.974\angle 342.35$

So $F_{23} = -F_{43} = 41.974\angle 342.35$



Considering equilibrium of link 2

Link 2 is subjected to forces F_{32} and F_{12}. For equilibrium these two forces must be equal and opposite. But as they are acting at A and O respectively they will form a couple which will try to rotate the link OA in anti-clock wise direction. Hence for static equilibrium a torque T must be applied in clockwise direction whose magnitude should be equal to the couple formed by these forces.

Now $F_{32} = -F_{23} = -41.974\angle 342.35 = -40\hat{i} + 12.7265\hat{j}$

$$T = -(R_{AO} \times F_{32}) = -(21.213\hat{i} + 21.213\hat{j}) \times (-40\hat{i} + 12.73\hat{j})$$
$$= -1118.56\hat{k}$$

Negative sign indicate the applied torque should be applied in clock-wise direction.

Static force analysis with friction

As we are considering only simple mechanisms with prismatic and revolute joints, the effect due to dry or Coulomb friction and greasy friction at the journals are discussed. Consider a pair of sliding surfaces as shown in figure **. When a force F is applied on the block to move it towards right, a friction force is generated which oppose this motion. According to Coulomb's law, the magnitude of this force for impending motion is μR, where R is the reaction force due to weight W.

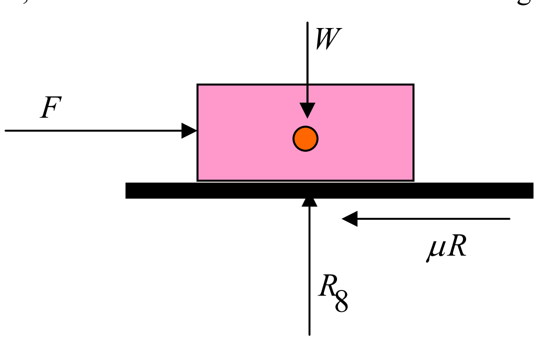


Figure 6

Greasy friction at a journal

Generally greasy or boundary lubrication type friction force occurs in heavily loaded, slow running bearings. Figure 7 (a) shows a journal inside a bearing during static condition. Here A is the contact point and the weight of the journal W and the reaction force R act in the vertical directions as shown in the figure. Now let us consider a torque T is applied to the journal in the clockwise direction. The friction force will now oppose this motion and so the contact point between the bearing and the journal shifts to point B as shown in figure (b). The resultant (R) of the normal reaction force (R_n) and the friction force (μR_n) at B should be equal and opposite to the weight as the journal is under static equilibrium condition. These two forces will form a couple in anticlockwise direction, which will oppose the applied torque.

Let OC be the perpendicular distance between W and R. If one draw a circle with radius OC and center at O, the reaction force will be tangent to that circle. This circle is known as friction circle. Now to find the radius of the friction circle, consider the triangle OBC. Here OC = OB $\sin\varphi$ where φ is the angle between the resultant and normal reaction force. Also the coefficient of friction $\mu = \tan\varphi$. Hence radius of the friction circle = $r_f = r\mu/(\sqrt{(1+\mu^2)})$. where r is the radius of the journal. For small value of μ,

$r_f = r\mu.$ Friction couple = $r_f W = \dfrac{Wr\mu}{\sqrt{1+\mu^2}} \simeq Wr\mu$.

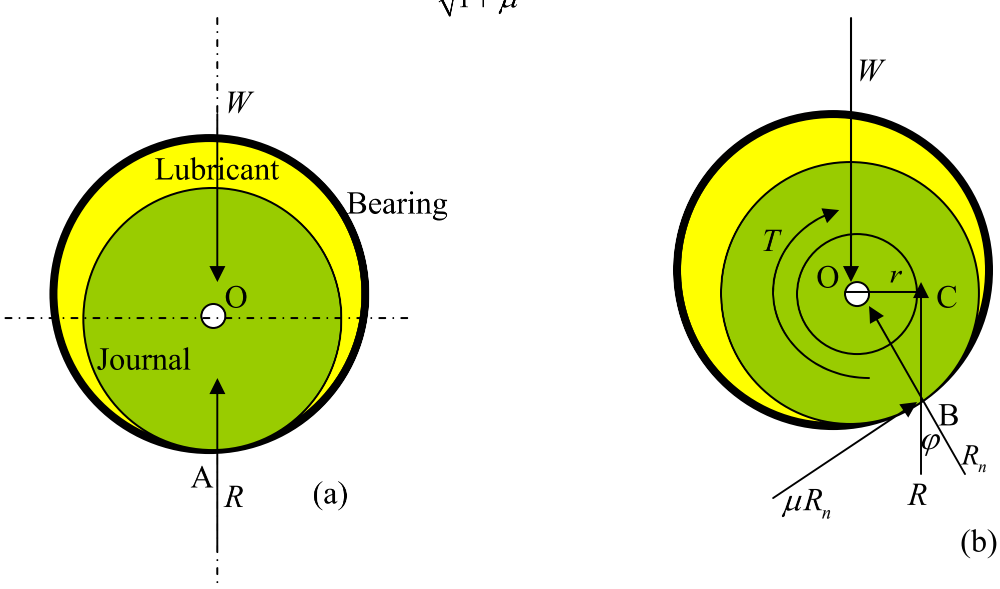
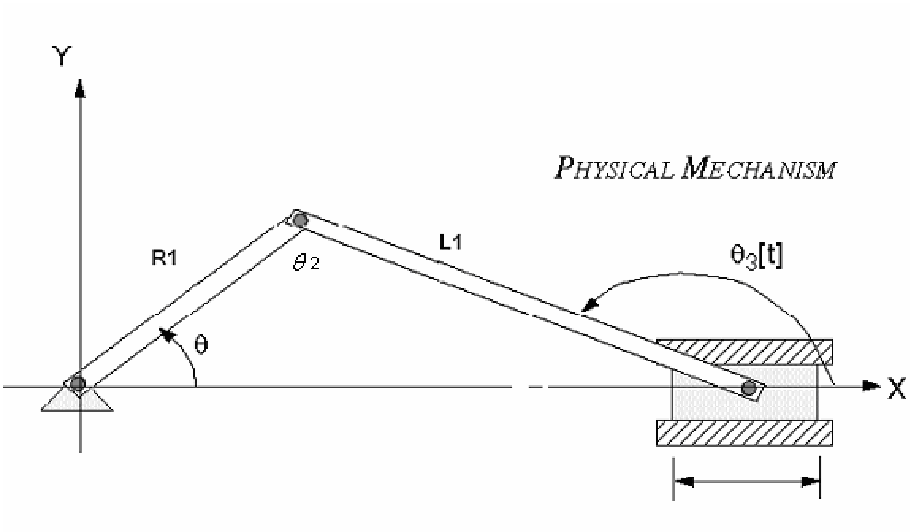


Figure 7 (a) Journal in static condition (b) Journal when a torque is applied to start the motion.

Example 3: Determine the torque required at the crank and also bearing forces in a slider-crank mechanism when the inertia forces are neglected. Also develop a matlab code for the same.



<u>User Specified Parameters</u>

Crank Length = R_1

Connecting Rod Length = L_1

Radius of Journal = R

Coefficient of friction = μ

Piston Force = P

Angle of the crank = θ

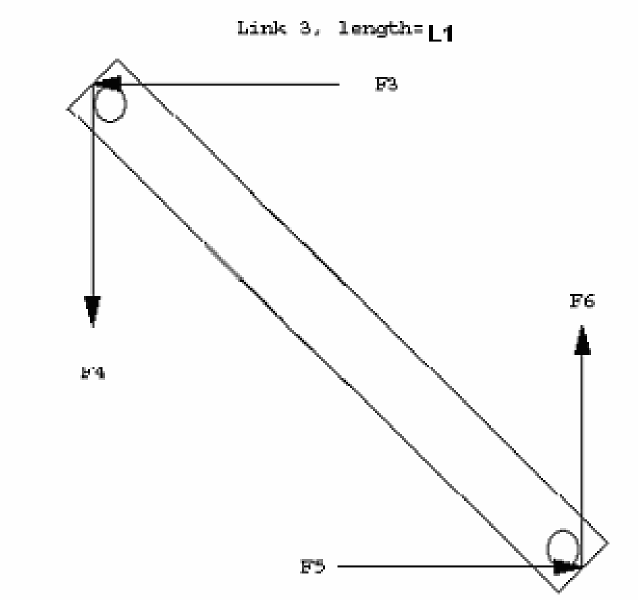
<u>Calculations:</u>

Radius of friction circle = $R_2 = \mu R / \sqrt{(1+\mu)^2}$

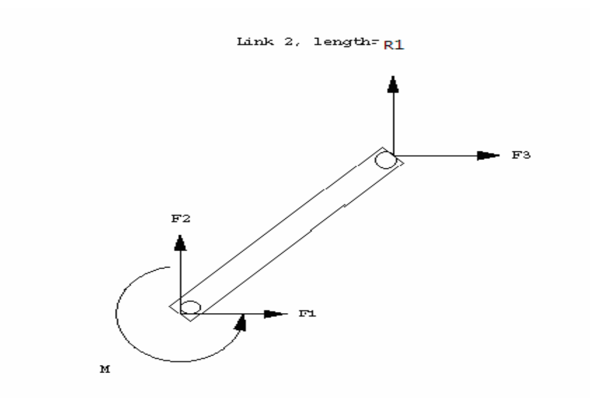
To find angle that connecting rod makes with horizontal θ_3

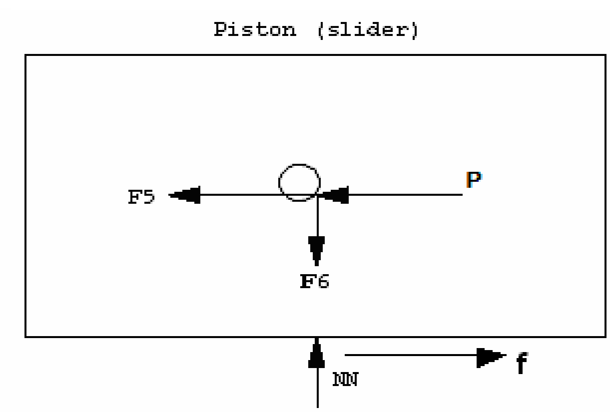
$L_1/\sin\theta = R_1/\sin(180-\theta_3)$

Performing Force analysis on the connecting rod :



Free body diagram of crank:





Free Body diagram of slider:

Force of reaction by connecting rod on slider = **F** (Combination of forces F5 & F6)

Reaction force because of friction between ground and slider = Rxn (combination of Normal reaction NN and friction f)

The following figures show the forces acting on the connecting rod, a combination of forces F5 and F6

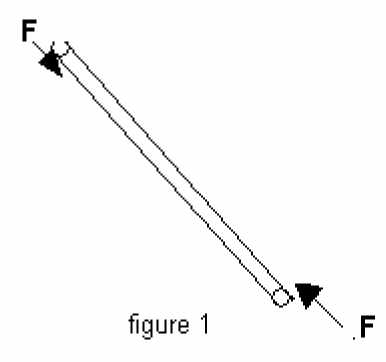


figure 1

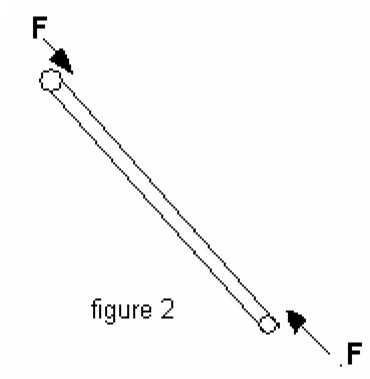


figure 2

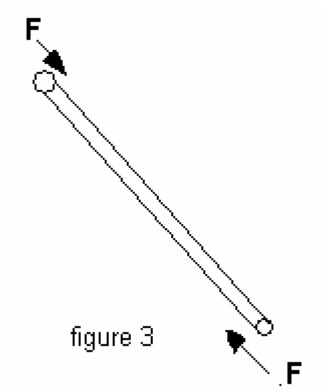


figure 3

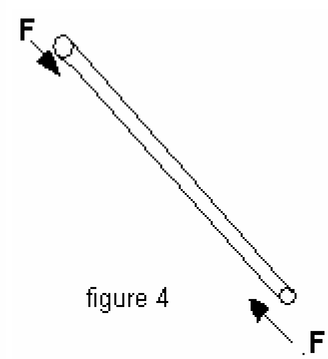


figure 4

Out of these only the figure 1 shows the correct direction of forces. This can be understood by consulting the initial figure. The tendency of the friction in the bearing is to increase ϑ_3 .the tendency of the friction in bearing connecting crank and connecting rod would be to increase θ_2.thus the force direction of the link reaction force can be found out. The angle made by reaction force with connecting rod α can be found out using friction circle radius.

tan $\alpha = R_2/(L_1/2)$

Angle made by reaction force with horizontal = $\theta_3 - \alpha$

Force on bearing connecting crank and connecting rod **F** (*Vector)*

From the code we find the magnitude of the link force and its direction.

Torque due to this force = $\mathbf{R_1}$ x **F** (*Vector Product)*

Matlab code:

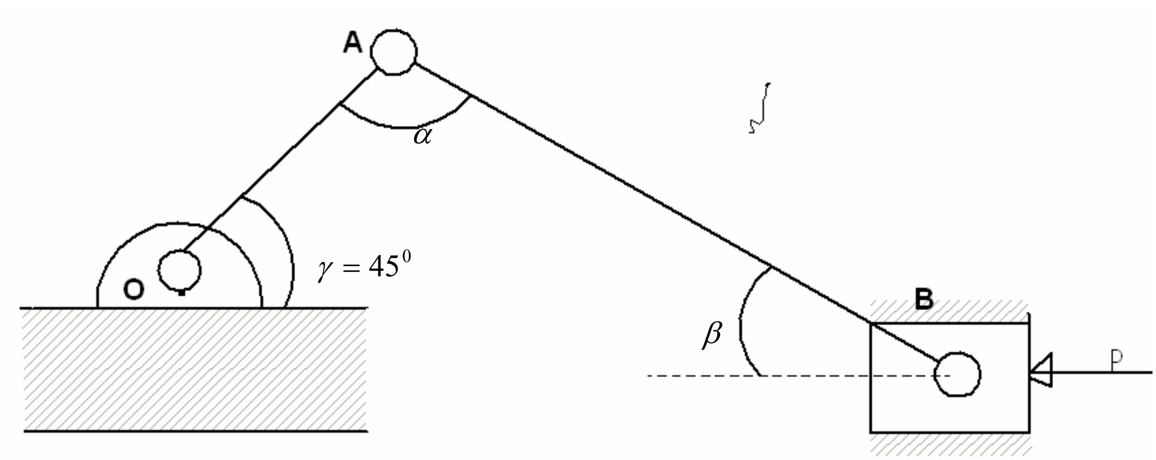
```
R1=0;
while (R1<=0)
R1 = input ('Enter the crank length=');
if (R1<=0) fprintf ('not acceptable value,enter agn')
else break
```

```
end
end;
L1=R1;
while (L1<=R1)
L1=input ('Enter the Connecting Rod Length=');
if (L1<=R1) fprintf ('not acceptable value,enter agn')
else break
end
end;
R=R1;
while (R>=(R1/10))
R=input ('enter the radius of journal=');
if (R>=(R1/10)) fprintf ('not acceptable value, enter agn')
else break
end
end;
C=1
while (C>=1)
C=input ('enter the coefficient of friction=');
if (C>=1) fprintf ('not acceptable value, enter agn')
else break
end
end;
P=input ('enter the piston force=');
Tht= input ('Enter the value of angle considered(degrees)=');
%R2=radius of journal bearing
Tht = Tht*pi/180;
R2= (C*R)/ sqrt (1+C^2)
A= atan ( (2*R2)/L1)
Tht3= asin ((R1/L1)*sin(Tht))
%Angle made with horizontal B
fprintf('the angle made by rxn force with horizontal')
B= 180*(Tht3-A)/pi
fprintf('the value of link rxn force=')
```

```
F=P/(cos(B)+sin(B)*C)

Rxn = (F*sin (Tht))/cos (atan(C));

fprintf('the value of torque=')

T= F*R1*sin (Tht + B)
```

Example 4. Calculate the torque required (assuming no friction in the bearing) for the static equilibrium of an in-line slider crank mechanism in the position when crank angle $\theta = 45^0$ (from the inner dead center). The dimensions are, Crank length =30 cm, Connecting rod length=70 cm and the piston force = 40N. Also find the torque required assuming that the co-efficient for all bearing is 0.1. The three journal bearings all have radii of 10 mm, and the crank is rotating in the clockwise.

Solutions:
Given data:--
$\theta = 45^0$
Crank length (link OA) =30 cm.
Connecting rod length (link AB)=70 cm.
And the piston force (P)= 40N.

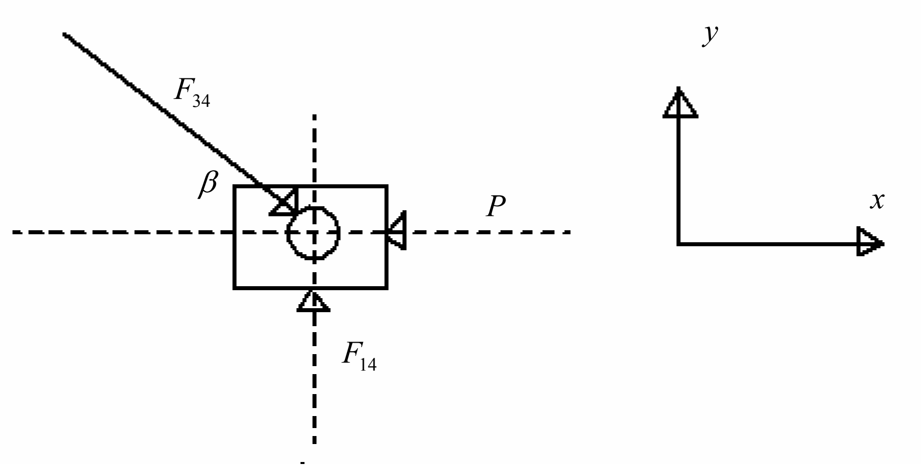


From the figure, and using the sine rule.
We can write,
$AB \sin\beta = OA \sin\gamma$

$$\Rightarrow \sin\beta = \frac{30}{70}\sin 45^0 = 0.303$$

$$\Rightarrow \beta = 17.64^0$$

Case (I):----



Without friction

Considering the link4, and using static force analysis,

$F_{34}\cos\beta = P = 40$

$\& F_{34}\sin\beta = F_{14}$

Therefore,

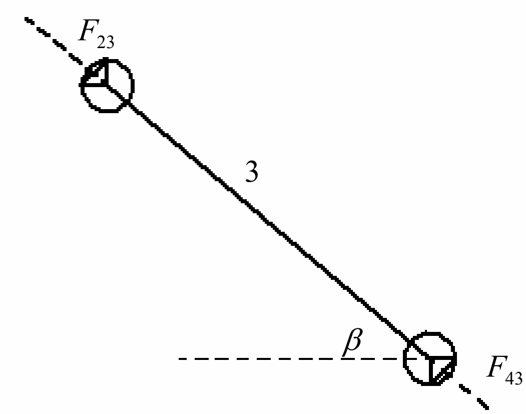
$F_{34} = 41.97N, \& F_{14} = 12.12N$

Also,

$F_{34} = -F_{43}$ (equal and opposite reaction)

$F_{43} = 41.97N = \left|\overline{F_{43}}\right|$

Considering the link 3.



Since link 3 is a 2-force member,

Therefore,

$\overline{F_{23}} = -\overline{F_{43}}$

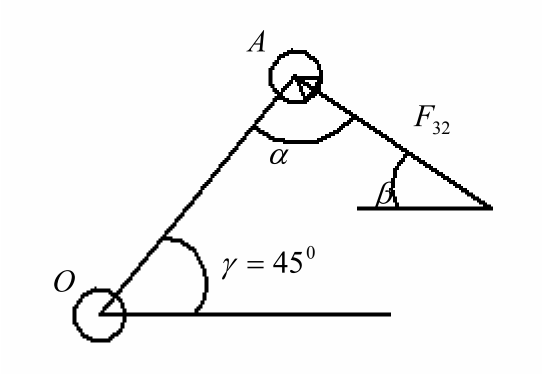
$\Rightarrow \left|\overline{F_{23}}\right| = 41.97N$

And also, we can write,(By equal and opposite reaction)

$\overline{F_{32}} = -\overline{F_{23}}$

$\Rightarrow F_{32} = 41.97N$

Considering link 2.



Now the torque due to reaction force is given by,

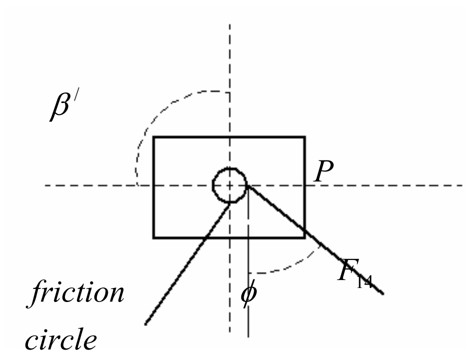
$\overline{T} = \overline{r} \times \overline{F_{32}}$

$\Rightarrow \overline{T} = 0.03(\cos 45^0 \hat{i} + \sin 45^0 \hat{j}) \times 41.97(-\cos\beta\hat{i} + \sin\beta\hat{j})$

$\Rightarrow \overline{T} = -1.112\hat{k}N - m$

Case (II):--

With friction,



Consediring the link 4.

Radius of friction circle, is given by $r_f = \dfrac{\mu r}{\sqrt{1+\mu^2}}$

Where, r= 10, and μ=0.1

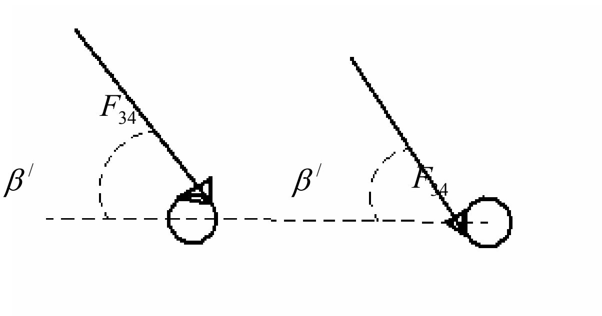
$\Rightarrow r_f = \dfrac{0.1\times 10}{\sqrt{1+(0.1)^2}} \simeq 0.99$

the angle 'ϕ' is given the angle by which the reaction force hift and is determined by $\phi=\tan^{-1}\mu=5.71°$

Now,

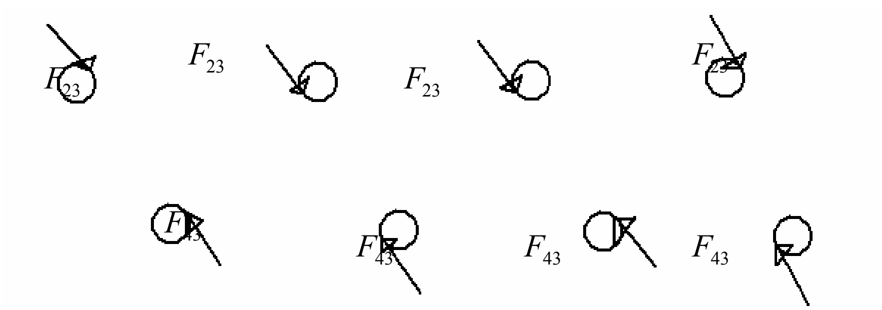
Since the rotation of the crank is clockwise direction, thus the angle r will decreased and simultaneously, angle α, angle β will increased and deceased. Also the piston (link 4)

Will move toward the X-axis (to the right). Thus the direction will be towards left & thus ϕ =+5.71(according to the figure)
Also the force F_{34} can act in two ways shown in below.

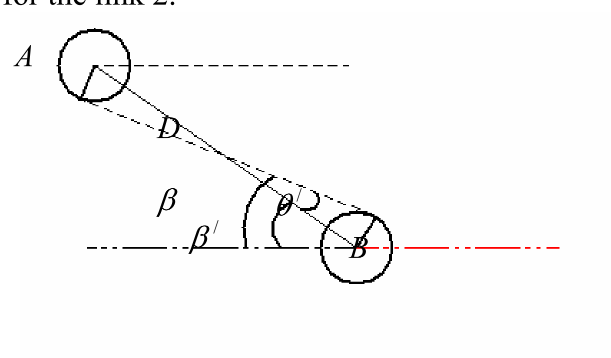


Since the angle β' is the decreasing and friction will tends to oppose it. Thus force F_{34} will act in the orientation (i).

Considering the link (3)
Since the link 3 is a 2-force member, then this link can be showed the four possible force situations.



Same way as above, the forces F_{43} and F_{23} will act in the orientation (iii).
Similarly, we can write for the link 2.



Now we have to find β'

$\beta - \theta' = \beta'$

$\Rightarrow \beta' = \beta - \theta' = 17.64 - \tan^{-1}(\frac{r_f}{DB}) = 17.64 - \tan^{-1}(\frac{0.99}{35}) = 16^0$

$\Rightarrow \beta' = 16^0$

Now considering link .4.

$F_{34} \cos\beta' = P + F_{14} \sin\phi$

&

$F_{34} \sin\beta' = F_{14} \cos\phi$

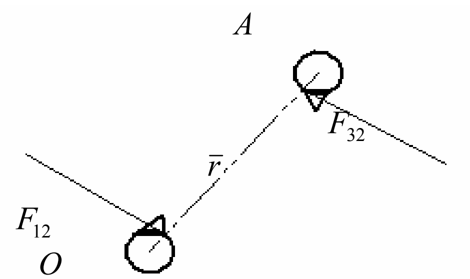
Therefore,

$F_{34} = 42.84N$

$F_{14} = 11.87N$

$|\overline{F}_{34}| = |\overline{F}_{43}| = |\overline{F}_{23}| = |\overline{F}_{32}| = 42.84N$ as in case (I) considering no friction.

Considering link.2.



$\overline{T} = \overline{r} \times \overline{F}_{32}$

Torque $\Rightarrow \overline{T} = 0.03(\cos 45^0 \hat{i} + \sin 45^0 \hat{j}) \times 42.84 \times (-\cos 16^0 \hat{i} + \sin 16^0)$

$\Rightarrow \overline{T} = -1.124\hat{k}N - m$

Example 5: Determine the driving torque available on the crank of a slider-crank mechanism, if a force of 2000 N pointing towards the main bearing is applied horizontally to the piston. Length of the crank and the connecting rod are 10 and 30 cm respectively. At the instant considered the crank has rotated 60 degree (CCW) from the inner dead center. Take coefficient of friction between all the pairing surfaces as 0.13. The diameter of the main bearing, crank pin and piston pin are respectively 10, 6 and 6 cm. Also find the driving torque in the absence of friction using virtual work principle.

Solution:

Considering the friction in all turning and sliding joints:

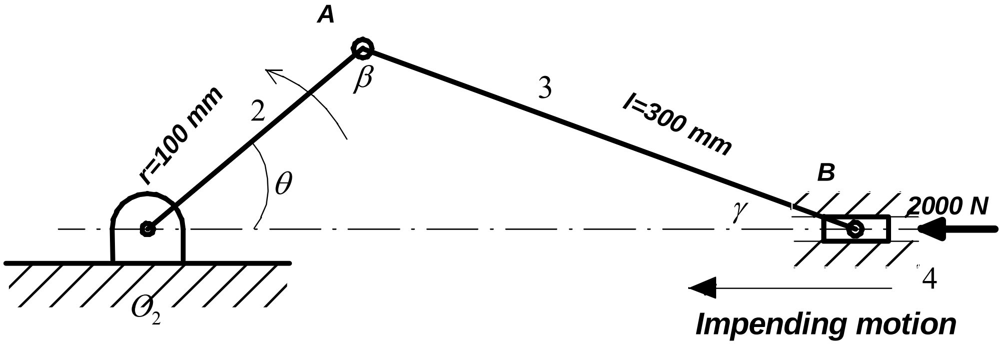


Figure 1

Now angle of friction in joints, $\phi = \tan^{-1}(\mu) = \tan^{-1}(0.13) = 7.407\,^{o}$.

Radius of friction circle, $r_f = h \sin\phi$

For the crank pin and gudgeon pin, $r_f = 60 \times \sin\phi = 7.735$ mm

For the main bearing of diameter 100 mm, $r_f = 12.89$ mm.

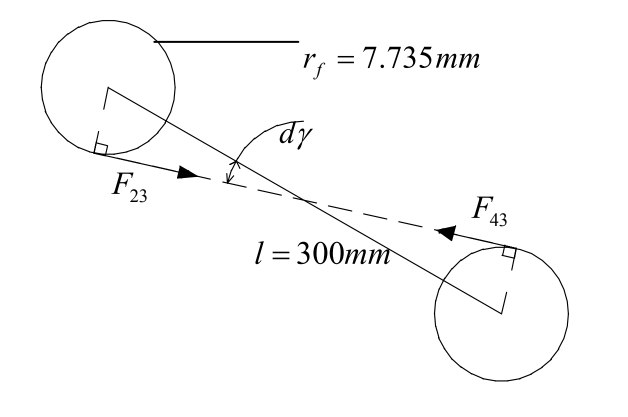


figure 2

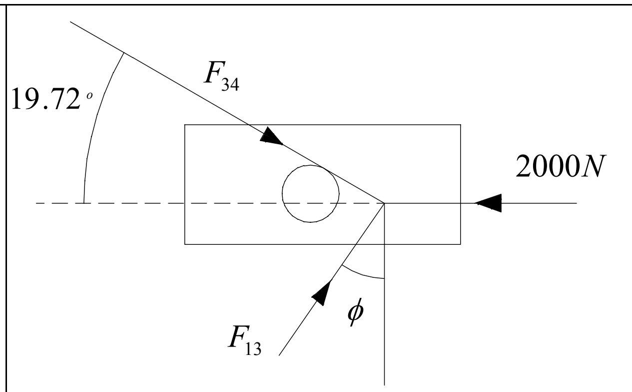


Figure 3

Consider the link 2 (figure 2),

$300/(\sin\theta) = 100/(\sin\gamma)$

⇨ $\gamma = 16.778\,^{o}$.

Now, from the figure, $d\gamma$ = tan-1(7.735/150) = 2.95 ^{o}

Now the Free body diagram of the piston is shown in figure 3,

The force equations are *(F_{14} can be divided into their frictional and normal components)*,

0.13 N $+F_{34}\cos19.72^{o} = 2000$ where N is the normal force acting

and $F_{34}\sin19.72^{o} = \text{N}$

$F_{34} = 2030\ N$

We know, $\beta = 180-(\theta+\gamma) = 103.23^o$.

(103.23+2.95)

From the figure 4,

Now the resisting couple, equal to $F_{32}\, d$ having a clockwise direction.

Torque = $F_{32}\, d = 2030 \times 0.10355$

$= 210.21$ Nm.

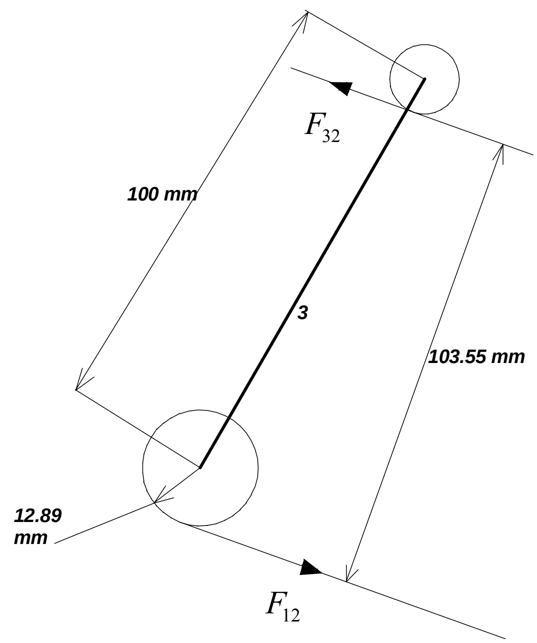
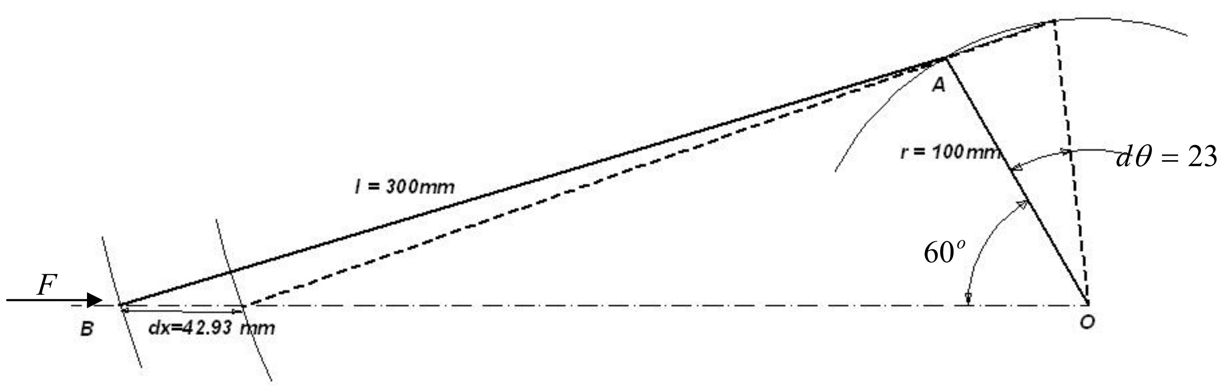


Figure 4

In the absence of friction using virtual work principle:



According to principle of virtual work,

$F\, dx = T\, d\theta$

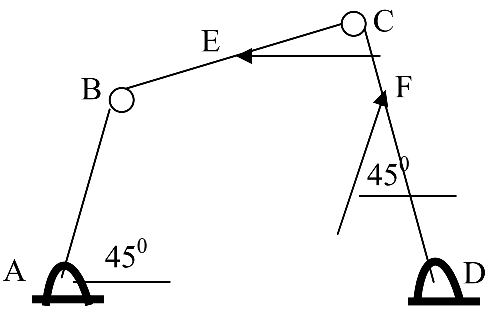
$T = (F\, dx) / d\theta$

From the figure, dx= 42.93 mm and $d\theta = 23^o = 23\times\pi/180$ rad.

Now, $T = (2000\times0.042) / (23\times\pi/180)$

$= 209.254$ Nm. ***(answer)***

Example 6: Determine the required input torque T_1 for the static equilibrium of the four bar mechanism shown in the figure. Forces F_2 and F_3 have magnitudes of 50 N and 75 N, respectively. Forces F_2 acts in the horizontal direction. Use both graphical and analytical methods. AB=30 cm, BC=40 cm, CD=50 cm and the fixed link AD=75 cm and CE=15 and CF= 20cm.



Solution:
Given data: --
AB=30 cm, F_2=50 N, F_3=75 N, BC=40 cm, CD=50 cm, AD=75 cm, CE=15 and CF= 20cm

Let link AB is rotating in counterclockwise with ω rad/s
Then, V_b=30ω cm/s. One may obtain the velocity of different points on the mechanism by using , graphical method, vector method, complex number methods etc. Here,

$$V_E = 13.33\omega\angle 28^0$$

$$V_c = 4.28\omega\angle 26^0$$

$$F_2 = -50\hat{i}$$

$$F_3 = 75\angle 45^0$$

Resolving the velocities at E and F, parallel and perpendicular to the applied forces at these positions respectively.

$V_e^1 = 11.67\omega$, Parallel to the F_2

$V_f^1 = 3.81\omega$, Parallel to the F_3.

Assuming T to be counter clockwise and applying principle of virtual work

$$T\times\omega + F_3\times 3.81 - F_3\times 3.81\omega = 0$$

$$\Rightarrow T = F_3\times 3.81 - F_3\times 3.81$$

$$\Rightarrow T = -297.75Nm = 297.75\text{Nm}$$

Alternatively, one may use dot product to find the virtual work done as follows.

$$T\omega = F_2.V_E + F_3.V_c$$
$$= (-50\hat{i}).(13.33\omega\cos 28\hat{i} + 13.33\omega\sin 28\hat{j})$$
$$+ (75\times 4.28\omega)(\cos 45\hat{i} + \sin 45\hat{j}).(\cos 26\hat{i} + \sin 26\hat{j})$$

$T\omega = 588.484 - 303.51$
$T = 284.97\text{Nm}$

Analytical Method:---

$$F_{23}\left[\cos 20^0 \hat{i} + \sin 20^0 \hat{j}\right] + F_{43}\left[\cos 64\hat{i} - \sin 64^0 \hat{j}\right] - F_2\hat{i} = 0$$

Equating the *i* component of the equations, we have,

$$F_{23}\cos 20^0 + F_{43}\cos 64^0 = F_2$$

Equating the j part of the equations:-

$$F_{23}\sin 20^0 = F_{43}\sin 64^0$$

$$F_{23}\left[\cos 20^0 + \sin 20^0 \cot 64^0\right] = 50$$

$$F_{23} = 45.187$$

$$r = \left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$T_a = r \times F_{23}$$

$$= \frac{30}{\sqrt{2}}\left(\hat{i} + \hat{j}\right) \times 45.187(-\cos 20^0 \hat{i} - \sin 20^0 \hat{j})$$

$$= -(\frac{30}{\sqrt{2}}45.187\sin 20^0) + \frac{30}{\sqrt{2}} \times 45.187\cos 20^0$$

$$= 572.9055$$

$$F_{34}\left[-\cos 33^0 \hat{i} - \sin 33^0 \hat{j}\right] + F_3\left[\cos 45\hat{i} + \sin 45^0 \hat{j}\right] - F_{14}\left[\cos 58.5\hat{i} + \sin 58.5^0 \hat{j}\right] = 0$$

$$F_{34}\cos 33^0 = \frac{F_3}{\sqrt{2}} - F_{14}\cos 58.5^0$$

$$F_{34}\sin 33^0 = \frac{F_3}{\sqrt{2}} - F_{14}\sin 58.5^0$$

$$F_{34}(\cos 33^0 - \sin 33^0) = F_{14}(\sin 58.5^0 - \cos 58.5^0)$$

Therefore,

$$F_{14} = 0.8906F_{34}$$

Therefore,

$$F_{34}\left[\cos 33^0 + 0.8906\cos 58.5^0\right] = \frac{75}{\sqrt{2}}$$

$$\Rightarrow F_{34} = 40.6689 \simeq 40N = F_{32}$$

We know that,

$r = \frac{30}{\sqrt{2}}(\hat{i} + \hat{j})$

Therefore,

$T_b = r \times F_{32} = \frac{30}{\sqrt{2}}(\hat{i} + \hat{j}) \times 40.7\left[\cos 33^0 + \sin 33^0\right]$

$T_b = \frac{30}{\sqrt{2}} \times 40.7\left[\sin 33^0 - \cos 33^0\right] = -253.86N$

By superposition principle,

$Total(T) = T_a + T_b = 572.9055 - 253.68 = 319.04$, Ans.

Gear force Analysis

The fundamental law of gearing states that in order to obtain a constant velocity ratio, the common normal to the tooth profile at the point of contact should always pass through a fixed point, called the pitch point. Thus the point of contact of the two gears has the same velocity. Applying Newton's third law, the force exerted by one gear to the other at the point of contact is same in magnitude but opposite in direction. In this section the forces in spur and helical gears are discussed.

Spur Gear In figure 8(a) shows the pitch circles of a pair of spur gears with center at a and b and rotating with angular velocities ω_2 and ω_3. The line of action and pressure angle ϕ are clearly shown in this figure. In figure 8(b) the pair of constraint forces (F_{23} and F_{32}) acting at the pitch point along the line of action are shown. Considering the freebody diagram of gear 2 as shown in figure 8(c), the force F_{32} is balanced by the reaction force F_{12} acing at the bearing. As these two forces are equal and opposite, they will form a couple. To overcome this reaction couple, the prime mover (say motor) should provide a torque equal in magnitude but opposite in direction, which is represented by T_{a2} in the figure.

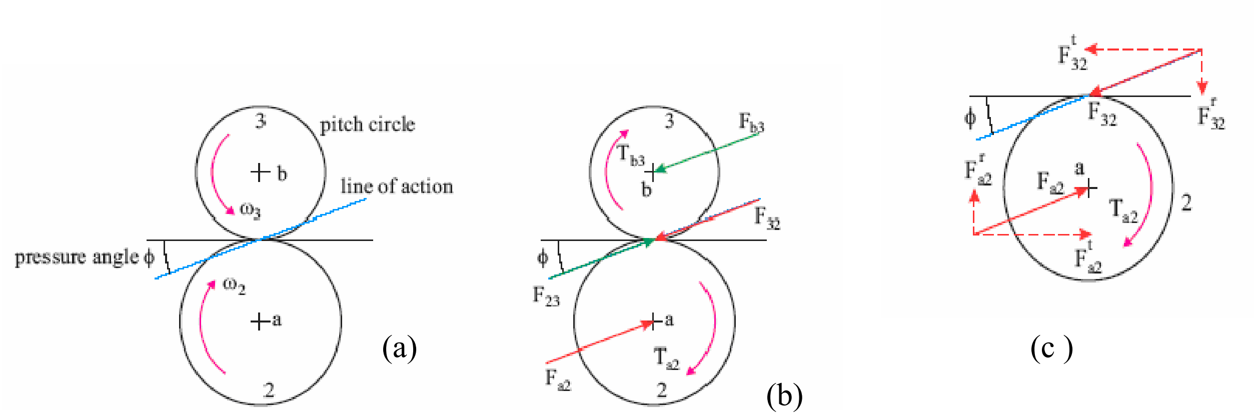


Figure 8 : Force analysis of Spur gear

One may resolve these forces in tangential and radial direction. Clearly, the tangential force is responsible for rotating the gear.

Now let us derive an expression for the gear forces form the given power and speed of operation. Let ‘ω’ be the speed of rotation (in rpm) of the gear with module ‘m’ and number of teeth ‘z’. The value of the pressure angle(in degrees) is ‘Φ’ and the power (in KW) it transmits be ‘P’. The diameter ‘D’ of the gear can be calculated as

$$D = m\ z \qquad \text{(a)}$$

and the torque ‘T’ transmitted by the gears is

$$T = \frac{60 \times 10^6 P}{2\pi\omega} \qquad \text{(b)}$$

From Figure 8(c), we see that the force that is responsible for transmitting the torque ‘T’ is the tangential component ($F^t_{32} = F^t_{23} = F_t$). The radial component ($F^r_{32} = F^r_{23} = F_r$) is the separating force, which always acts towards the center of the gear. Thus we get

$$F_t \frac{D}{2} = T, \text{ or, } F_t = \frac{2T}{D} \qquad \text{(c)}$$

So, using equation (c) one may obtain the tangential force F_t from known value of D and T.

From Figure 8(c) the radial component can be obtained as

$$F_r = F_t \ \tan \Phi \qquad \text{(d)}$$

Hence the resultant force acting on the gear or on the bearing equals to $F = \sqrt{(F_t^2 + F_r^2)}$

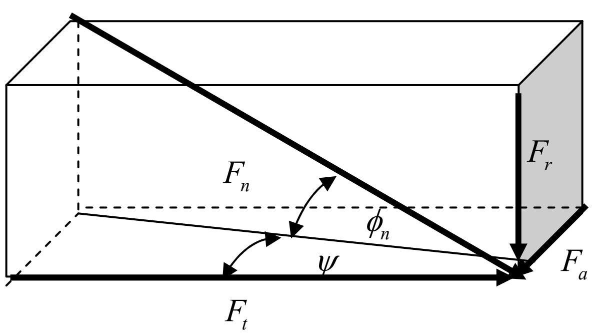
Also one may note that $F_t = F\cos\phi$ and $F_r = F\sin\phi$

Thus the net force ‘F’ can be easily calculated using the above expressions. This analysis of the gear tooth force is based on certain assumptions which are as follows

1. As the point of contact moves, the value of the resultant force ‘F’ changes, which is neglected in the above analysis.
2. It is assumed that only one pair of teeth take the entire load. At times there are two pairs which simultaneously are in contact and share the load. This aspect is also neglected in this case.
3. The analysis is valid under static conditions, when the gears are running at very low velocities. In practice there are dynamic forces also due to the power transmission. The effect of these dynamic forces are neglected in the analysis.

Helical gears

Like the spur gears, the helical gears also connect parallel shafts. But a major difference between a helical gear and a spur gear is that the teeth in case of a helical gear are cut in the form of a helix on the pitch cylinder. In these types of gears the contact between the meshing teeth begins with a point on the leading edge of the tooth and gradually extends along the diagonal line across the tooth. There is a gradual pick up of the load by the tooth resulting in a smooth engagement and a quiet operation even at very high speeds.



In a helical gear
ϕ_n =normal pressure angle
ψ =helix angle

Figure 9: Force analysis of helical gear

In helical gear the normal force F_n consists of three components viz., tangential component F_t, radial component F_r and the axial or thrust component F_a as shown in the figure 9. They are related as follows.

Tangential force $F_t = F_n \cos\phi_n \cos\varphi$ (a)

Radial force $F_r = F_n \sin\phi_n$ (b)

Thrust or axial force $= F_a = F_n \cos\phi_n \sin\varphi = F_t \tan\psi$ (c)

Let N be the speed of rotation in rpm of the gear with module m and number of teeth z, α is the transverse pressure angle and ψ is the helix angle. Now the diameter D of the gear can be determined from the relation

$$D = m\ z \qquad \text{(d)}$$

The angular velocity of the gear $= \omega = \dfrac{2\pi N}{60}$ (e)

The torque T transmitted by the gears can be calculated from the power P from the relation

$$P = T\omega \qquad \text{(f)}$$

From Figure 9, we see that the force that is responsible for transmitting the torque 'T' is the tangential component F_t. The radial component F_r is the separating force, which always acts towards the center of the gear, and F_a is the axial or thrust component. The direction of this axial component depends upon whether the gear is left or right handed, the direction of rotation and on whether the driving or driven gear is under consideration. Thus we get

$$T = F_t \frac{D}{2} \text{ or } F_t = \frac{2T}{D} \qquad \text{(g)}$$

It may be recalled that the normal pressure angle ϕ_n, helix angle ψ and transverse pressure angle ϕ are related by

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi} \qquad \text{(h)}$$

Once we have calculated the value of the normal pressure angle from (h), we can calculate the tangential force from equations (f and g), axial or the thrust force using equation (c) and radial component from expression (b). The resultant force on the helical gear can now be calculated using the three components as

$$F_n = \sqrt{(F_t^2 + F_r^2 + F_a^2)} \qquad \text{(i)}$$

The following guidelines should be followed while calculating the axial or thrust component F_a

1. Select the driving gear from the pair.
2. Use right hand for right-handed helix and left hand for left handed helix.
3. Keep the fingers in the direction of rotation of the gear and the thumb will indicate the direction of the thrust component of the driving gear.

4. The direction of the thrust component of the driven gear will be the opposite to that for the driving gear.

Example 7 Two helical gears on the parallel shafts have a normal pressure angle of 20 degrees and a normal module of 6 mm. The centre distance is 200 mm and the assembly has 20 and 40 teeth. The gear set transmits 50 KW at a pinion speed of 1200 rev/min. Determine the tangential, radial and thrust loads on the gear teeth, and show these forces on the gears. The pinion is handed and rotates clockwise.

Solution:

Given data: ---

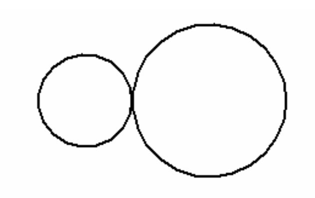
Normal pressure angle $=\varphi_n=20^0$

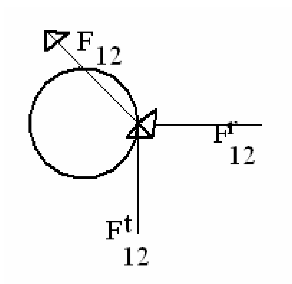
Normal module $=m_1=m_2=m_n=6$mm.

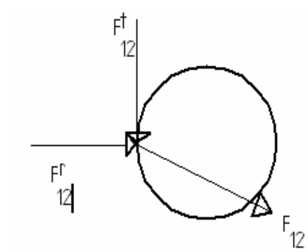
Center distance=C=200mm

No of teeth of the pinion=20,

No of the teeth of the gear=40.







$$\frac{r_1}{r_2}=\frac{20}{40}=\frac{1}{2}\Rightarrow 2r_1=r_2$$

$$2r_1+r_1=200\Rightarrow 3r_1=200.\text{ Hence, } r_1=\frac{200}{3}.$$

Therefore, $r_2=\dfrac{400}{3}$

Now as C=r_1+r_2

$$c = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}(m_1 T_1 + m_2 T_2) = \frac{1}{2}\frac{m_n}{\cos\psi}(T_1 + T_2)$$

So, $\cos\psi = 25.8419^0$

$P = 50\text{kw},$

$$w = 2\pi(\frac{1200}{60}) = 125.6\text{rad/sec}$$

$$T_1 w_1 = P \Rightarrow T_1 = \frac{50 \times 1000}{125.6} = 398.089\text{Nm}$$

$$F_{21}^T = \frac{T}{r_1} = \frac{398.089 \times 1000}{(200/3)} = 5971.335N$$

We know that

$$F_{12}^T = -F_{21}^T, \text{Hence, } \left|F_{21}^T\right| = \left|F_{12}^T\right| = 5971.335\text{N}$$

$$F_n = F_n^T \cos\phi_n, F_n = F_n^T \cos\phi_n$$

$$F_r = F_n \sin\phi_n, F_a = F_n \sin\psi$$

$$F_{12}^a = F_{12}^T \tan\psi = 5971.335 \times \tan 25.8419^0 = 2892.04532\text{N}$$

$$F_{21}^a = -F_{12}^a,$$

$$F_{12}^r = F_{12}^T \sin\phi_n = 5971.335 \times \tan 20^0 = 2173.388\text{N} \qquad \text{Ans.}$$

Summery

The following points are learned in this chapter

- Classification of forces: external and constraint forces
- Determination of moments
- Freebody diagram
- Two and three force members
- Static force analysis using graphical method
- Static force analysis using analytical method (vector method)
- Use of superposition theory for multiple external forces acting on a mechanism
- Static force analysis with sliding and grease friction (concept of friction circle)
- Application of virtual work principle for static force analysis.

Exercise Problems

1. Draw the constraint forces in all the six types of lower pairs, viz., (i) revolute or turning pair (ii) prismatic or sliding pair, (iii) cylindrical pair, (iv) screw or helical pair, (v) planar or flat pair, and (vi) globular or spherical pair.

2. Explain with neatly drawn free-body diagram the effect of friction in the bearings on the torque required by the crankshaft in a slider-crank mechanism when the crank is rotating in (i) clock wise direction (ii) anti-clock wise direction.

3. Calculate the torque required for static equilibrium of an in-line slider crank mechanism in the position when crank angle θ = 60 deg (from inner dead center). The dimensions are crank length r = 100 mm, connecting rod length L = 175 mm, and the piston force is P = 50 N. Assume crank to be rotating in anticlockwise direction. Use, graphical, analytical and virtual work principle to find the result.

4. Taking same data as in problem 3, also find the torque required assuming that the coefficient of friction for all bearings is 0.1. The three journal bearings all have radii of 20 mm, and the crank is rotating in the clockwise direction.

5. Figure below shows a mechanism used to crush rocks. The mechanism is moving slowly, so the inertia forces may be neglected. In the position shown, determine the torque required to drive the input link AB when the crushing force acting in the horizontal direction is 5000N. Here, AB = 50 cm, BC=100 cm, CD=120 cm and the fixed link AD=150 cm and CE=25 cm and the angle CED of the ternary link CED is 90^{0}. Use (a) graphical method, (b) analytical method and (c) virtual work principle to determine the bearing forces and required torque.