



DEPARTMENT OF MATHEMATICS

UNIT – III TWO DIMENSIONAL RANDOM VARIABLES

DISCRETE RANDOM VARIABLES

PROBLEMS: [DISCRETE RANDOM VARIABLE]

(i) From the following table for bivariate distribution of (X, Y) find

(i) $P(X \leq 1)$; (ii) $P(Y \leq 3)$; (iii) $P(X \leq 1, Y \leq 3)$

(iv) $P(X \leq 1 / Y \leq 3)$; (v) $P(Y \leq 3 / X \leq 1)$; (vi) $P(X + Y \leq 4)$

(vii) The Marginal distribution of x or Marginal PMF of x

(viii) The Marginal distribution of y or Marginal PMF of y

(ix) The conditional distribution of x given $y=2$

(x) Examine x and y are independent.

$X \backslash Y \rightarrow$	1	2	3	4	5	6
\downarrow						
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$



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$X \setminus Y \rightarrow$	1	2	3	4	5	6
\downarrow						
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

Soln:

$X \setminus Y \rightarrow$	1	2	3	4	5	6	$P(X=n): P(X)$
\downarrow							
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$P(X=0) : \frac{8}{32} : \frac{8}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$P(X=1) : \frac{10}{16} : \frac{20}{32}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$P(X=2) : \frac{8}{64} : \frac{4}{32}$
$P(Y=y)$	$P(Y=1)$	$P(Y=2)$	$P(Y=3)$	$P(Y=4)$	$P(Y=5)$	$P(Y=6)$	
$P(Y)$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{6}{32}$	$\frac{16}{64}$	$\frac{30}{32}$
	$\frac{6}{64}$	$\frac{6}{64}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{12}{64}$	$\frac{16}{64}$	$\frac{64}{64} \rightarrow 1$

(i) $P(X \leq 1) = P(X=0) + P(X=1) = \frac{8}{32} + \frac{20}{32} = \frac{28}{32}$

(ii) $P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3) = \frac{6}{64} + \frac{6}{64} + \frac{11}{64} = \frac{23}{64}$



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$$\begin{aligned} \text{(iii)} \quad P(x \leq 1, y \leq 3) &= P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3) \\ &= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} \\ &= \frac{1+2+2+4}{32} = \frac{9}{32} \end{aligned}$$

$$\text{(iv)} \quad P(x \leq 1 | y \leq 3) = \frac{P(x \leq 1, y \leq 3)}{P(y \leq 3)} = \frac{9/32}{28/64} = 18/28$$

$$\text{(v)} \quad P(y \leq 3 | x \leq 1) = \frac{P(x \leq 1, y \leq 3)}{P(x \leq 1)} = \frac{9/32}{28/32} = 9/28$$

$$\begin{aligned} \text{(vi)} \quad P(x+y \leq 4) &= P(0,1) + P(0,2) + P(0,3) + P(0,4) + P(1,1) + P(1,2) + \\ &P(1,3) + P(2,1) + P(2,2) \\ &= 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32} \\ &= \frac{13}{32} \end{aligned}$$



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(vii) Marginal distribution of $X: P(X=x)$

$$X: \quad 0 \quad 1 \quad 2$$

$$P(X=x): \quad 8/32 \quad 20/32 \quad 4/32$$

(viii) Marginal distribution of $Y: P(Y=y)$

$$Y: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

$$P(Y=y): \quad 6/64 \quad 6/64 \quad 11/64 \quad 13/64 \quad 12/64 \quad 16/64$$

(ix) $P(X \neq y=2) = P(X=x_i / Y=2) \quad x_i \rightarrow i=0,1,2, \dots$

$$\text{Now } P(X=0 / Y=2) = \frac{P(X=0, Y=2)}{P(Y=2)} = \frac{0}{6/64} = 0$$

$$P(X=1 / Y=2) = \frac{P(X=1, Y=2)}{P(Y=2)} = \frac{4/16}{6/64} = \frac{2}{3}$$

$$P(X=2 / Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)} = \frac{4/32}{6/64} = \frac{4}{3}$$

(x) Are X and Y are independent:

$$P(X=i) \cdot P(Y=j) = P(i, j)$$

$$\text{Now } P(0,1) = P(X=0) \cdot P(Y=1)$$

$$0 \neq 8/32 \times 3/32, \text{ not independent}$$



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(2) The joint probability mass function of (x, y) is given by

$p(x, y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal & conditional probability distributions. Also find the probability distribution of $(x+y)$ & $p(x+y > 3)$.

Soln:

$X/Y \rightarrow$	1	2	3
\downarrow			
0	$3k$	$6k$	$9k$
1	$5k$	$8k$	$11k$
2	$7k$	$10k$	$13k$

$$\sum_{i=1}^n \sum_{j=1}^n p(x_i, y_j) = 1$$

$$\Rightarrow 3k + 6k + 9k + 5k + 8k + 11k + 7k + 10k + 13k = 1$$

$$72k = 1$$

$$k = \frac{1}{72}$$



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Soln:

$X/Y \rightarrow$	1	2	3
\downarrow			
0	$3k$	$6k$	$9k$
1	$5k$	$8k$	$11k$
2	$7k$	$10k$	$13k$

$$\sum_{i=1}^n \sum_{j=1}^n p(x_i, y_j) = 1$$

$$\Rightarrow 3k + 6k + 9k + 5k + 8k + 11k + 7k + 10k + 13k = 1$$

$$72k = 1$$

$$k = \frac{1}{72}$$

$X/Y \rightarrow$	1	2	3	$P(X=x)$
\downarrow				
0	$3/72$	$6/72$	$9/72$	$18/72$
1	$5/72$	$8/72$	$11/72$	$24/72$
2	$7/72$	$10/72$	$13/72$	$30/72$

$$P(Y=y) \quad 15/72 \quad 24/72 \quad 38/72 \rightarrow \frac{1}{1}$$

Marginal distribution of X:

$X :$	0	1	2
$P(X=x) :$	$18/72$	$24/72$	$30/72$



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Marginal distribution of y :

$$y: \quad 1 \quad 2 \quad 3$$
$$p(y=y_j): \quad 15/72 \quad 24/72 \quad 33/72$$

Condition probability: $p(x/y)$

$$p(x=x_i / y=y_j); \quad x_i \rightarrow i=0,1,2; \quad y_j \rightarrow j=1,2,3$$

$$\text{Now } p(x=0 / y=1) = \frac{p(x=0, y=1)}{p(y=1)} = 3/15$$

$$p(x=1 / y=1) = \frac{p(x=1, y=1)}{p(y=1)} = 5/15$$

$$p(x=2 / y=1) = \frac{p(x=2, y=1)}{p(y=1)} = 7/15$$

$$p(x=0 / y=2) = \frac{p(x=0, y=2)}{p(y=2)} = 6/24$$

$$p(x=1 / y=2) = \frac{p(x=1, y=2)}{p(y=2)} = 8/24$$

$$p(x=2 / y=2) = \frac{p(x=2, y=2)}{p(y=2)} = 10/24$$

$$p(x=0 / y=3) = \frac{p(x=0, y=3)}{p(y=3)} = 9/33$$



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$$P(X=1/Y=3) = \frac{P(X=1, Y=3)}{P(Y=3)} = 11/33$$

$$P(X=2/Y=3) = \frac{P(X=2, Y=3)}{P(Y=3)} = 13/33$$

Conditional probability of $Y/X : P(Y/X)$

$$P(Y=Y_j/X=X_i) : X_i \rightarrow i=0,1,2 \quad Y_j \rightarrow j=1,2,3$$

$$\text{Now } P(Y=1/X=0) = \frac{P(Y=1, X=0)}{P(X=0)} = 3/18$$

$$P(Y=2/X=1) = \frac{P(Y=2, X=1)}{P(X=1)} = 6/18$$

$$P(Y=3/X=0) = \frac{P(Y=3, X=0)}{P(X=0)} = 9/18$$

$$P(Y=1/X=1) = \frac{P(Y=1, X=1)}{P(X=1)} = 5/24$$

$$P(Y=2/X=1) = \frac{P(Y=2, X=1)}{P(X=1)} = 8/24$$

$$P(Y=3/X=1) = \frac{P(Y=3, X=1)}{P(X=1)} = 11/24$$

$$P(Y=1/X=2) = \frac{P(Y=1, X=2)}{P(X=2)} = 7/30$$

$$P(Y=2/X=2) = \frac{P(Y=2, X=2)}{P(X=2)} = 10/30$$

$$P(Y=3/X=2) = \frac{P(Y=3, X=2)}{P(X=2)} = 13/30$$



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probability distribution of $x+y$:

$x+y$	$P(x+y)$
1	$P(0,1) : 3/72$
2	$P(0,2)+P(1,1) : 5/72+6/72 : 11/72$
3	$P(0,3)+P(1,2)+P(2,1) : 9/72+8/72+7/72 : 24/72$
4	$P(1,3)+P(2,2) : 10/72+11/72 : 21/72$
5	$P(2,3) : 13/72$

$$\begin{aligned}P[x+y > 3] &= P[x+y=4] + P[x+y=5] \\&= 21/72 + 13/72 \\&= 34/72\end{aligned}$$