TWO DIMENSIONAL RANDOM VARIABLES

DISCRETE RANDOM VARIABLE

(1) It is finite, infinite countable.

(2) Joint probability mass function (or) Joint probability function [(or) distribution]:

(i) $p(x_i, y_j) \geq 0$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots n$.

(ii) $\sum_{i=1}^{n} \sum_{j=1}^{n} p(x_i, y_j) \geq 1$

(3) To find constants [k, a, c, . . .]

$$\sum_{i=1}^{n} \sum_{j=1}^{n} p(x_i, y_j) = 1$$

(4) Marginal distribution function:

(i) Marginal distribution function of X:

$$P(X=x) = p(x) = \sum_{j=1}^{n} p(x_i, y_j), \quad i = 1, 2, \ldots n; \; j = 1, 2, \ldots n$$

(ii) Marginal distribution function of Y:

$$P(Y=y) = P(y) = \sum_{i=1}^{n} p(x_i, y_j), \quad i = 1, 2, \ldots n; \; j = 1, 2, \ldots n$$

(5) Joint Cumulative distribution function of x & y:

$$F(x,y) = p(X \le x, Y \le y)$$

(6) To find x & y are independent :

$$p(x=i) \cdot p(y=j) = p(i,j) \quad , \quad i = 1, 2, \ldots n \quad j = 1, 2, \ldots n$$

(7) Mean:

(i) Mean of x :

$$E(x) = \sum_{i=1}^{n} x_i \, p(x_i)$$

(ii) Mean of y :

$$E(y) = \sum_{j=1}^{n} y_j \, p(y_j)$$

(8) $E(x^2) = \sum_{i=1}^{n} x_i^2 \, p(x_i)$

$$E(y^2) = \sum_{j=1}^{n} y_j^2 \, p(y_j)$$

(9) Variance :

(i) $Var(x) = E(x^2) - (E(x))^2$

$$= \sum_{i=1}^{n} x_i^2 p(x_i) - \left[\sum_{i=1}^{n} x_i p(x_i)\right]^2$$

(ii) $Var(y) = E(y^2) - [E(y)]^2$

$$= \sum_{j=1}^{n} y_j^2 p(y_j) - \left[\sum_{j=1}^{n} y_j p(y_j)\right]^2$$

(10) Conditional probability :

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

CONTINUOUS RANDOM VARIABLE

(1) It is infinite uncountable.

(2) Joint probability density function.

(i) $f(x,y) \geq 0$

(ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$

(3) To find constants [k, a, c, ...]

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$$

(4) Marginal distribution function:

(i) Marginal distribution function of x:

$$P(X=x) = f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

(ii) Marginal distribution function of y:

$$P(Y=y) = f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

(5) Joint cumulative distribution function of x & y:

$$F(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(x,y)\,dx\,dy$$

If Joint cumulative distribution is given then to find Joint probability function,

$$f(x,y) = \frac{\partial^2}{\partial x \partial y} F[x,y]$$

(6) To find x & y are independent:

$$f(x) \cdot f(y) = f(x,y)$$

(7) Mean:

(i) Mean of x:

$$E(x) = \int_{-\infty}^{\infty} x\, f(x,y)\,dx \quad \text{(or)} \quad E(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, f(x,y)\,dy\,dx$$

(ii) Mean of y:

$$E(y) = \int_{-\infty}^{\infty} y\, f(x,y)\,dy \quad \text{(or)} \quad E(y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\, f(x,y)\,dx\,dy$$

(8) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x,y)\,dx$ (or) $E(x^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y)\,dy\,dx$

$E(y^2) = \int_{-\infty}^{\infty} y^2 f(x,y)\,dy$ (or) $E(y^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 f(x,y)\,dx\,dy$

(9) Variance :

(i) $Var(x) = E(x^2) - (E(x))^2$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f(x,y)\,dy\,dx - \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\,dy\,dx\right]^2$$

(ii) $Var(y) = E(y^2) - (E(y))^2$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y^2 f(x,y)\,dx\,dy - \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f(x,y)\,dx\,dy\right]^2$$

(10) Conditional probability :

(i) Conditional probability of x/y

$$f(x/y) = \frac{f(x,y)}{f(y)}$$

(ii) Conditional probability of y/x

$$f(y/x) = \frac{f(x,y)}{f(x)}$$

NOTE:

(i) $p(x<a)$ or $p(x\le a) = \int\int_{x}^{a} f(x,y)\,dx\,dy$

(ii) $p(x>a)$ or $p(x\ge a) = \int\int_{a} f(x,y)\,dx\,dy$

(iii) $p(y<a)$ or $p(y\le a) = \int\int^{a} f(x,y)\,dy\,dx$

(iv) $p(y>a)$ or $p(y\ge a) = \int\int_{a} f(x,y)\,dy\,dx$

(v) $p(x\le a,\ y\le a) = \int^{a}\int^{a} f(x,y)\,dy\,dx$

(vi) $p(x\ge a,\ y\ge a) = \int_{a}\int_{a} f(x,y)\,dy\,dx$

(vii) $p(x\le a,\ y\ge a) = \int^{a}\int_{a} f(x,y)\,dy\,dx$

(viii) $p(x\ge a,\ y\le a) = \int_{a}\int^{a} f(x,y)\,dy\,dx$

(ix) $p(a<x<b,\ a<y<b) = \int_{a}^{b}\int_{a}^{b} f(x,y)\,dy\,dx$.

PROPERTIES:

(i) $F(-\infty,\infty) = 0$; $F(-\infty,y) = 0$; $F(x,-\infty) = 0$

(ii) $F(\infty,\infty) = 1$

(iii) $0 \le F(x,y) \le 1$

(iv) $F(x,\infty) = F(x)$; $F(\infty,y) = F(y)$

(v) $E(ax \pm by) = aE(x) \pm bE(y)$

(vi) $var(ax \pm by) = a^2 var(x) \pm b^2 var(y)$