



DEPARTMENT OF MATHEMATICS

UNIT – I PROBABILITY AND RANDOM VARIABLES

Normal Distribution:

If x is a continuous random variable which follows normal distribution,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Note:

$$M_x(t) = e^{t\mu + \frac{t^2\sigma^2}{2}}$$

$$E(x) = \mu$$

$$\text{Var}(x) = \sigma^2$$

$$SD = \sqrt{\text{var}} = \sqrt{\sigma^2} = \sigma$$

Properties/characteristics of Normal Distribution

- * The curve is bell shaped and symmetric about the origin.
- * The curve has a single peak i.e., unimodal.
- * The mean of the normal distribution lies at the center of the curve.
- * In normal distribution, mean, median & mode are coincide.



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Note:

$$* z = \frac{x - \mu}{\sigma}$$

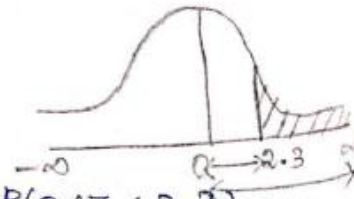
$$* P(-\infty < z < 0) = -0.5$$

$$* P(0 < z < \infty) = 0.5$$

Example:

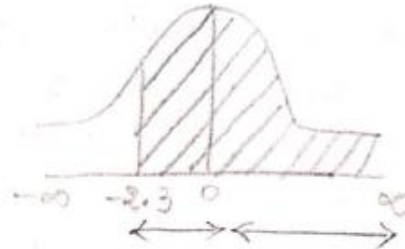
1]. $P(z > 2.3)$

$$\begin{aligned} P(z > 2.3) &= P(0 < z < \infty) - P(0 < z < 2.3) \\ &= 0.5 - P(0 < z < 2.3) \end{aligned}$$



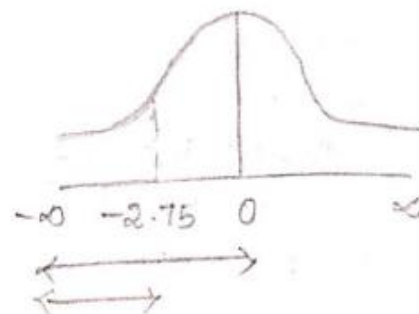
2]. $P(z > -2.3)$

$$\begin{aligned} P(z > -2.3) &= P(-2.3 < z < 0) \\ &\quad + P(0 < z < \infty) \\ &= P(0 < z < 2.3) + 0.5 \end{aligned}$$



3]. $P(z < -2.75)$

$$\begin{aligned} P(z < -2.75) &= P(-\infty < z < 0) \\ &\quad - P(-2.75 < z < 0) \\ &= P(0 < z < \infty) + P(0 < z < 2.75) \\ &= 0.5 + P(0 < z < 2.75) \end{aligned}$$





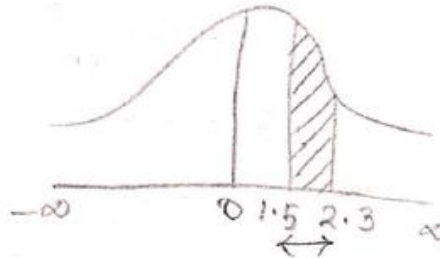
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A). $P(1.5 < Z < 2.3)$

$$P(1.5 < Z < 2.3)$$

$$= P(0 < Z < 2.3) - P(0 < Z < 1.5)$$



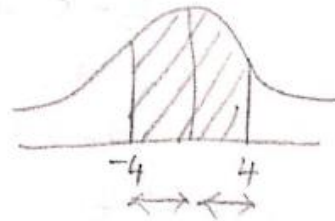
B). $P(-4 < Z < 4)$

$$P(-4 < Z < 4)$$

$$= P(-4 < Z < 0) + P(0 < Z < 4)$$

$$= P(0 < Z < 4) + P(0 < Z < 4)$$

$$= 2 P(0 < Z < 4)$$





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In a test on 2000 electric bulbs, it was found that the life of the particular make, was normally distributed with an ^{mean} average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for i). more than 2150 hours, ii). less than 1950 hours and iii). more than 1920 hours but less than 2160 hours.

Soln.:

$$\text{Mean: } \mu = 2040 \text{ hours}$$

$$\text{S.D: } \sigma = 60 \text{ hours}$$

$$\text{i). } P(X \geq 2150)$$

$$\begin{aligned} \text{Let } Z &= \frac{X - \mu}{\sigma} \\ &= \frac{2150 - 2040}{60} \end{aligned}$$

$$Z = 1.833$$

$$\therefore P(X \geq 2150) = P(Z \geq 1.833)$$

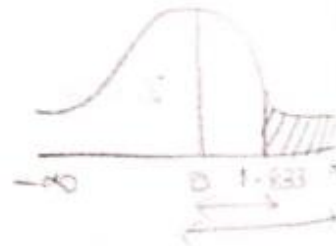
$$= P(0 < Z < \infty) - P(0 < Z < 1.833)$$

$$= 0.5 - 0.4664$$

$$= 0.034$$

$$= 0.034 \times 2000$$

$$= 67.2$$





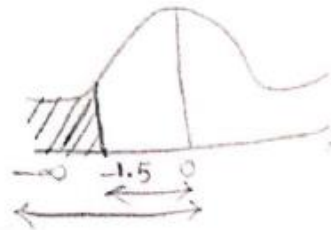
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$$ii) P(X \leq 1950)$$

$$\text{Let } z = \frac{x - \mu}{\sigma}$$
$$= \frac{1950 - 2040}{60}$$

$$z = -1.5$$



$$P(X \leq 1950) = P(Z \leq -1.5)$$

$$= P(-\infty < Z < 0) - P(-1.5 < Z < 0)$$

$$= P(0 < Z < \infty) - P(0 < Z < 1.5)$$

$$= 0.5 - 0.4332$$

$$= 0.067$$

$$= 0.067 \times 2000$$

$$= 134$$

iii) More than 1920 hrs and less than 2160 hrs.

$$P(1920 \leq X \leq 2160)$$

$$\text{Let } X = 1920 \Rightarrow z = \frac{1920 - 2040}{60} = -2$$

$$X = 2160 \Rightarrow z = \frac{2160 - 2040}{60} = 2$$

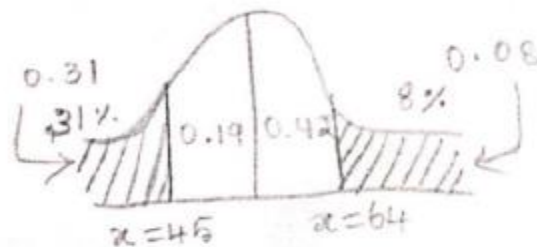


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$$\begin{aligned}
 P(1990 \leq x \leq 2160) &= P(-2 \leq z \leq 2) \\
 &= 2P(0 \leq z \leq 2) \\
 &= 2[0.4772] \\
 &= 0.954 \\
 &= 0.954 \times 2000 \\
 &= 1908
 \end{aligned}$$

5]. In a Normal distribution, 31% of the items are under 45 and 8% are over 64. Find mean & S.D.



$$\text{Let } z = \frac{x - \mu}{\sigma}$$

when $x = 45$

$$\textcircled{-0.5} = \frac{45 - \mu}{\sigma}$$

0.19 table value

$$-0.5\sigma = 45 - \mu$$

$$\mu - 0.5\sigma = 45$$

↳ (1)

$x = 64$

$$\textcircled{1.41} = \frac{64 - \mu}{\sigma}$$

0.42 table value

$$1.41\sigma = 64 - \mu$$

$$\mu + 1.41\sigma = 64$$

↳ (2)



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Solving (1) and (2),

$$-1.9\sigma = -19$$

$$\sigma = \frac{-19}{-1.9}$$

$$\sigma = 10$$

Subs. $\sigma = 10$ in (1)

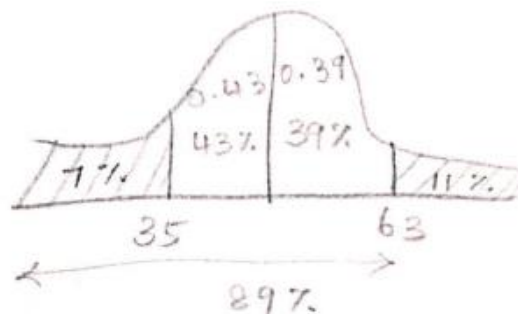
$$\mu = 45 + 0.5\sigma$$

$$= 45 + 0.5(10)$$

$$= 45 + 5$$

$$\mu = 50$$

3]. In a distribution exactly normal 7% of one items are under 35 and 89% are under 63. What are the mean & SD of distribution.





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when $x = 35$

$$z = \frac{x - \mu}{\sigma}$$

$$\textcircled{-1.48} = \frac{35 - \mu}{\sigma}$$

0.43 table value σ

$$\mu - 1.48\sigma = 35$$

\hookrightarrow (1)

$x = 63$

$$z = \frac{x - \mu}{\sigma}$$

$$\textcircled{1.23} = \frac{63 - \mu}{\sigma}$$

0.39 table value σ

$$\mu + 1.23\sigma = 63$$

\hookrightarrow (2)

Solving (1) & (2), we get

$$\mu = 50.29$$

$$\sigma = 10.33$$



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AREA UNDER NORMAL CURVE
TABLE OF AREAS

$ Z \rightarrow$	0	1	2	3	4	5	6	7	8	9
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0539	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4603	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4879	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4905	.4906	.4907	.4908	.4909
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4950	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000