DISCRETE RANDOM VARIABLE

DISCRETE

1) DEFINITION:

A random variable x is discrete if it assumes only discrete values [finite (or) countably infinite]

(2) PROBABILITY MASS FUNCTION [PMF]

If x is a discrete random variable then the function $p(x) = p(x=x)$ is called p.m.f. of x provided satisfy the following conditions:

(i) $p(x_i) \geq 0 \quad , \forall \ i = 1, 2, 3, \ldots$

(ii) $\sum_{i=1}^{\infty} p(x_i) = 1$

(3) TO FIND CONSTANTS [k, a, c ...]

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

(4) CUMULATIVE DISTRIBUTION FUNCTION (or) DISTRIBUTION FUNCTIO

$$F(x) = P(X \le x) = \sum_{x_i \le x} p(x_i)$$

If cumulative distribution is given then to find p.m.f, $P[X = x_i] = F[x_i] - F[x_i - 1]$

(5) TO FIND MEAN (or) FIRST MOMENT :

$$E(X) = \mu_1' = \sum_{i=1}^{\infty} x_i\, p(x_i)$$

(6) TO FIND SECOND MOMENT :

$$E(X^2) = \mu_2' = \sum_{i=1}^{\infty} x_i^2\, p(x_i)$$

(7) TO FIND VARIANCE:

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \sum_{i=1}^{\infty} x_i^2 p(x_i) - \left[\sum_{i=1}^{\infty} x_i\, p(x_i)\right]^2$$

(8) TO FIND MOMENTS :

$$\mu_1' = E(x) = \sum_{i=1}^{\infty} x_i p(x_i) \text{ , first moment}$$

$$\mu_2' = E(x^2) = \sum_{i=1}^{\infty} x_i^2 p(x_i) \text{ , second moment}$$

$$\mu_3' = E(x^3) = \text{ , third moment}$$

$$\mu_4' = E(x^4) = \text{ , fourth moment.}$$

.....

$$\mu_r' = E(x^r) = \sum_{i=1}^{\infty} x_i^r p(x_i) \text{ , } r^{th} \text{ moment.}$$

9) MOMENT GENERATING FUNCTION [MGF] :

$$M_x(t) = E[e^{tx}] = \sum_{x=0}^{\infty} e^{tx} p(x)$$

A random variable X has the following probability function:

$X = x$:	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	$3k$

(i) Find the value of k

(ii) Find $P(X<2)$; $P(-2<x<2)$; $P(0<x\leq 3)$; $P(-1\leq x\leq 3)$

(iii) Find the distribution function of x.

(iv) Find Mean & Variance.

(v) Find 3rd moment.

(vi) Find moment generating function.

Soln:

(i) To Find k:

$$\sum_{i=1}^{\infty} P(x_i) = 1$$

$$0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$$

$$0.6 + 6k = 1$$

$$6k = 0.4$$

$$k = 4/60$$

(i) $k = 1/15$

$x = x$:	-2	-1	0	1	2	3
$P(x)$:	$1/10$	$4/15$	$2/10$	$2/15$	$3/10$	$3/15$

(a) $P(x<2) = P(x=-2) + P(x=-1) + P(x=0) + P(x=1)$

$= 1/10 + 4/15 + 2/10 + 2/15$

$= 4/2$

(b) $P(-2<x<2) = P(x=-1) + P(x=0) + P(x=1)$

$= 4/15 + 2/10 + 2/15$

$= 2/5$

(c) $P(0<x\leq 3) = P(x=1) + P(x=2) + P(x=3)$

$= 2/15 + 3/10 + 3/15$

$= 19/30$

(d) $P(-1\leq x\leq 3) = P(x=-1) + P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 4/15 + 2/10 + 2/15 + 3/10 + 3/15$

$= 9/10$

(iii) Distribution function of x:

x :	-2	-1	0	1	2	3
F(x):	1/10	1/6	11/30	1/2	4/5	1

x : x	$F(x) = p(x \le x)$
-2	$F(-2) = p(x \le -2) = 1/10$
-1	$F(-1) = p(x \le -1) = p(x=-2) + p(x=-1) = 1/10 + 1/15 = 1/6$
0	$F(0) = p(x \le 0) = p(x=-2) + p(x=-1) + p(x=0) = 11/30$
1	$F(1) = p(x \le 1) = p(x=-2) + p(x=-1) + p(x=0) + p(x=1) = 1/2$
2	$F(2) = p(x \le 2) = p(x=-2) + p(x=-1) + p(x=0) + p(x=1) + p(x=2)$
3	$F(3) = p(x \le 3) = p(x=-2) + p(x=-1) + p(x=0) + p(x=1) + p(x=2) + p(x=3) = 1$

To find Mean & Variance:

$$E(x) = \sum_i x_i\, p(x_i)$$

$$= (-2) \times 1/10 + (-1) \times 1/15 + 0 + 1 \times 2/15 + 2 \times 3/10 + 3 \times 3/15$$

$$= -\frac{1}{5} - \frac{1}{15} + \frac{2}{15} + \frac{3}{5} + \frac{3}{5}$$

$$= 16/15$$

$$E(x^2) = \sum_i x_i^2 p(x_i)$$

$$= (-2)^2 \times 1/10 + (-1)^2 \times 1/15 + 0 + 1 \times 2/15 + (2)^2 \times 3/10 + (3)^2 \times 3/15$$

$$= \frac{4}{10} + \frac{1}{15} + \frac{2}{15} + \frac{12}{10} + \frac{27}{15}$$

$$= 18/5$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= 18/5 - (16/15)^2$$

(v) To find 3rd moment:

$$E(x^3) = \sum_i x_i^3 p(x_i)$$

$$= (-2)^3 \times 1/10 + (-1)^3 \times 1/15 + 0 + 2/15 + (2)^3 \times 3/10 + (3)^3 \times 3/15$$

$$= -\frac{4}{5} - \frac{1}{15} + \frac{2}{15} + \frac{12}{5} + \frac{27}{5} = \frac{106}{15}$$

(vi) To find Moment generating function:

$$M_x(t) = E(e^{tx}) = \sum_{x=-2}^{3} e^{tx} p(x)$$

$$= e^{-2t}(1/10) + e^{-t}(1/15) + 2/10 + e^{t}(2/15) + e^{2t}(3/10) + e^{3t}(3/15)$$

A discrete random variable x has the probability function given below:

x:	0	1	2	3	4	5	6	7
$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) the value of k.

(ii) $p(x<6)$, $p(x\geq 6)$, $p(0<x<4)$

(iii) Distribution function of x

(iv) Find the smallest value of x, if $p(x\leq x)>1/2$

(v) Find the probability function of $Y=2x+5$

(vi) $P(1/2<x<5/2 / x>1)$

Soln:

(i) WKT $\sum_{i=1}^{\infty} p(x_i)=1$

$$p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)+p(x=5)+p(x=6)+p(x=7)=1$$

$$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$\Rightarrow 10k^2+9k-1=0$$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0$

$\Rightarrow 10k(k+1)-(k+1)=0$

$\Rightarrow (k+1)(10k-1)=0$

$\Rightarrow k=-1 ; k=1/10$

$k=-1$ is impossible. we choose $k=1/10$ since probability value ≥ 0.

$\therefore$

x:	0	1	2	3	4	5	6	7
$p(x)$:	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

(ii) (a) $P(X<6) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)$

$= 0 + 1/10 + 2/10 + 2/10 + 3/10 + 1/100$

$= \frac{10+20+20+30+1}{100}$

$= \frac{81}{100}$

(b) $P(X \geq 6) = 1-P(X<6)$

$= 1 - \frac{81}{100}$

$= \frac{19}{100}$.

(c) $P(0<x<4) = P(x=1) + P(x=2) + P(x=3)$

$= 1/10 + 2/10 + 2/10 = 5/10$

(iii) Distribution function of x:

x :	0	1	2	3	4	5	6	7
F(x) :	0	1/10	3/10	5/10	8/10	81/100	83/100	100/100 = 1

(iv) The smallest value of x, if $P(x \le x) > 1/2$

x :	0	1	2	3	4	5	6	7
F(x) :	0	0.1	0.3	0.5	0.8	0.81	0.83	1

Since $P(x \le x) > 1/2 = 0.5$, this is true for $x = 4, 5, 6, 7$

the smallest value of x is 4. $\Rightarrow x = 4$.

(v) probability function of $Y = 2x + 5$

x = x :	0	1	2	3	4	5	6	7
Y = 2x+5 :	5	7	9	11	13	15	17	19

Y	:	5	7	9	11	13	15	17	19
$P(Y)$	:	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$

(vi) $P(1/2 < x < 5/2 \,/\, x > 1)$

WKT $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{P\left(\left(1/2 < x < 5/2\right) \cap (x > 1)\right)}{P(x > 1)}$$

$$= \frac{P[(0.5 < x < 2.5) \cap (x > 1)]}{P(x > 1)}$$

$$= \frac{P\left[[(x=1),(x=2)] \cap [(x=2),(x=3),(x=4),(x=5),(x=6),(x=7)]\right]}{P[(x=2),(x=3),(x=4),(x=5),(x=6),(x=7)]}$$

$$= \frac{P(x=2)}{P((x=2),(x=3),(x=4),(x=5),(x=6),(x=7))}$$

$$= \frac{2/10}{2/10 + 2/10 + 3/10 + 1/100 + 2/100 + 17/100} = \frac{2/10}{90/100} = 2/9$$