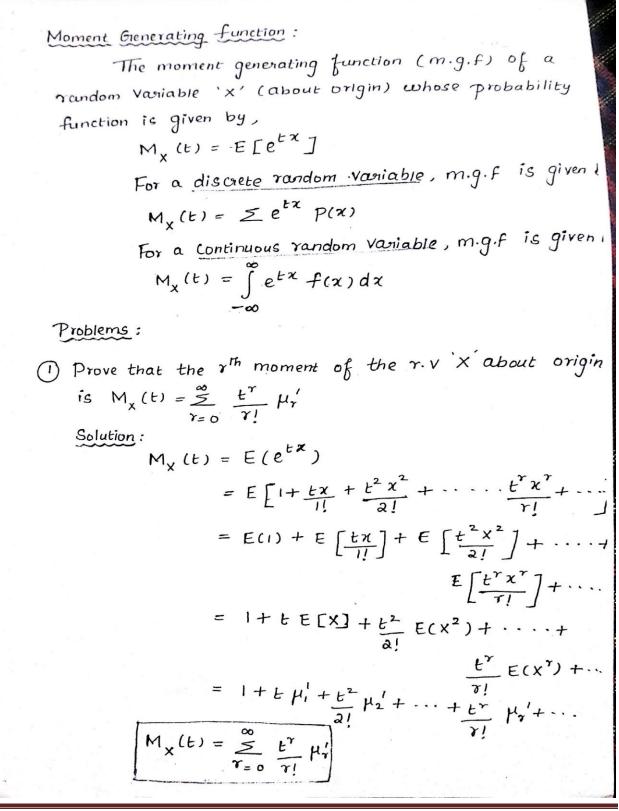


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) A random variable x has the probability function  

$$f(x) = \frac{1}{a^{x}}, x = 1, 2, 3, \dots \infty$$
Find its (i) M.G.F (ii) Mean & Variance (iii) P(x is even  
Solution: (iv) P(x ≥ 5) and  $\pounds P(x \text{ is divisible by } 3)$   
(i) M.G.F = M<sub>x</sub>(t) =  $E(e^{tx})$   

$$= \frac{\infty}{x_{e_1}} e^{tx} \cdot \frac{1}{a^{x}}$$

$$= \frac{\infty}{x_{e_1}} \left(\frac{e^{t}}{a}\right)^{x}$$

$$= \frac{e^{t}}{a} + \left(\frac{e^{t}}{a}\right)^{2} + \left(\frac{e^{t}}{a}\right)^{3} + \dots$$

$$= \frac{e^{t}}{a} + \left(\frac{e^{t}}{a}\right)^{2} + \left(\frac{e^{t}}{a}\right)^{2} + \dots$$

$$= \frac{e^{t}}{a} \left(1 - \frac{e^{t}}{a}\right)^{-1}$$

$$= \frac{e^{t}}{a} \cdot \frac{x}{a - e^{t}}$$
(ii) Mean =  $E(x) = M_{x}'(0)$   
 $M_{x}'(t) = \frac{(a - e^{t})e^{t} - e^{t}(-e^{t})}{(a - e^{t})^{2}}$ 

$$= \frac{ae^{t} - e^{2t} + e^{2t}}{(a - e^{t})^{2}}$$

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$$E(x) = M_{x}'(0) = \frac{2}{x^{k_{1}}} = \frac{1}{2} 2$$

$$E(x) = \frac{1}{2k}$$

$$M_{x}''(t) = \frac{(2-e^{t})^{2}(2e^{t}) - 2e^{t} \cdot 2(2-e^{t})(-e^{t})}{(2-e^{t})^{2}}$$

$$= \frac{2e^{t}(2-e^{t})[2-e^{t} + 2e^{t}]}{(2-e^{t})^{4}}$$

$$= \frac{2e^{t}(2+e^{t})}{(2-e^{t})^{3}}$$

$$M_{x}''(0) = \frac{2(2+1)}{(2-1)^{3}} = 2(3) = 6$$

$$\frac{E(x^{2}) = 6}{(2-e^{t})^{3}}$$

$$\frac{E(x^{2}) = 6}{(2-e^{t})^{3}}$$

$$\frac{E(x^{2}) = 6}{(2-e^{t})^{3}}$$

$$\frac{E(x^{2}) = 6}{(2-e^{t})^{3}} = 2(3) = 6$$

$$\frac{1}{(2-1)^{3}} = 2(3) = 6$$

$$\frac{1}{(2-1)^{3$$

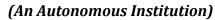
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(iii)

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(iv) 
$$P(X \ge 5) = P(X = 5) + P(X = 6) + P(X = 7) + \dots \infty$$
  

$$= \frac{1}{2^5} + \frac{1}{2^4} + \frac{1}{2^7} + \dots \infty$$

$$= \frac{(\frac{1}{2})^5}{(\frac{1}{2})^5} + (\frac{1}{2})^6 + (\frac{1}{2})^7 + \dots \infty$$

$$= \frac{(\frac{1}{2})^5}{(\frac{1}{2})^6} = \frac{1}{\frac{1}{2^2}} = \frac{1}{16}$$

$$P(X \ge 5) = \frac{1}{16}$$

$$P(X \ge 5)$$

$$P($$

x = 0  $= e^{0} f(0) + e^{t} f(1) + e^{2t} f(2) + \cdots \infty$   $= 0 + e^{t} \cdot \frac{2}{3} + e^{2t} \left(\frac{1}{3}\right) + 0$   $M_{x}(t) = \frac{2e^{t}}{3} + \frac{e^{2t}}{3}$ 

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Find the MGF of a random variable 'x' having the density function sity function  $f(x) = \begin{cases} \frac{x}{2}, & 0 \le x \le 2 \\ 0, & 0 \end{cases}$  KUsing the generating fn find the first four moments for the origin for the origiSolution : (i)  $M_x(t) = \int e^{tx} f(x) dx$ =  $\int_{a}^{a} e^{tx} \cdot \frac{x}{a} dx$  $=\frac{1}{a}\int xe^{tx}dx$  $u = x, v = e^{t}$  $u' = 1, v_{i} = \frac{e}{1}$  $= \frac{1}{2} \left[ \frac{x e^{tx}}{t} - \frac{e^{tx}}{L^2} \right]^2$  $V_{a} = e^{t}$  $U_{v_{1}} - u'_{v_{2}}$  $=\frac{1}{2}\left[\frac{2e^{2t}}{L}-\frac{e^{2t}}{L^{2}}+\frac{1}{L^{2}}\right]$  $M_{x}(t) = \frac{1}{2t^{2}} \left[ 2te^{2t} - e^{2t} + 1 \right]$  $M_{X}^{2}(t) = \frac{2te^{2t}}{2t^{2}} - \frac{e^{2t}}{2t^{2}} + \frac{1}{2t^{2}}$ (ii)  $= t^{-1}e^{2t} - \frac{1}{2}t^{-2}e^{2t} + \frac{1}{2}t^{-2}$  $M_{\chi}'(t) = e^{2t} \left(\frac{-1}{t^2}\right) + t^{-1} \frac{e^{2t}}{2} - \frac{1}{2} \left[\frac{e^{2t}}{2} t^{-2} + \frac{e^{2t}}{2}\right]$  $+ \frac{1}{2} (-2) \frac{1}{t^3}$  $= -\frac{e^{2t}}{L^2} + \frac{e^{2t}}{2t} - \frac{e^{2t}}{4t^2} + \frac{e^{2t}}{L^3} - \frac{1}{t^3}$ 09.07.2019 S.GOWRI/AP/MATHEMATICS MOMENTS Page 5 OF 7



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$$\begin{array}{l} \textcircled{\bullet} \quad \text{let 'x' be a random Variable with pd f} \\ f(x) = \begin{cases} \frac{1}{3} e^{-x/3}, & x \neq 0 \\ 0, & \text{otherwise} \end{cases} \\ \hline f(x) = \begin{cases} \frac{1}{3} e^{-x/3}, & x \neq 0 \\ 0, & \text{otherwise} \end{cases} \\ \hline find (3) \quad p(x \neq 3) \quad \text{in MGF of } x \quad (\text{in }) \quad E(x) \quad \text{and } Var(x). \end{cases} \\ \hline \text{Solution }: \\ \hline \text{Griven }: \quad f(x) = \begin{cases} \frac{1}{3} e^{-x/3}, & x \neq 0 \\ 0, & \text{otherwise}. \end{cases} \\ (i) \quad P(x \neq 3) = \int_{3}^{\infty} f(x) \, dx \\ &= \int_{3}^{\infty} \frac{1}{3} e^{-x/3} \\ \frac{1}{3} e^{-x/3} \end{bmatrix}_{3}^{\infty} \\ &= \frac{e^{-1}}{\frac{1}{3}} \left[ \frac{e^{-x/3}}{-1/3} \right]_{3}^{\infty} \\ &= \frac{e^{-1}}{\frac{1}{3}} \left[ \frac{P(x \neq 3) = 0.3679}{e^{1} x + f(x) \, dx} \\ &= \int_{0}^{\infty} e^{1x} f(x) \, dx \\ &= \int_{0}^{\infty} e^{1x} f(x) \, dx \\ &= \int_{0}^{\infty} e^{1x} \frac{1}{3} e^{-x/3} \, dx \\ &= \int_{0}^{\infty} e^{1x} \frac{1}{3} e^{-x/3} \, dx \\ &= \frac{1}{3} \int_{0}^{\infty} e^{\left(\frac{1}{3} - \frac{1}{3}\right)x} \, dx \\ &= \frac{1}{3} \left[ \frac{e^{-\left(\frac{1}{3} - \frac{1}{3}\right)}}{-\left(\frac{1}{3} - \frac{1}{3}\right)} \right]_{0}^{\infty} \end{array}$$



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 $M_{x}(t) = -\frac{1}{3} \left[ 0 - \frac{1}{\frac{1}{3} - t} \right] = +\frac{1}{3} \left( \frac{1}{\frac{1 - 3t}{3}} \right)$  $M_{\chi}(t) = \frac{1}{1-3t}$ (iii)  $M_x(t) = \frac{1}{1-3t} = (1-3t)^{-1}$  $M_{\chi}^{\prime}(t) = -\frac{1}{(1-3t)^{a}}(-3) = \frac{3}{(1-3t)^{2}}$  $M'_{x}(0) = \frac{3}{1-0} = 3$  $E(x) = Mean = M'_{x}(0) = 3$  $M_{x}''(t) = -6(1-3t)^{-3}(-3)$  $= 18(1-3t)^{-3}$  $M_{x}^{n}(0) = 18$  $E(x^2) = M_x''(0) = 18$  $Var(x) = E(x^2) - [E(x)]^2$  $= 18 - (3)^{2}$ = 18 - 9 Var(X) = 9