Continuous Random Variable :

A random variable 'X' is called a continuous random variable if it takes all possible values in a given interval.

Examples : Age , Height and Weight

Distribution function (or) Cumulative Distribution function of the random variable X :

The C.D.F of a continuous random variable X is defined as,

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\, \cancel{dt}\, dx$$

Probability Density function : (P.D.f)

Let X be a continuous random variable. The function $f(x)$ is called the p.d.f of the random variable X if it satisfies the following conditions :

(i) $f(x) \ge 0 \ , \ -\infty < x < \infty$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Remark :

1. $P(a < X < b) = P(a \le X \le b) = \int_{a}^{b} f(x)\, dx$

2. $P(X > a) = \int_{a}^{\infty} f(x)\, dx$

3. $P(X < a) = \int_{-\infty}^{a} f(x)\, dx$

4. $P(X > a / X > b) = \frac{P(X > a)}{P(X > b)}$

(4) If 'x' is a Continuous random variable whose p.d.f is given by,

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find (a) What is the value of 'c'? (b) Find $P(X > 1)$

Solution :

(a) Given: $f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases} \longrightarrow (1)$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\therefore \int_0^2 c(4x - 2x^2)\,dx = 1$$

$$c\left[4\frac{x^2}{2} - 2\frac{x^3}{3}\right]_0^2 = 1$$

$$c\left[2(2^2) - \frac{2}{3}(2^3)\right] = 1$$

$$c\left[8 - \frac{16}{3}\right] = 1 \Rightarrow c\left(\frac{24-16}{3}\right) = 1$$

$$c\left(\frac{8}{3}\right) = 1$$

$$\boxed{c = \frac{3}{8}}$$

Put $c = \frac{3}{8}$ in (1),

$$f(x) = \begin{cases} \frac{3}{8}(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{Otherwise} \end{cases}$$

(b) $P(X>1) = \int_1^\infty f(x)\,dx$

$$= \int_1^2 f(x)\,dx + \int_2^\infty f(x)\,dx$$

$$= \int_1^2 \frac{3}{8}(4x-2x^2)\,dx + 0$$

$$= \frac{3}{8} \times 2 \int_1^2 (2x-x^2)\,dx$$

$$= \frac{3}{4}\left[2\frac{x^2}{2} - \frac{x^3}{3}\right]_1^2$$

$$= \frac{3}{4}\left[(4-1) - \frac{1}{3}(8-1)\right]$$

$$= \frac{3}{4}\left[3-\frac{7}{3}\right] = \frac{\cancel{3}}{4}\left[\frac{9-7}{\cancel{3}}\right] = \frac{2}{4} = \frac{1}{2}$$

$$\boxed{P(X>1) = \frac{1}{2}}$$

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5) The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with probability density function given by,

$$f(x) = \begin{cases} \lambda e^{-x/100}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

What is the probability that (a) a computer will function between 50 and 150 hrs, before breaking down (b) it will function less than 500 hours.

Solution:

Given: $f(x) = \begin{cases} \lambda e^{-x/100} & , x \geq 0 \\ 0 & , x < 0 \end{cases} \longrightarrow (1)$

Since $f(x)$ is a p.d.f of 'x',

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{0}^{\infty} \lambda e^{-x/100}\,dx = 1$$

$$\lambda \left[\frac{e^{-x/100}}{-1/100} \right]_0^{\infty} = 1$$

$$\lambda(-100)\left[e^{-\infty} - e^{0}\right] = 1$$

$$\lambda(-100)\left[0 - 1\right] = 1$$

$$100\lambda = 1$$

$$\boxed{\lambda = \frac{1}{100}}$$

(a) We know that,

$$P(a < x < b) = \int_a^b f(x)\,dx$$

$$P(50 < x < 150) = \int_{50}^{150} f(x)\,dx$$

$$= \int_{50}^{150} \frac{1}{100} e^{-x/100}\,dx$$

$$= \frac{1}{100}\left[\frac{e^{-x/100}}{-1/100}\right]_{50}^{150}$$

$$= -\left[e^{-150/100} - e^{-50/100}\right]$$

$$= -e^{-1.5} + e^{-0.5}$$

$$= -0.2231 + 0.6065$$

$$= 0.3834$$

(b) $P(X < 500) = \int_0^{500} f(x)\, dx$

$$= \int_0^{500} \frac{1}{100} e^{-x/100}\, dx$$

$$= \frac{1}{100}\left[\frac{e^{-x/100}}{-1/100}\right]_0^{500}$$

$$= -\left[e^{-500/100} - e^{0}\right]$$

$$= -\left[e^{-5} - 1\right]$$

$$= 1 - e^{-5}$$

$$= 1 - 0.0067$$

$$\boxed{P(X < 500) = 0.9933}$$