



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

DEPARTMENT OF MATHEMATICS

RANDOM VARIABLES

Definition: A real variable 'X' whose value is determined by the outcome of a random experiment is called a random variable.

Example: A random experiment consists of two tosses of a coin. Consider the random variable which is the number of heads (0, 1 or 2).

Outcome :	HH	HT	TH	TT
Value of X :	2	1	1	0

Types of Random Variables :

There are two types of random variables :

1. Discrete random variable
2. Continuous random variable.

Discrete random variable :

A random variable which can assume only a countable number of real values is called a discrete random variable.

Examples :

1. Number of telephone calls per unit time.
2. Marks obtained in a test.

Distribution function (or) Cumulative Distribution Function of the random variable X :

The C.D.F of a random variable X is defined as,

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i)$$

in $(-\infty, \infty)$



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PROBLEMS :

- ① Find the constant 'k' from the following probability distribution of a discrete random variable X.

Values of $X=x$	1	2	3	4	5	Total
$p(x)$	0.1	0.2	k	2k	0.1	1

Solution: We know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\therefore \sum_{i=1}^5 p(x_i) = 1$$

$$p(1) + p(2) + p(3) + p(4) + p(5) = 1$$

$$0.1 + 0.2 + k + 2k + 0.1 = 1$$

$$3k + 0.4 = 1$$

$$3k = 1 - 0.4 = 0.6$$

$$3k = 0.6$$

$$k = \frac{0.6}{3} = 0.2$$

$$\boxed{k = 0.2}$$

- ② For the following probability distribution,
- Find the distribution function of X,
 - What is the smallest value of 'x' for which $P(X \leq x) > 0.5$

Solution:

- (i) The distribution function of 'X' is given by,



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$x = x$	$F(x) = P(X \leq x)$
0	$F(0) = P(X \leq 0) = P(X=0) = \frac{1}{4} = 0.25$
1	$F(1) = P(X \leq 1) = P(X=0) + P(X=1)$ $= \frac{1}{4} + \frac{2}{4} = \frac{3}{4} = 0.75$
2	$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$ $= \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$

(ii) The smallest value of 'x' for which $P(X \leq x) > 0.5$ is 1.

③ A random variable 'x' has the following probability function

Value of $x = x$	0	1	2	3	4	5	6	7
$P(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

- (i) Find k (ii) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$.
 (iii) If $P(X \leq k) > \frac{1}{2}$, find the minimum value of k and determine the distribution function of X.

Solution:

(i) We know that,

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

$$\sum_{i=0}^7 p(x_i) = 1$$

$$p(0) + p(1) + p(2) + \dots + p(7) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$



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$10k^2 + 9k - 1 = 0$

$\therefore k = \frac{-9 \pm \sqrt{81 + 40}}{20}$ $-b \pm \sqrt{b^2 - 4ac}$ $a=10$
 $k = \frac{\quad}{20}$ $b=9$
 $c=-1$

$= \frac{-9 \pm \sqrt{121}}{20} = \frac{-9 \pm 11}{20} = \frac{-9+11}{20} \text{ (or)} \frac{-9-11}{20}$

$\therefore k = \frac{2}{20}, \frac{-20}{20}$

$k = \frac{1}{10} \text{ or } k = -1$

$k = -1$ is not possible, Since probability cannot be a negative value.

$\therefore \boxed{k = \frac{1}{10}}$

Subs $k = \frac{1}{10}$ in the given table,

$X = x$	0	1	2	3	4	5	6	7
$p(x)$	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{7}{100} + \frac{1}{10}$ $= \frac{17}{100}$

(ii) $P(X < 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$

$= \frac{10 + 20 + 20 + 30 + 1}{100}$

$\boxed{P(X < 6) = \frac{81}{100}}$



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$$P(X \geq 6) = 1 - P(X < 6)$$

$$= 1 - \frac{81}{100} = \frac{100 - 81}{100}$$

$$P(X \geq 6) = \frac{19}{100}$$

$$P(0 < X < 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10} = \frac{4}{5}$$

$$P(0 < X < 5) = \frac{4}{5}$$

(iii) Given: $P(X \leq k) > \frac{1}{2}$

By trial, $k = 4$

Distribution function of X:

$$P(X \leq 3) = P(X=0) + \dots + P(X=3)$$

$$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10}$$

$$= 0.5$$

$$P(X \leq 4) = P(X=0) + \dots + P(X=4)$$

$$= 0.8$$

$X=x$	$F(x) = P(X \leq x)$
0	$F(0) = P(X \leq 0) = P(X=0) = 0$
1	$F(1) = P(X \leq 1) = P(X=0) + P(X=1)$ $= 0 + \frac{1}{10} = \frac{1}{10}$
2	$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$ $= 0 + \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$
3	$F(3) = P(X \leq 3) = P(X=0) + P(X=1) + P(X=2)$ $+ P(X=3)$ $= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} = \frac{5}{10}$



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$$\begin{aligned} 4 \quad F(4) &= P(X \leq 4) = P(X=0) + \dots + P(X=4) \\ &= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} \\ &= \frac{8}{10} \end{aligned}$$

$$\begin{aligned} 5 \quad F(5) &= P(X \leq 5) = P(X=0) + \dots + P(X=5) \\ &= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} \\ &= \frac{10 + 20 + 20 + 30 + 1}{100} \\ &= \frac{81}{100} \end{aligned}$$

$$\begin{aligned} 6 \quad F(6) &= P(X \leq 6) = P(X=0) + \dots + P(X=6) \\ &= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100} \\ &= \frac{10 + 20 + 20 + 30 + 1 + 2}{100} \\ &= \frac{83}{100} \end{aligned}$$

$$\begin{aligned} 7 \quad F(7) &= P(X \leq 7) = P(X=0) + \dots + P(X=7) \\ &= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100} \\ &\quad + \frac{2}{100} + \frac{17}{100} \\ &= \frac{83}{100} + \frac{17}{100} = 1 \end{aligned}$$