



UNIT 5 - Multiple Integrals

Change of order of integration.

① Change the order of integration for $\int_0^1 \int_0^x f(x,y) dy dx$

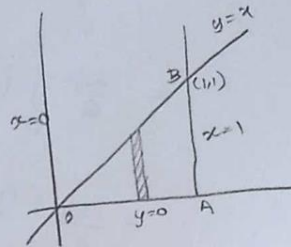
Solution:-

Given Integral is not in the correct order

let us rearrange it

$$I = \int_0^1 \int_0^x f(x,y) dy dx.$$

Given:- $y=0$ to $y=x$
 $x=0$ to $x=1$

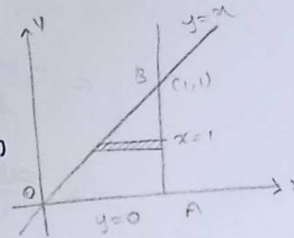


Inner limit is w.r.t y

∴ It is a vertical strip

Now to change the order of integration

we have to draw a horizontal strip



x limits : $x=y$ to $x=1$

y limits : $y=0$ to $y=1$

$$\therefore I = \int_0^1 \int_y^1 f(x,y) dx dy.$$

2. Change the order of integration in $\int_0^1 \int_0^y f(x,y) dx dy$

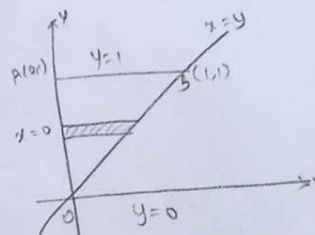
Given limits:

x limit : $x=0$ to $x=y$

y limit : $y=0$ to $y=1$

Inner limit is w.r.t 'x'

∴ Given limit is a horizontal strip





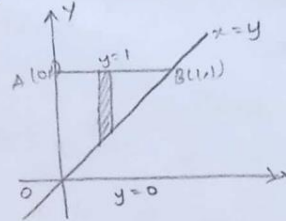
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Now to change the order of integration we have to draw a vertical strip

x limit : $x=0$ to $x=1$

y limit : $y=x$ to $y=1$

$$I = \int_0^1 \int_x^1 f(x,y) dy dx.$$



3. Evaluate by changing the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy dx$

Solution:-

$$I = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy dx$$

Given limits:

x limits $x=0$ to $x=4$

y limits $y = \frac{x^2}{4}$ to $y = 2\sqrt{x}$

(i) $x^2=4y$ to $y^2=4x$

Inner limit is w.r.t 'y'

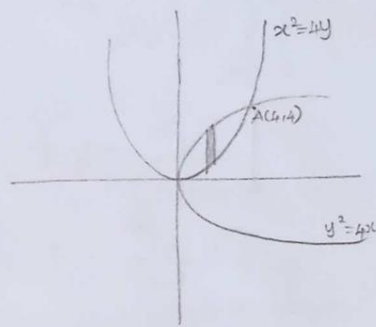
∴ It is a vertical strip

To find point A:

Solving $x^2=4y$ and $y^2=4x$

Squaring, $x^4 = 16y^2$
 $x^4 = 16(4x)$
 $x^4 = 64x$
 $x^3 = 64$
 $x = 4$
A(4,4)

Sub x in (i)
 $y^2 = 4x$
 $y^2 = 4(4)$
 $y^2 = 16$
 $y = 4$

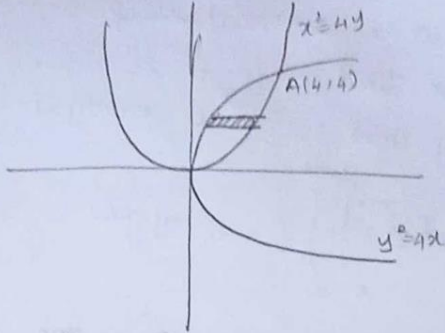




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Now to change the order of integration draw a horizontal strip

x limits :
 $x = \frac{y^2}{4}$ to $x = 2\sqrt{y}$
 $y = 0$ to $y = 4$



$\therefore I = \int_0^4 \int_{\frac{y^2}{4}}^{2\sqrt{y}} dx dy$

$= \int_0^4 \left[x \right]_{\frac{y^2}{4}}^{2\sqrt{y}} dy = \int_0^4 [2\sqrt{y} - \frac{y^2}{4}] dy$

$= \left[\frac{2y^{3/2}}{3/2} - \frac{1}{4} \frac{y^3}{3} \right]_0^4$

$= \left[\frac{2(4)^{3/2}}{3/2} - \frac{1}{4} \frac{(4)^3}{3} \right]$

$= \left[\frac{4}{3}(8) - \frac{16}{3} \right] = \frac{32-16}{3}$

$I = \frac{16}{3}$

4. Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy dy dx$ and then evaluate. $\frac{3}{8} a^4$

The region of integration R is bounded by the curve $y = \frac{x^2}{a}$ i.e. the parabola $x^2 = ay$, the line $y = 2a - x$, i.e. $x + y = 2a$ and the lines $x = 0$ and $x = a$.