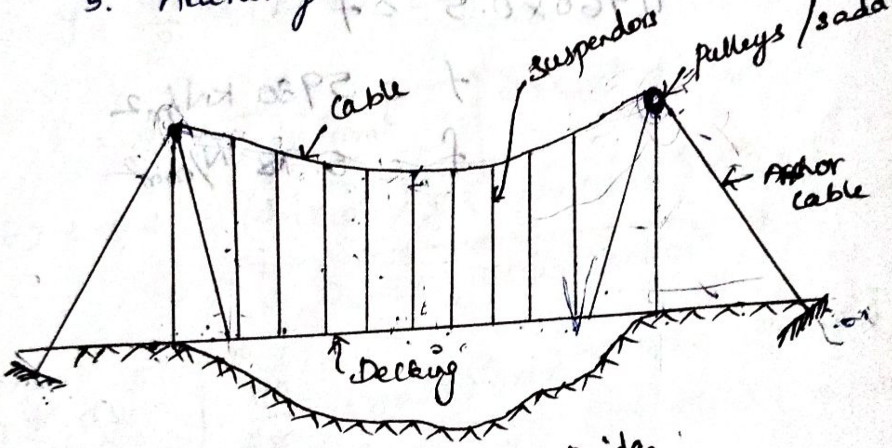
Unit - 4

Cables and Suspension Bridges

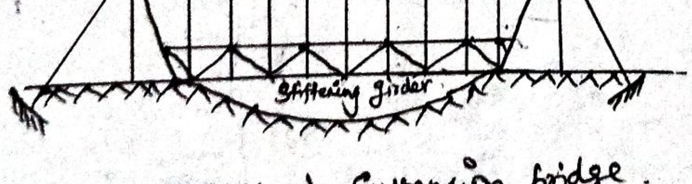
Suspension Bridges:

Suspension bridges are used for highways where the span of a bridge is more than 200m. Suspension bridges consists of the following.

1. The cable
2. Suspenders
3. Decking, including the stiffening girders
4. Supporting tower
5. Anchorage.



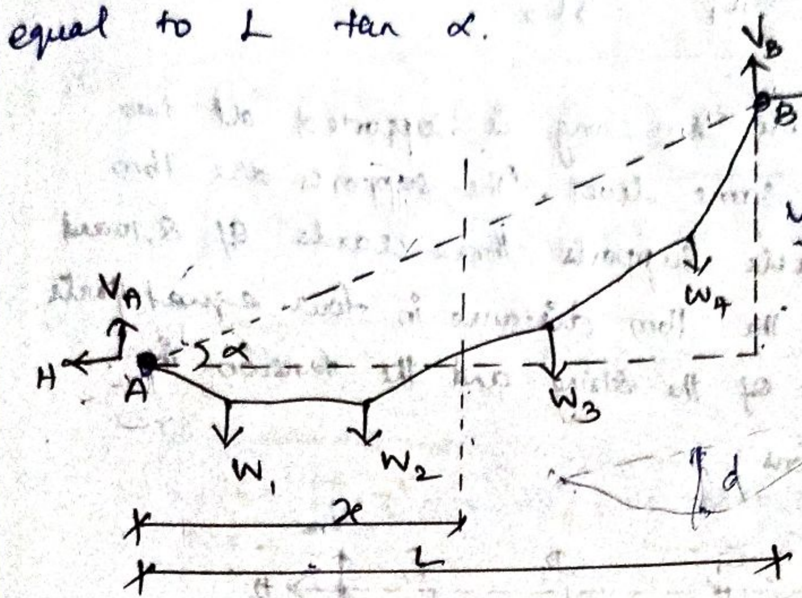
Unstiffened Suspension Bridge



Stiffened Suspension bridge.

Equillibrium of Light cable - General cable theorem:

A light cord (or) cable suspended from two points A and B and subjected to a number of point loads $W_1, W_2 \dots W_n$. Let L be the horizontal span of the cable and α between the inclination of line AB with horizontal. The difference in elevation B is equal to $L \tan \alpha$.



$\tan \alpha = \frac{y}{L}$

$y = L \tan \alpha$

Assume, that the cable is perfectly flexible, so that the bending moment at any point on the cable is zero.

$$Hy = \frac{x}{L} \Sigma M_B - \Sigma M_x$$

Uniformly loaded cable:

1. Horizontal Reaction:

$$H = \frac{PL^2}{8d}$$

2. Cable tension at the ends:

$$\tan \beta = \frac{L}{4d}$$

3. Shape of the cable:

$$y = \frac{4dx}{L^2}(L - x)$$

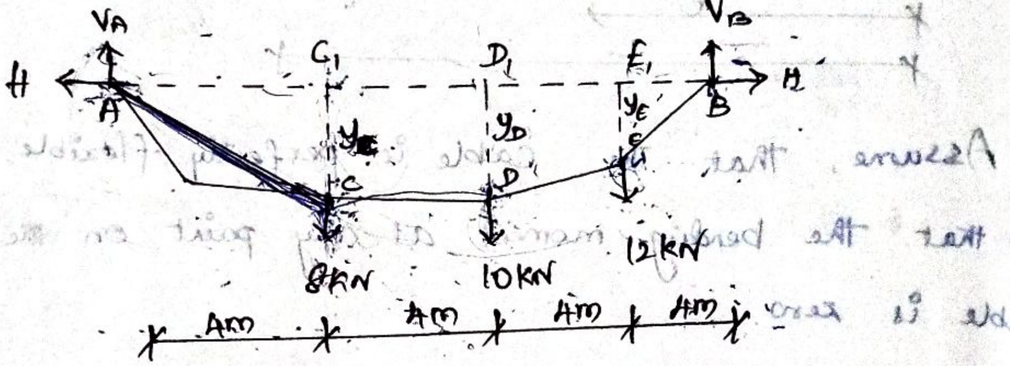
4. Length of the cable : Both ends at same level :

$$S = L + \frac{8d^2}{3L}$$

5. Length of the cable : Ends at different level :

$$S = L + \frac{2d_1^2}{3L_1} + \frac{2d_2^2}{3L_2}$$

1. A light cable, 18m long is supported at two ends at the same level. The supports are 16m apart. The cable supports three loads of 8, 10 and 12 kN dividing the 16m distance in four equal parts. Find the shape of the string and the tension in various portions.



Soln :-

Let, $CC_1 = y_c$, $DD_1 = y_D$, $EE_1 = y_E$

$\Sigma M_B = 0 \Rightarrow 16 \times V_A - [(8 \times 12) + (10 \times 8) + (12 \times 4)] = 0$

$V_A = 14 kN$

The cable is in equillibrium, the shape taken by it is that of a funicular polygon. Thus the shape represents the bending moment diagram to some scale.

$y_c : y_D : y_E = (14 \times 4) : (14 \times 8 - 8 \times 4) : (14 \times 12 - 8 \times 8 - 10 \times 4)$

$= 56 : 80 : 64 = 7 : 10 : 8$

$= 1 : 10/7 : 8/7$

$y_D = {}^{10}/_7 \, y_c$, $y_E = {}^{8}/_7 \, y_c$, $y_c = 1$

$AC = \sqrt{4^2 + y_c^2} = \sqrt{16 + y_c^2} = 4\sqrt{1 + 0.0625 y_c^2}$

$CD = \sqrt{4^2 + (y_D - y_c)^2} = \sqrt{16 + ({}^{10}/_7 - 1)^2 y_c^2}$ $\boxed{(1+x)^n = 1 + \frac{xn}{1!}}$

$= \sqrt{16 + {}^{9}/_{49} \, y_c^2} = 4\sqrt{1 + 0.0115 y_c^2}$

$DE = \sqrt{16 + (y_D - y_E)^2} = \sqrt{16 + \left(\frac{10}{7} - \frac{8}{7}\right)^2 y_c^2}$

$= \sqrt{16 + {}^{4}/_{49} \, y_c^2} = 4\sqrt{1 + 0.0051 y_c^2}$

$EB = \sqrt{16 + y_E^2} = \sqrt{16 + \frac{8}{7} y_c^2}$

$= \sqrt{16 + \frac{64}{49} y_c^2} = 4\sqrt{1 + 0.0816 y_c^2}$

Total length $AB = AC + CD + DE + EB$

$18 = 4\left[(1 + 0.0625 y_c^2)^{1/2} + (1 + 0.0115 y_c^2)^{1/2} + (1 + 0.0051 y_c^2)^{1/2} + (1 + 0.0816 y_c^2)^{1/2}\right]$

$(1+x)^n = 1 + \frac{nx}{1!}$

$4.5 = \left[\left(1 + \frac{0.0625}{2} y_c^2\right) + \left(1 + \frac{0.0115}{2} y_c^2\right) + \left(1 + \frac{0.0051}{2} y_c^2\right) + \left(1 + \frac{0.0816}{2}\right) y_c^2\right]$

$4.5 = 4 + 0.08 y_c^2$

$y_c = 2.5 \, m$,

$y_D = {}^{10}/_7 \times 2.5 = 3.57 \, m$

$y_E = \frac{8}{7} y_c = {}^{8}/_7 \times 2.5$

$= 2.86 \, m$

Soln :

$$V_A = \frac{5+5+5+5+5}{2} = 12.5\,kN$$

y_C: $CC_1 = GG_1 = M_C = 12.5 \times 5 = 62.5$

y_D: $DD_1 = FF_1 = M_D = (12.5 \times 10) - 5\times5 = 100$

y_E: $EE_1 = M_E = (12.5 \times 15) - (5\times10) - (5\times5) = 112.5$

$EE_1 : DD_1 : CC_1 = 112.5 : 100 : 62.5$

$y_E : y_D : y_C = 1 : 0.89 : 0.556$

But $y_E = 2.5m$.

$y_D = y_F = 2.5 \times 0.89 = 2.22\,m$

$y_C = y_G = 2.5 \times 0.556 = 1.39m$

The length of the cable $= 2(AC + CD + DE)$

$$= 2\left[5\left(1+\frac{1.39^2}{25}\right)^{1/2} + 5\left\{1+\frac{(2.22-1.39)^2}{25}\right\}^{1/2} + 5\left\{1+\frac{(2.5-2.22)^2}{25}\right\}^{1/2}\right]$$

$(1+x)^n = 1 + \frac{xn}{1!} + \dots$

$$= 10\left\{1+\frac{1.932}{50}+1+\frac{0.69}{50}+1+\frac{0.08}{50}\right\}$$

$= 30.54m.$

$$S = L + \frac{8d^2}{3L} = 30 + \frac{8}{3}\,\frac{2.5^2}{30}$$

$= 30.56m$

$M_C = 0 \Rightarrow H \times 1.39 - V_A \times 5 = (1.39H) - (5 \times 12.5)$

$H = 45\,kN$

Max tension in AC $= \sqrt{4x^2 + (12.5)^2}$

$= 46.6$ kN

Area required $= \frac{46.6 \times 1000}{140}$

$= 333\ mm^2$

Three Hinged Stiffening Girder:

The cable of the suspension bridge is the main load bearing member, the curvature of the cable of an unstiffened bridge changes as the load moves on the decking. To avoid this, the decking is stiffened by provision of either a three hinged stiffening girder or a two hinged stiffening girder.

1. Equillibrium of the cable:

$$y = \frac{4dx}{L^2}(L - x)$$

2. Equillibrium of the girder:

$$V_D = (1 - \alpha)$$

$$V_B = \alpha$$

3. Bending moment diagram:

$$M_x = \mu_x - Hy$$

4. Influence line for H:

$$H = \frac{\alpha L}{2d}$$

5. Influence line for P:

$$P = 2/L$$

6. Influence line for Bending moment :

$$M_x = -\frac{x}{L}(L-x)$$

7. Maximum Bending moment due to single point load W :

(i) Maximum positive BM :

$$M_{max}(+) = 0.096\,WL$$

(ii) Maximum Negative BM :

$$M_{max}(-) = -0.0625\,WL$$

8. Maximum BM due to UDL :

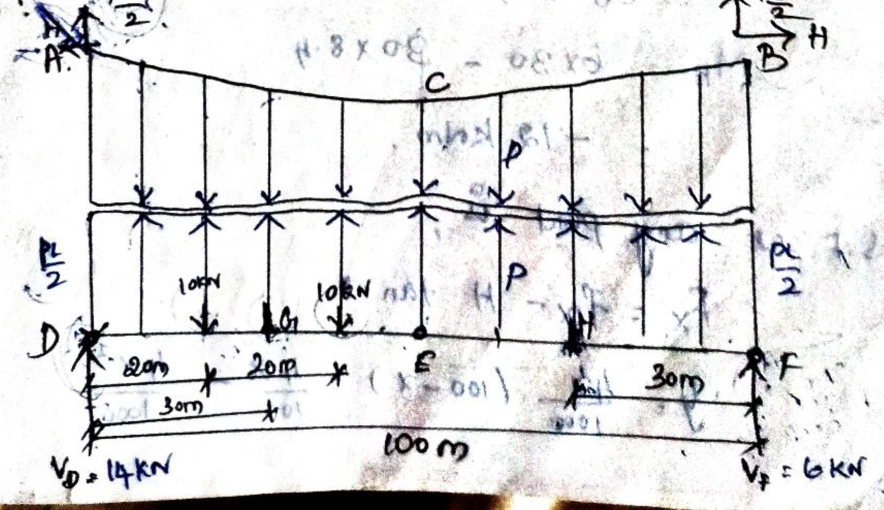
$$M_{max}(\pm) = \pm\,0.01883\,WL^2$$

9. Influence line for Shear force :

$$H \tan\theta = \left(1 - \frac{2x}{L}\right)$$

3. The three hinged stiffening girder of a suspension bridge of 100m span is subjected to two point loads of 10kN each placed at 20m and 40m respectively from the left hand hinge. Determine the BM and SF in the girder at section 30m from each end. Also determine the maximum tension in the cable which has a central dip of 10m.

Soln :



$V_F = \left(\frac{10\times 40 + 10\times 20}{100}\right) = 6\,kN$

$V_D = \frac{1}{100}(10\times 60 + 10\times 80) = 14\,KN$

For find H, Take moment about E.

$M_E = 0, = \mu_E - Hd$

$\mu_E = (6\times 50).$

$M_E = (6\times 50) - 10\times H = 0$

$H = 30\,kN$

The equation of The parabola is,

$y = \frac{4dx}{L^2}(L-x)$

$= \frac{4\times 10\times x}{100^2}(100-x)$

$y = \frac{4x}{1000}(100-x)$

At $x = 30m$,

$y = \frac{4\times 30}{1000}(100-30)$

$= 8.4m.$

Points G and H from D and E at 30m

B.M at any point is;

$M = M_x - Hy$

$M_G = ((14\times 30) - (10\times 10)) - (30\times 8.4)$

$= 68\,KNm$

$M_H = 6\times 30 - 30\times 8.4$

$= -72\,KNm.$

S.F at any point is,

$F_x = f_x - H \tan\theta$

$y = \frac{4x}{1000}(100-x) = \frac{4x}{100} - \frac{4x^2}{1000}$

$$\frac{dy}{dx} = \tan\theta = \frac{4}{10} - \frac{8x}{1000}$$

At $x = 30m$,

$$\tan\theta = \frac{4}{10} - \frac{8\times 30}{1000}$$

$$= 0.16$$

on H, at $x = 70$ (30 from B)

$\tan\theta = -0.16$ (Anticlockwise)

$$F_G = (14-10) - (30\times 0.16) = -0.8\ kN$$

$$F_H = (-6) - (30\times -0.16) = -1.2\ kN$$

$$H = \frac{pL^2}{8d}$$

$$p = \frac{8d}{L^2}H$$

$$= \frac{8\times 10}{100\times 100}\times 30$$

$$= 0.24\ kN/m$$

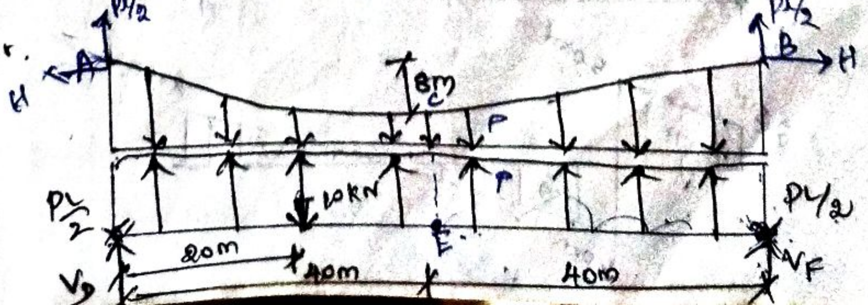
Vertical reaction at ends of the cable,

$$\frac{pL}{2} = \frac{0.24\times 100}{2} = 12\ kN$$

Maximum tension in the cable,

$$= \sqrt{12^2 + 30^2} = 32.4\ kN$$

4. A suspension bridge cable of span 80m and Central dip 8m is suspended from the same level at two towers. The bridge cable is stiffened by a three hinged stiffening girder which carries a single concentrated load of 10kN at a point 20m from one end. Sketch the SF diagram for the girder.



Soln :

$V_D + V_F = 10\ kN$

$\Sigma M_F = 0,\ 80 V_D - 10 \times 60 = 0$

$$V_D = \frac{10 \times 60}{80}$$

$V_D = 7.5\ kN$

$V_F = 2.5\ kN$

Considering the equillibrium of stiffening girder and find the B.M @ E.

$$M_E = 0 \Rightarrow (2.5 \times 40) + (P \times 40 \times 20) - \left(\frac{P \times 80}{2} \times 40\right) = 0$$

$$P = \frac{100}{800} = 0.125\ kN/m$$

$$\frac{PL}{2} = \frac{0.125 \times 80}{2} = 5\ kN$$

At any section distance x from D

The S.F is given by.

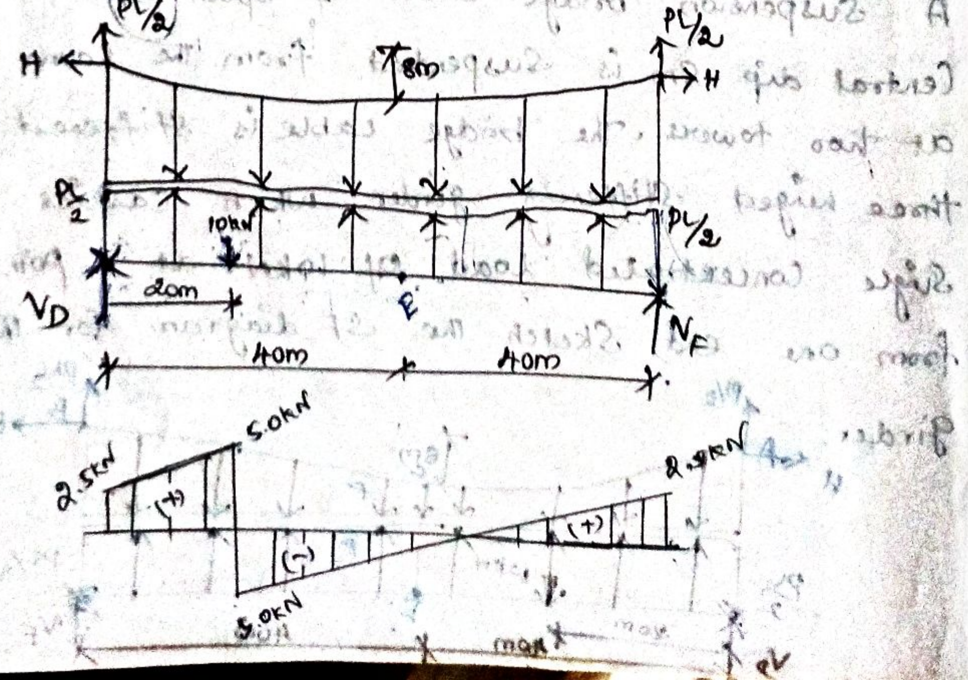
$$F_x = 7.5 - 5 + 0.125x \Big| - 10 \rightarrow (1)$$

At $x = 0$, $F_x = 7.5 - 5 = 2.5$

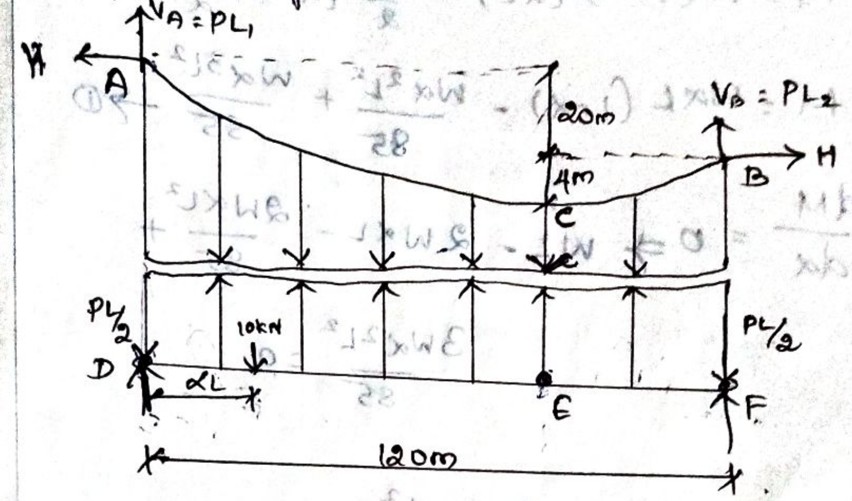
At $x = 20m$, $F_x = 7.5 - 5 + (0.125 \times 20) \Big| -10 = \pm 5$

At $x = 80m$, $F_x = 7.5 - 5 + (0.125 \times 80) - 10 = 2.5$

SFD :



5. A Suspension bridge cable hangs between two points A and B separated horizontally by 120m and with B 20m above A. The lowest point in the cable is 4m below B. ~~The cable supports a stiffening girder weighing 1/3 kN/m run which is hinged vertically below A, B and~~ The cable supports a stiffening girder which is hinged vertically below A, B and the lowest point in the cable. Find the position and magnitude of the largest BM which a point load of 10 KN can induce in the girder together with the position of the load.



Soln:

$$d_1 = 20+4 = 24$$

$$d_2 = 4m, \quad L = 120m$$

$$\frac{L_1}{L_2} = \sqrt{d_1/d_2} = \sqrt{24/4} = 2.45 \Rightarrow L_1 = 2.45 L_2$$

$$L_1 + L_2 = 120m$$

$$2.45L_2 + L_2 = 120$$

$$3.45 L_2 = 120$$

$$L_2 = 34.78 \simeq 35m$$

$$L_1 = 120 - 35 = 85m$$

Point load W be at a distance αL from D.

The S.S Reactions are $V_D = W(1-\alpha)$

$V_F = W\alpha$.

B.M at any point is,

$M = \mu - Hy$

Max B.M occurs under the load. so the load is between D and E

$M_E = 0, \; W\alpha(35) - \frac{P \times 120}{2} \times 35 + \frac{P}{2}(35^2) = 0$

$P = \frac{W\alpha}{42.5}$

Max. BM under the load is,

$M = W(1-\alpha)(\alpha L) - \frac{PL}{2}(\alpha L) + \frac{P}{2}(\alpha L)^2$

$M = W\alpha L(1-\alpha) - \frac{W\alpha^2 L^2}{85} + \frac{W\alpha^3 L^2}{85} \rightarrow ①$

$\frac{dM}{d\alpha} = 0 \Rightarrow WL - 2W\alpha L - \frac{2W\alpha L^2}{85} + \frac{3W\alpha^2 L^2}{85} = 0$

$-1 + 2\alpha + \frac{2 \times 120\alpha}{85} - \frac{3 \times 120\alpha^2}{85} = 0$

$\alpha^2 - 1.14\alpha + 0.236 = 0$

$\alpha = 0.18, \quad \alpha L = 0.18 \times 120 = 21.6m$

① $\Rightarrow$

$M_{max} = 10 \times 0.18 \times 120(1-0.18) - \frac{10(0.18 \times 120)^2}{85} + \frac{10 \times 0.18 \times (0.18 \times 120)^2}{85}$

$= 177 - 54.9 + 9.9$

$= 132$ KNm from 21.6m from left support.

For horizontal reaction,

$$Hy_c = \frac{4}{16} \Sigma M_B - \Sigma M_c$$

$$\Sigma M_B = (8\times12) + (10\times8) + (12\times4) = 224 KNm$$

ΣM_c = Sum of moments of external loads about C = 0,

$$Hy_c = \frac{x}{L} \Sigma M_B - \Sigma M_x$$

$$2.5H = \frac{4}{16}(224)$$

$$H = 22.4 KN$$

Hence tension in the part AC is given by,

$$T_{AC} = \sqrt{V_A^2 + H^2} = \sqrt{14^2 + 22.4^2} = 26.4 N$$

The tensions in parts CD, DE and EB will be such that their horizontal component is equal to H.

2. A cable is used to support five equal and equidistant loads over a span of 30m. Find the length of the cable required and its Sectional area if the safe tensile stress is 140 N/mm². The central dip of the cable is 2.5m and loads are 5kN each.