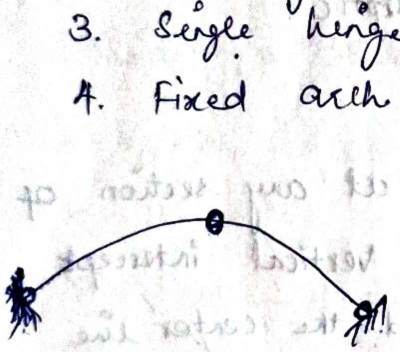
Unit-3 : Arches.

An arch is an curved girder, either a solid rib (or) braced, supported at its ends and carrying transverse loads which are frequently vertical. Since the transverse loading at any section normal to the axis of the girder is at an angle to the normal face, an arch is subjected to three restraining forces.

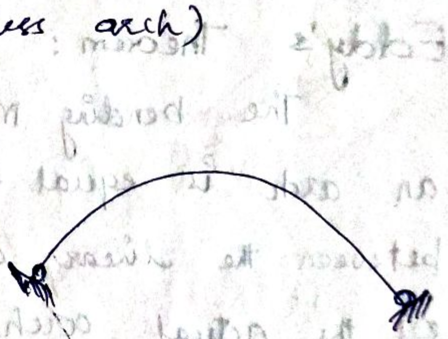
(i) Thrust
(ii) Shear Force
(iii) Bending moment

Types of Arches:

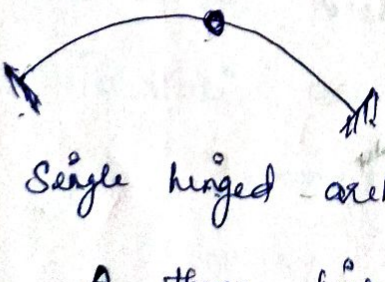
1. Three hinged arch
2. Two hinged arch
3. Single hinged arch
4. Fixed arch (hingeless arch)



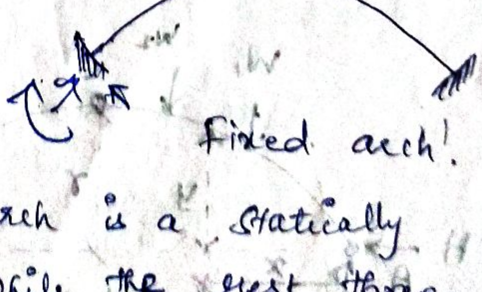
Three hinged arch



Two hinged arch



Single hinged arch

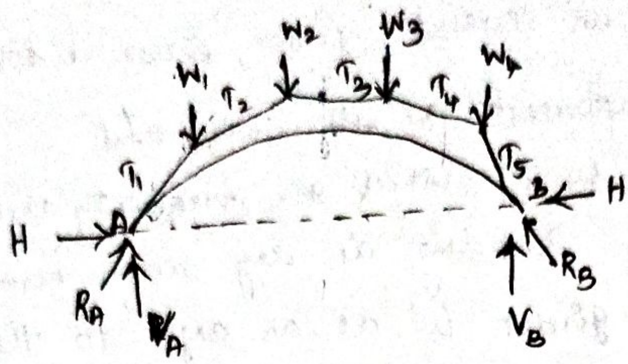


Fixed arch.

A three hinged arch is a statically determinate structure while the rest three arches are statically indeterminate.

In bridge construction (rail road bridges) two hinged and fixed arches are used.

Linear Arch (Theoretical Arch) :



It is not possible to construct the actual arch of the shape of theoretical arch. The moving loads will change the shape of the theoretical arch, and it cannot be made to change its shape to suit the varying load positions.

Arch is made as,

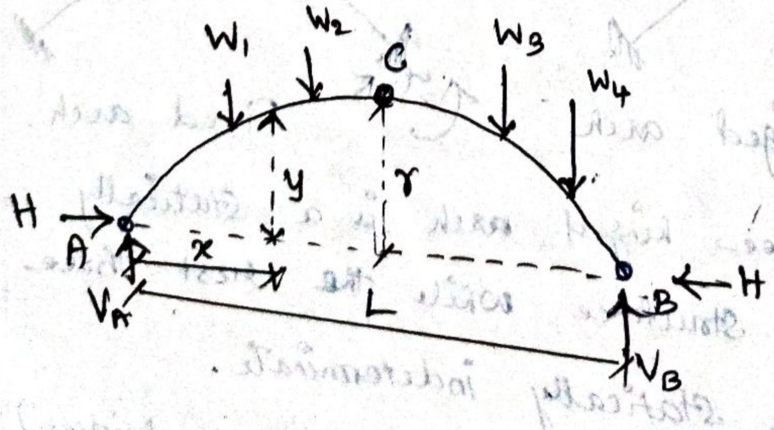
(i) parabolic

(ii) Circular (or) elliptic.

Eddy's Theorem :

The bending moment at any section of an arch is equal to the vertical intercept between the linear arch and the center line of the actual arch.

Three hinged Arch :



Three hinged arch is a statically determinate structure, having a hinge at each abutment and also at the crown.

Unknown reactions = V_A, H_A, V_B, H_B

$= 4.$

Equillibrium equations = 3

Additional equation = $\Sigma M_c = 0$

$4 - 4 = 0 \Rightarrow$ Determinate

$\Sigma M_c = 0$

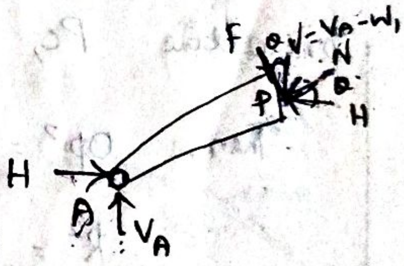
$M_c = V_1 x - W_1(x-a) - W_2(x-b) - H \times y$

$M_c = \mu_c - Hy$

$\mu_c - Hy = 0$

$\therefore H = \mu_c / y$

Consider a section, PA



$V = V_A - W_1$

$H = H$

$F = V\cos\theta - H\sin\theta.$

$N = V\sin\theta + H\cos\theta.$

Three hinged Parabolic arch:

Parabolic arch,

$y = kx(L-x)$, k is constant

$x = L/2$, $y = r$ = Central rise.

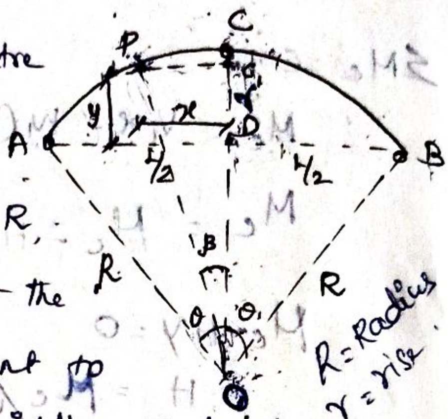
$r = k\,L/2\,(L - L/2)$

$r = k(L^2/4) \Rightarrow k = \frac{4r}{L^2}$

$y = \frac{4r}{L^2} x (L-x) \Rightarrow$ Parabolic arch equation.

According to Eddy's theorem, the vertical intercept between the linear arch and actual arch gives the B.M at a section. Due to UDL linear arch and actual arch both are parabolic and pass through the hinge at the crown. Therefore a parabolic arch will not have B.M due to UDL. It will be subjected to pure compression.

Three hinged ~~Parabolic~~ arch: Circular arch!



Let us consider the centre line of the arch to be segment of a circle of radius R, subtending an angle of 2θ at the centre. It is always convenient to have the origin at D, the middle of the span. Draw line PC_1 parallel to AB.

Then, $OP^2 = OC_1^2 + PC_1^2$

$$R^2 = [y + (R-r)]^2 + x^2$$

$$r(2R-r) = L/2 \cdot L/2 = L^2/4$$

If OP makes an angle β with OC,

$x \Rightarrow \sin\beta = \frac{PC_1}{OP}$

$$\sin\beta = \frac{x}{OP} = \frac{x}{R}$$

$$x = OP\sin\beta = R\sin\beta$$

$y \Rightarrow y = C_1D = (OC_1 - OD)$

$$= R\cos\beta - R\cos\theta$$

$$= R(\cos\beta - \cos\theta)$$

1. A parabolic arch hinged at the springings and crown has a span of 20m. The central rise of the arch is 4m. It is loaded with a UDL of intensity 2kN/m on the left of 8m length. Calculate

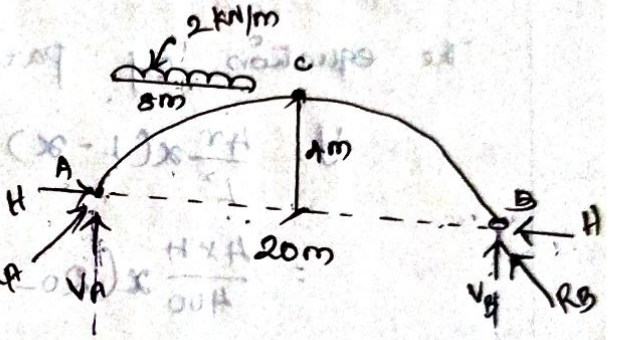
(i) The direction and magnitude of reactions at the hinges.

(ii) The bending moment, normal thrust and shear at 4m and 15m from the left end.

(iii) Maximum positive and negative bending moments.

Soln :

(i) Reaction at the hinges:



$\Sigma M_B = 0$

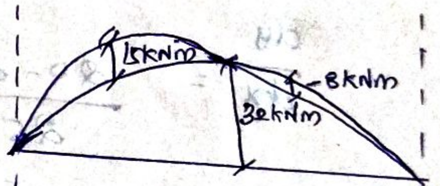
$V_A \times 20 - 2 \times 8 \times (20 - 8/2) = 0$

$V_A = 12.8 \text{ kN}$

$V_B = (2 \times 8) - 12.8$

$= 3.2 \text{ kN}$

For parabolic three hinged arches B.M due to UDL is 0.



BMD.

$\Sigma M_c = 0$ from right

$(3.2 \times 10) - (H \times 4) = 0$

$H = 8 \text{ kN}$

Reaction at A,

$R_A = \sqrt{V_A^2 + H^2}$

$= \sqrt{12.8^2 + 8^2}$

$= 15.09 \text{ kN}$

Inclination with horizontal,

$\tan \theta_A = \frac{V_A}{H} = \frac{12.8}{8}$

$\theta_A = \tan^{-1} 1.6 = 58^\circ$

Reaction at B,

$$R_B = \sqrt{V_B^2 + H^2}$$

$$= \sqrt{3.2^2 + 8^2}$$

$$= 8.62 \text{ KN}$$

Inclination with horizontal

$$\tan \theta_B = \frac{V_B}{H} = \frac{3.2}{8}$$

$$\theta_B = \tan^{-1} 0.4 = 21^\circ 48''$$

(ii) B.M, Thrust and shear:

The equation of parabola is,

$$y = \frac{4r}{L^2} x(L-x)$$

$$= \frac{4 \times H}{400} x(20-x)$$

$$y = \frac{x}{25}(20-x) = \frac{20x - x^2}{25} \rightarrow \text{(A)}$$

$$\frac{dy}{dx} = \frac{20-2x}{25} \rightarrow \text{(1)}$$

At $x = 4\text{ m}$, $y = \frac{4}{25}(20-4) = 2.56\text{m}$

Sub in (1)

$$\tan\theta = \frac{dy}{dx} = \frac{20-2\times4}{25}$$

$$\theta = \tan^{-1} 0.48 = 25^\circ 38'$$

$\Sigma M_x = 0$, $x = 4$,

$$M_4 = (12.8\times4) - (4\times2\times4/2) - (8\times2.56)$$

$$= 14.72 \text{ kNm}$$

Vertical Shear,

$$V_4 = 12.8 - (2\times4)$$

$$= 4.8\text{ kN}, \quad H = 8\text{ kN}$$

$F = V\cos\theta - H\sin\theta$

$= (4.8 \times 0.901) - (8 \times 0.433)$

$= 0.861$ kN

$N = V\sin\theta + H\cos\theta$

$= (4.8 \times 0.433) + (8 \times 0.901)$

$= 9.286$ kN

At $x = 15$ m,

$y = \frac{15}{20}(20-15)$

$= 3$ m

$\frac{dy}{dx} = \tan\theta = \frac{20-(2\times 15)}{25}$

$\theta = \tan^{-1} 0.4 = 21^\circ 48'$ (Inclination with BA)

$\sin\theta = 0.3714, \quad \cos\theta = 0.9285$

$M_{15} = (3.2 \times 5) - (8 \times 3)$

$= -8$ kNm

$F = V\cos\theta - H\sin\theta$

$= (3.2 \times 0.9285) - (8 \times 0.3714)$

$= 0$

$N = V\sin\theta + H\cos\theta$

$= (3.2 \times 0.3174) + (8 \times 0.9285)$

$= 8.616$ kN

(iii) Max. Negative and positive BM :

Max. positive B.M will occurs at somewhere under the UDL. Let it occur at x from A.

$M_x = (12.8 \times x) - \frac{2x^2}{2} - 8y$

From (A)

$M_x = 12.8x - x^2 - \frac{8x}{25}(20-x)$

$$\frac{dM_x}{dx} = 0 \Rightarrow 12.8 - 2x - \frac{32}{5} + \frac{16}{25}x = 0$$

$$x = 4.7m$$

$$M_{max}(+ve) = (12.8 \times 4.7) - 4.7^2 - \frac{8}{25}\left(4.7\left[20 - 4.7\right]\right)$$

$$= 15\ KNm.$$

The max. (-ve) B.M will occur some where in the span BC.

from (A)

$$M_x = 3.2x - 8y = 3.2x - \frac{8x}{25}(20 - x)$$

$$\frac{dM_x}{dx} = 0 \Rightarrow 3.2 - \frac{32}{5} + \frac{16x}{25} = 0$$

$$x = 5m \text{ . from (B)}$$

$$M_{max}(-ve) = 3.2 \times 5 - \frac{8 \times 5}{25}(20 - 5)$$

$$= -8\ kNm.$$

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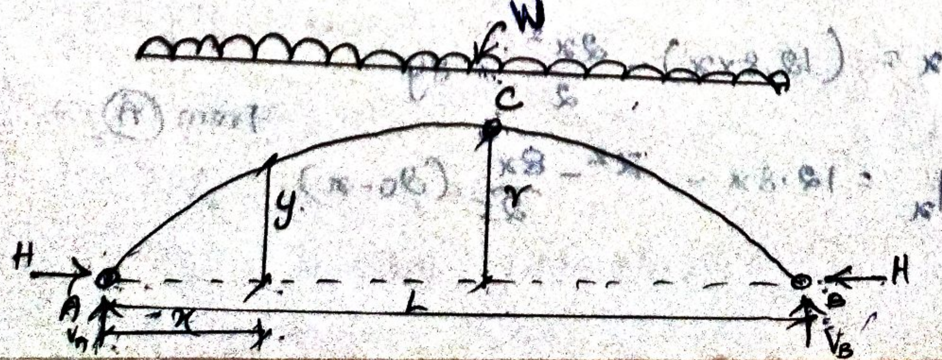
2. A symmetrical parabolic arch with a central hinge of rise r and span L is supported at its ends on pins at the same level. What is the value of the horizontal thrust when a load W which is Uniformly Distributed horizontally covers the whole span? Show also that with this loading there is no bending moment at any point in the arch rib.

Soln:



Vertical reaction at A and B will be

$$V_A = V_B = W/2$$

$$\Sigma M_c = 0$$

$$H \times r = W/2 \times L/2 - W/2 \times L/4$$

$$H = \frac{WL}{8r}$$

At a distance x from A consider a section,

Parabolic equation $y = \frac{4r}{L^2} x(L-x)$

$$M_x = -H \times y + V_A x - \frac{W}{L}\frac{x^2}{2}$$

$$= -\frac{WL}{8r} \cdot \frac{4r}{L^2} x(L-x) + \frac{W}{2}x - \frac{Wx^2}{2L}$$

$$= -\frac{Wx}{2} + \frac{Wx^2}{2L} + \frac{Wx}{2} - \frac{Wx^2}{2L}$$

$$= 0.$$

Hence bending moment at any point in the arch rib is Zero.

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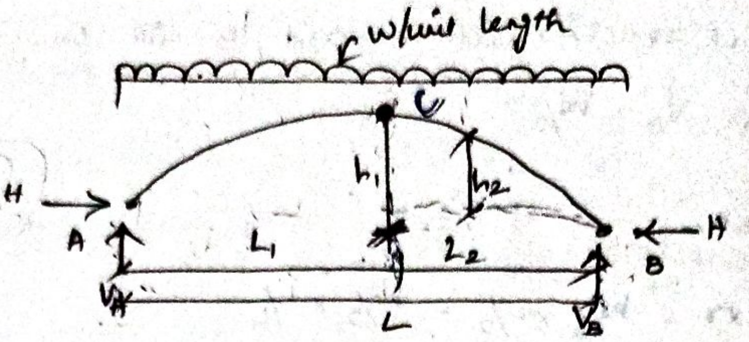
3. An arch in the form of a parabola with axis vertical has hinges at the abutments and the vertex. The abutments are at different levels, the horizontal span being L and the heights of vertex above the abutments being h_1 and h_2. Show that the horizontal thrust due to a load w/unit length uniformly distributed across the span is

$$\frac{WL^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$



<u>Soln</u> :

Let L_1 be the distance of the vertex from the left hand abutment. With C as origin, the equation of parabola is $y = kx^2$

For CA, $h_1 = kL_1^2$ (or) $k = \dfrac{h_1}{L_1^2} \rightarrow ①$

For CB, $h_2 = kL_2^2 = k(L - L_1)^2$

$$k = \frac{h_2}{(L - L_1)^2} \rightarrow ②$$

① = ②

$$\frac{h_1}{L_1^2} = \frac{h_2}{(L - L_1)^2}$$

$$L_1^2 h_2 = (L - L_1)^2 h_1$$

$$L_1 \sqrt{h_2} = (L - L_1)\sqrt{h_1}$$

$$L_1\sqrt{h_2} + L_1\sqrt{h_1} = L\sqrt{h_1}$$

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}$$

$\Sigma M_C = 0$

$$H h_1 = V_A \frac{L\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}} - \frac{W}{2} \frac{h_1 L^2}{(\sqrt{h_1} + \sqrt{h_2})^2} \rightarrow ③$$

$\Sigma M_B = 0$

$$H(h_1 - h_2) + \frac{WL^2}{2} = V_A L \rightarrow ④$$

$$V_A = \frac{H(h_1-h_2)}{L} + \frac{WL}{2}$$

Sub V_A in (3)

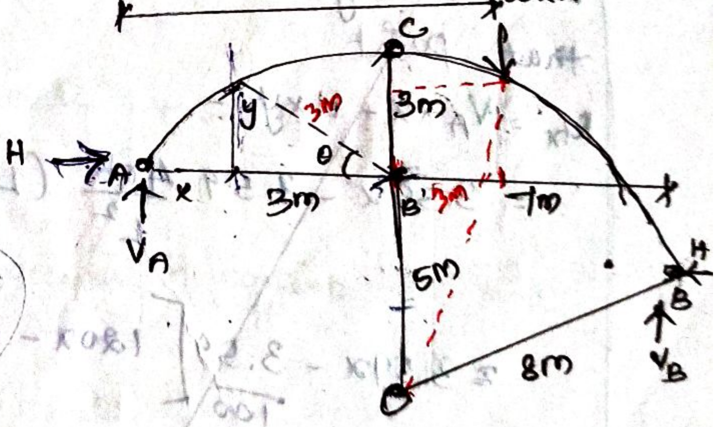
$$Hh_1 = \left[\frac{H(h_1-h_2)}{L} + \frac{WL}{2}\right]\frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} - \frac{W}{2}\,\frac{h_1L^2}{(\sqrt{h_1}+\sqrt{h_2})^2}$$

$$H\left[h_1 - \frac{(h_1-h_2)\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right] = \frac{WL^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}\sqrt{h_1h_2}$$

$$H = \frac{WL^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

4. A three higed circular arch consists of a portion AC of radius 3m and rise of the hige C with respect to the left abutment is 3m. The right hand portion CB is of radius 8m and the horizontal distance BC is 7m. If a concentrated load of 10kN acts at 6m from the left hand end, Determine the reactions at the higes and max. BM on the arch.

Soln.:



The rise of the crown above the hinge B,

$= 8 - \sqrt{8^2-7^2} = 4.13\,m$

$\Sigma M_B = 0,$

$H(4.13-3) + (V_A \times 10) - 10\times 4 = 0$

$H\times 1.13 + 10V_A - 40 = 0 \rightarrow (1)$

$\Sigma M_C = 0$

$-H\times 3 + V_A \times 3 = 0$

$V_A = H \rightarrow (2)$

Sub (2) in (1)

$V_A = H = 3.59\ kN$

$V_B = 10 - 3.59$

$= 6.41 \text{ kN}$

Reaction at A,

$R_A = \sqrt{V_A^2 + H^2} = \sqrt{3.59^2 + 3.59^2} = 5.08 \text{ kN}$

Its inclination with horizontal.

$\tan\theta = \frac{V_A}{H} = \frac{3.59}{3.59} = 1$

$\theta = \tan^{-1} 1 \qquad \theta = 45°.$

Reaction at B,

$R_B = \sqrt{6.41^2 + 3.59^2} = 7.35 \text{ kN}$

Inclination with the horizontal

$\tan\theta = \frac{6.41}{3.59} = 1.783$

$\theta = \tan^{-1} 1.783$

$\theta \simeq 60°48'$

Consider a section x distance from A and then Bending moment will be maximum at that point.

$y = \frac{4rx}{L^2}(L-x)$

$= \frac{4 \times 3x}{100}(10-x)$

$= \frac{12x}{100}(10-x)$

$= \frac{120x}{100} - \frac{12x^2}{100}$

$= \frac{1}{100}(120x - 12x^2)$

$M_x = V_A \times x - H \times y$

$= 3.59x - 3.59 \frac{4rh}{L^2}(L-x)$

$= 3.59x - \frac{3.59}{100}\left[120x - 12x^2\right)$

$\frac{dM_x}{dx} = 0 \Rightarrow$

$3.59 - \frac{3.59 \times 120}{100} + \frac{3.59 \times 24x}{100} = 0$

$3.59 - 4.308 + 0.86x = 0$

$0.86x = 0.718$

$x = 0.83 \text{m}$

$M_{x\,max(+)} = (3.59 \times 0.83) - \frac{3.59}{100}\left[120 \times 0.83 - 12 \times 0.83^2\right]$

$= 2.979 - 0.0359\left[99.6 - 8.266\right]$

$= 2.979 - 3.27 \text{ kNm}$

$= -0.291 \text{ kNm}.$

$\cos\theta = \frac{3-x}{3}$

$3\cos\theta - 3 = -x$

$x = 3 - 3\cos\theta$

$x = 3(1-\cos\theta)$

$\sin\theta = y/3 \quad y = 3\sin\theta$

$\tan\theta = \frac{y}{3-x}$

$\frac{\sin\theta}{\cos\theta} = \frac{3\sin\theta}{3-x}$

$3 - x = 3\cos\theta$

$x = 3 - 3\cos\theta$

$= 3(1-\cos\theta)$

$M_{x\,max(-)} \Rightarrow$

Let the max negative BM in portion AC, Occur at any point at horizontal distance x from A,

$x = 3(1-\cos\theta), \quad y = 3\sin\theta$

$M_x = -3.59 \times 3\sin\theta + 3.59 \times 3(1-\cos\theta)$

$= -10.77\sin\theta + 10.77 - 10.77\cos\theta.$

$= -10.77(\sin\theta - 1 + \cos\theta)$

$\frac{dM}{d\theta} = -10.77(\cos\theta - \sin\theta) = 0.$

$\cos\theta - \sin\theta = 0$

$\cos\theta = \sin\theta$

$\tan\theta = 1 \quad \theta = 45°$

$\theta = 45°.$

$M_{max(-)} = -10.77(\sin 45° - 1 + \cos 45°)$

$= -4.45 \text{ kNm}$

Max. positive B.M occurs in portion BC will occur below the load.

Height of load point above O $y = \sqrt{8^2 - 3^2} = 7.42 \text{ m}.$

Distance of load above B $= 7.42 - 3.87$

$= 3.55 \text{ m}$

$M_{max(+ve)} = 6.41 \times 4 - 3.59 \times 3.55 = 12.90 \text{ kNm}.$

Two hinged Arch:

A Two hinged arch is statically indeterminate to single degree, since there are four reactions to be determined while the no of equilibrium equations are three, Then H is considered as a redundant.

$$\frac{\partial U}{\partial H} = 0$$

U is the total strain energy stored in the arch

$$U = \int \frac{M^2}{2EI}\, ds.$$

$$\frac{\partial U}{\partial H} = \int \frac{2M}{2EI} \frac{\partial M}{\partial H} \cdot ds \qquad \frac{\partial U}{\partial H} = \frac{\partial U}{\partial M} \cdot \frac{\partial M}{\partial H}$$

$$= \int \frac{M}{EI} \frac{\partial M}{\partial H} \cdot ds$$

$$M = \mu - Hy \quad , \quad \frac{\partial M}{\partial H} = -y.$$

$$\frac{\partial U}{\partial H} = 0 = \int \frac{(\mu - Hy)\ (-y)}{EI} \cdot ds$$

$$H \int \frac{y^2 ds}{EI} = \int \frac{\mu y\, ds}{EI}$$

$$H = \int \frac{\mu y . ds}{EI} \Big/ \frac{y^2 ds}{EI}$$

$$H = \frac{\int \mu . y\, dx}{\int y^2 . dx}$$

If the two hinges are forced a distance λ apart by the thrust, λ must be added to the right hand side of equation

$$\int \frac{(\mu - Hy)(-y)}{EI} . ds = \lambda$$

$$H = \frac{\int M y \, ds - \lambda E I}{\int y^2 . ds}$$

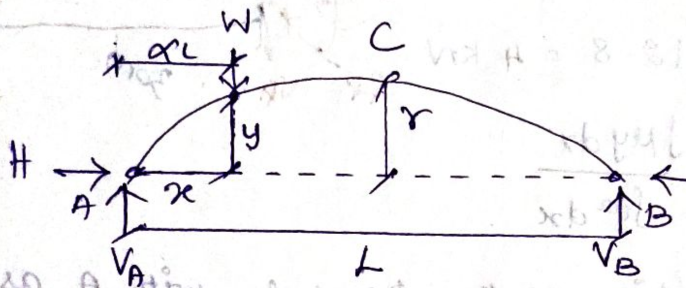
Two hinged parabolic Arch :

Consider a two hinged Parabolic arch of horizontal span L and central rise r, Subjected to a point load W at a distance αL from the left support.

The equation of arch is

$$y = \frac{4r}{L^2} x (L - x)$$

$\frac{\alpha L}{L} = \frac{10}{30} = 0.3$



$\alpha L = 10$

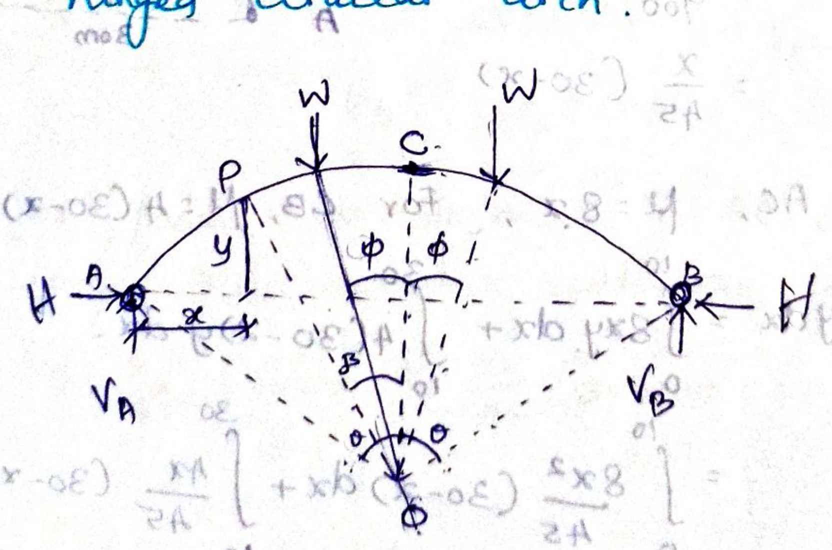
$L > 30$

$V_A = W(1-\alpha)$

$V_B = W\alpha .$

$$H = \frac{5}{8} W \frac{L}{r} \alpha (1-\alpha)(1+\alpha-\alpha^2)$$

Two hinged circular arch !



For Semi circular arch,

$$H = \frac{W \cos^2 \phi}{\pi}$$

If load is applied at the center, $H = \frac{W}{\pi} = 0.318W$

5. A parabolic arch, hinged at the ends has a Span 30m and rise 5m. A concentrated load of 12kN acts at 10m from the left hinge. The Second moment of area varies as the secant of the slope of the rib axis. Calculate the horizontal thrust and the reactions at the hinges. Also, Calculate the maximum bending moment anywhere on the arch.

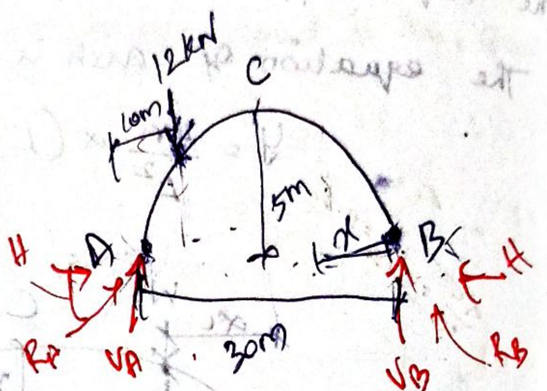
Soln:

$\Sigma M_B = 0$ $\quad V_A + V_B = 12$

$30 \times V_A = 12 \times 20$

$V_A = 8\ kN$

$V_B = 12 - 8 = 4\ kN$



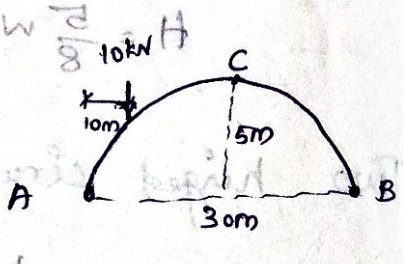
$$H = \frac{\int \mu y\, dx}{\int y^2\, dx}$$

The equation of the parabola with A as origin, is

$$y = \frac{4r}{L^2} x (L - x)$$

$$= \frac{4 \times 5}{900} x (30 - x)$$

$$= \frac{x}{45} (30 - x)$$



For AC, $\mu = 8x$, For CB, $\mu = 4(30 - x)$

$$\int_0^{30} \mu y\, dx = \int_0^{10} 8xy\, dx + \int_{10}^{30} 4(30-x) y\, dx.$$

$$= \int_0^{10} \frac{8x^2}{45} (30 - x)\, dx + \int_{10}^{30} \frac{4x}{45} (30 - x)^2\, dx$$

$$= \frac{44000}{109}$$

$$\int_0^{30} y^2\, dx = \int_0^{30} \frac{x^2 (30-x)^2}{45^2}\, dx$$

$$\int_0^{30} y^2 \, dx = \frac{1}{45^2} \int_0^{30} (900x^2 + x^4 - 60x^3) \, dx$$

$$= \frac{1}{45^2} \left[300x^3 + \frac{x^5}{5} - 15x^4 \right]_0^{30}$$

$$= 400$$

$$H = \frac{44000}{9 \times 400}$$

$$= 12.22 \text{ kN}$$

Reaction at A, $R_A = \sqrt{8^2 + 12.22^2}$ $\left(\sqrt{V_A^2 + H^2}\right)$

$$= 14.61 \text{ kN}$$

$\tan \theta_A = \frac{8}{12.22} = 0.655$ $\tan \theta = \frac{V_A}{H}$

$$\theta_A = 33^\circ 14'$$

Reation at B, $R_B = \sqrt{4^2 + 12.22^2}$

$$= 12.85 \text{ kN}$$

$$\tan \theta_B = \frac{4}{12.22}$$

$$\theta_B = 18^\circ 6'$$

Max. positive BM will occur in AC, just below the load. Rise of arch under the load.

$$y = \frac{x}{45}(30 - x) = \frac{10}{45}(30 + 10)$$

$$= 40/9 \text{ m}.$$

Max (+ve) BM = $\left(-12.22 \times \frac{40}{9}\right) + (8 \times 10)$

$$= 25.69 \text{ kNm}.$$

Let the maximum negative moment occur at a distance x from B,

$$M_x = 4x - 12.22 \times \frac{x(30-x)}{45}$$

$$\frac{dM_x}{dx} = 0 \Rightarrow 4 - \frac{12.22 \times 30}{45} + \frac{12.22}{45} \times 2x = 0$$

$$x = 7.65m$$

$$\text{Max (-ve) BM} = 4 \times 7.65 - 12.22 \times \frac{7.65(30-7.65)}{45}$$

$$= -15.80 \text{ kNm.}$$

Max. B.M is 25.69 kNm which occurs below the load.

6. A parabolic two hinged arch has a span of 32m and a rise of 8m. A UDL of 1kN/m covers 8m horizontal length of the left side of the arch. If $I = I_0 \sec\theta$ where θ is the inclination of the arch of the section to the horizontal, and I_0 is the moment of inertia of the section at that crown, find out the horizontal thrust at hinges and bending moment at 8m from the left hinge. Also find out normal thrust and radial shear at this section.

Soln:

$\Sigma M_B = 0.$

$V_A \times 32 = 8 \times 1 \times 28$

$V_A = 7kN$

$V_B = 8 - 7 = 1kN$

$M_1 = 7x - 1 \times x^2/2 = 7x - x^2/2$

$M_2 = 1(32-x)$

The rise of the arch at any distant x from A is,

$$y = \frac{4 \times 8}{32^2} x(32-x) = \frac{x}{32}(32-x)$$

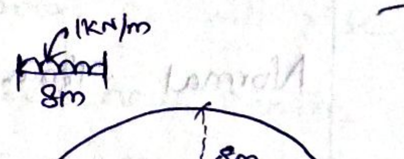
$$\int_0^L My \cdot dx = \int_0^8 \left(7x - \frac{x^2}{2}\right) \frac{x(32-x)}{32}\, dx + \int_8^{32} 1(32-x)\frac{x(32-x)}{32}\, dx$$

$$= \int_0^8 \left(7x^2 - \frac{x^3}{2} - \frac{7}{32}x^3 + \frac{x^4}{64}\right) dx + \int_8^{32} \left(32x + \frac{x^3}{32} - 2x^2\right) dx$$

$$= \left[\frac{7x^3}{3} - \frac{x^4}{8} - \frac{7x^4}{128} + \frac{x^5}{320}\right]_0^8 + \left[16x^2 + \frac{x^4}{128} - \frac{2x^3}{3}\right]_8^{32}$$

$$= 8^3\left[\frac{7}{3} - 1 - \frac{7\times 8}{128} + \frac{8^2}{320}\right] + \left[\left(16\times 32^2 + \frac{32^4}{128} - \frac{2\times 32^2}{3}\right) - \left(16\times 8^2 + \frac{8^4}{128} - \frac{2\times 8^2}{3}\right)\right]$$

$$= 2477.07$$



$$\int_0^L y^2 \cdot dx = \int_0^{32} \frac{x^2(32-x)^2}{32^2}\, dx$$

$$= \frac{1}{32^2}\int_0^{32} (1024x^2 + x^4 - 64x^3) \cdot dx$$

$$= \frac{1}{32^2}\left[\frac{1024x^3}{3} + \frac{x^5}{5} - \frac{64x^4}{4}\right]_0^{32}$$

$$= \frac{1024\times 32}{3} + \frac{32^3}{5} - \frac{64\times 32^2}{3}$$

$$= 1092.16$$

$$H = \frac{\int_0^L M y . dx}{\int_0^L y^2 . dx} = \frac{2477.07}{1092.16} = 2.27 \text{ kN}$$

Now, $y = \frac{x}{32}(32 - x)$

$$\frac{dy}{dx} = \tan\theta = \frac{32 - 2x}{32} = 1 - \frac{x}{16}$$

At $x = 8m$, $y = \frac{8}{32}(32 - 8) = 6m$

$$\tan\theta = 1 - \frac{8}{16} = 0.5$$

$$\theta = 26^\circ 34'$$

$\sin\theta = 0.447$, $\cos\theta = 0.894$

BM at $x = 8$,

$$M_{x=8} = 1 \times 24 - 2.27 \times 6$$

$$= 10.38 \text{ kNm.}$$

Vertical shear, $V = -1$ kN

Normal Thrust, $N = H\cos\theta + V\sin\theta$

$$= 2.27 \times 0.894 - (1 \times 0.447)$$

$$= 1.583 \text{ kN}$$

Radial Shear, $F_r = -H\sin\theta + V\cos\theta$

$$= -2.27 \times 0.447 - 1 \times 0.894$$

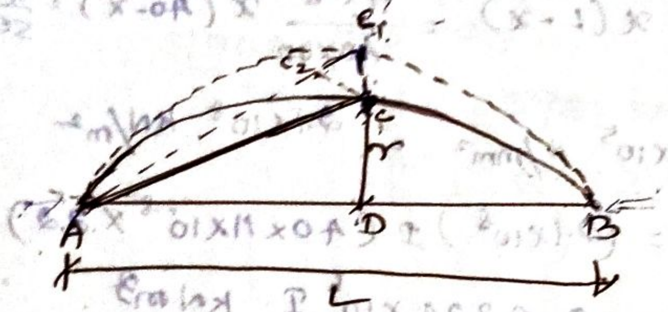
$$= -1.909 \text{ kN.}$$

Fixed Arch:

In fixed arch, there are six reaction components to be determined. Since only three equations are available from static equilibrium, Hence the fixed arch is statically indeterminate to three degree.

Temperature Effects:

(i) Three Hinged Arch:



Due to increase in temperature, the length of the arch ACB will increase. Since the two end hinges are rigidly fixed, the crown C will rise from C to C_1'. Thus AC_1B will be new centre line of the arch. No temperature stresses will therefore be induced.

$$CC_1 = \left[\frac{L^2 + 4r^2}{4r}\right] \alpha t$$

(ii) Two Hinged Arch:

Since there is no central hinge, the end hinges will exert a horizontal thrust on the arch to prevent the ends from moving out when the temperature of the arch increases. Due to this horizontal thrust, there will be bending moment at all the sections

A.

$$H_t = \frac{L\alpha t}{\int_0^L \frac{y^2 ds}{EI}} = \frac{EI \cdot L\alpha t}{\int_0^L y^2 ds}$$

7\. A two hinged parabolic arch of span 40m and rise 8m is subjected to a temperature rise of 22K. Calculate the maximum bending stress at the crown due to the temperature rise of $\alpha = 11\times10^{-8}/^\circ K$ and $E = 2.1\times10^5$ N/mm². The rib section is symmetrical and 1m deep.

Soln :

$$y = \frac{4r}{L^2}x(L-x) = \frac{4\times 8}{40\times40}x(40-x) = \frac{x}{50}(40-x)$$

$$E = 2.1\times10^5 \text{ N/mm}^2 = 2.1\times10^8 \text{ kN/m}^2$$

$$EI\,L\alpha t = (2.1\times10^8)\,I\,(40\times11\times10^{-8}\times22)$$

$$= 2.0328\times10^6\, I \text{ kN m}^3.$$

$$\int_0^L y^2 ds = \int_0^{40}\frac{x^2}{2500}(40-x)^2 dx$$

$$= \int_0^{40}\frac{x^2}{2500}(1600+x^2-80x)\,dx$$

$$= \frac{1}{2500}\int_0^{40}\left[1600x^2+x^4-80x^3\right]dx$$

$$= \frac{1}{2500}\left[\frac{1600x^3}{3}+\frac{x^5}{5}-\frac{80x^4}{4}\right]_0^{40}$$

$$= \frac{1}{2500}\left[\frac{1600\times40^3}{3}+\frac{40^5}{5}-\frac{80\times40^4}{4}\right]$$

$$= \frac{40^3}{2500}\left[\frac{1600}{3}+\frac{40^2}{5}-\frac{80\times40}{4}\right]$$

$$1360 \text{ m}^3$$

$$H_t = \frac{2.0328 \times 10^6 \, I \text{ kNm}^2}{1360} = 1495 \, I \text{ kN.}$$

$$M_c = H_t \times y = 14952 \times 8 = 11960 \, I \text{ kNm.}$$

Bending Equation,

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}$$

$$M/I = f/y \Rightarrow \frac{My}{I} = f$$

$$\frac{M}{(I/y)} = f \qquad y = d/2 = 1/2 = 0.5\text{m}$$

$$\frac{M}{Z} = f \Rightarrow \frac{11960 Z}{Z/0.5} = f$$

$$11960 \times 0.5 = f$$

$$f = 5980 \text{ kN/m}^2$$

$$f = 5.98 \text{ N/mm}^2$$