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## DEPARTMENT OF COMPUTER SCIENCE ENGINEERING

## 19ECB231 - DIGITAL ELECTRONICS

II YEAR/ III SEMESTER

## UNIT 4 - DESIGN OF SEQUENTIAL CIRCUITS

TOPIC -Design of synchronous sequential circuits

## Characteristic Equations

D flip-flop Characteristic Equations

$$
\mathrm{Q}(\mathrm{t}+1)=\mathrm{D}
$$

JK flip-flop Characteristic Equations

$$
Q(t+1)=J Q^{`}+K^{`} Q
$$

T flip-flop Characteristic Equations

$$
\mathrm{Q}(\mathrm{t}+1)=\mathrm{T} \bigoplus \mathrm{Q}=\mathrm{TQ}^{`}+\mathrm{T}^{`} \mathrm{Q}
$$

## 5-4 Analysis of Clocked Sequential Cunturas

The analysis of a sequential circuit consists of obtaining a able or a diagram for the time sequence of inputs, outputs, nd internal states. It is also possible to write Boolean xpressions that describe the behavior of the sequential rcuit. These expressions must include the necessary time equence, either directly or indirectly.

## State Equations

The behavior of a clocked sequential circuit can be lescribed algebraically by means of state equations. A state equation specifies the next state as a function of the present state and inputs. Consider the sequential circuit hown in Fig. 5-15. It consists of two D flip-flops A and B, in input $x$ and an output $y$.

## 



## State Equation

$$
\begin{aligned}
& A(t+1)=A(t) x(t)+B(t) x(t) \\
& B(t+1)=A^{\prime}(t) x(t)
\end{aligned}
$$

state equation is an algebraic expression that specifies he condition for a flip-flop state transition. The left side of he equation with ( $\mathrm{t}+1$ ) denotes the next state of the fliplop one clock edge later. The right side of the equation is boolean expression that specifies the present state and pput conditions that make the next state equal to 1 .

$$
Y(\mathrm{t})=(\mathrm{A}(\mathrm{t})+\mathrm{B}(\mathrm{t})) \mathrm{x}(\mathrm{t})^{`}
$$

## State Table

## The time sequence of inputs, outputs, and flip-flop states can be enumerated in a state table (sometimes called transition table).

Table 5-2
State Table for the Circuit of Fig. 5-15

| Present <br> State | Input |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## State Diagram

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The information available in a state table can be represented graphically in the form of a state diagram. In this type of diagram, a state is represented by a circle, and the transitions between states are indicated by directed lines connecting the circles.
$1 / 0$ : means input =1
output=0


## Flip-Flop Input Equations

The part of the combinational circuit that generates external outputs is descirbed algebraically by a set of Boolean functions called output equations. The part of the circuit that generates the inputs to flip-flops is described algebraically by a set of Boolean functions called flip-flop input equations. The sequential circuit of Fig. 5-15 consists of two D flip-flops A and B, an input x, and an output y. The logic diagram of the circuit can be expressed algebraically with two flip-flop input equations and an output equation:

$$
\begin{aligned}
& D A=A x+B x \\
& D B=A^{`} x \\
& y=(A+B) x^{`}
\end{aligned}
$$

## Analysis with D Flip-Flop

The circuit we want to analyze is described by the input quation $\quad D_{A}=A \bigoplus x \bigoplus y$
The DA symbol implies a D flip-flop with output $A$. The $x$ and $y$ variables are the inputs to the circuit. No output equations are given, so the output is implied to come from the output of the flip-flop.

(a) Circuit diagram

## Analysis with D Flip-Flop

he binary numbers under Axy are listed from 000 through 1 as shown in Fig. 5-17(b). The next state values are ptained from the state equation $A(t+1)=A \bigoplus x \bigoplus y$
he state diagram consists of two circles-one for each state shown in Fig. 5-17(c)

| Present <br> state | Inputs | Next <br> state |  |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{A}$ | $\boldsymbol{x}$ | $\boldsymbol{y}$ | $\boldsymbol{A}$ |
| O | O | O | O |
| O | O | 1 | 1 |
| O | 1 | O | 1 |
| O | 1 | 1 | O |
| 1 | O | O | 1 |
| 1 | O | 1 | O |
| 1 | 1 | O | O |
| 1 | 1 | 1 | 1 |

(b) State table

(c) State diagram


Fig. 5-18 Sequential Circuit with $J K$ Flip-Flop

The circuit can be specified by the flip-flop input equations

$$
\begin{array}{ll}
J_{A}=B & K_{A}=B x^{`} \\
J_{B}=x^{`} & K_{B}=A^{`} x+A x^{`}=A_{\oplus}{ }^{x}
\end{array}
$$

Table 5-4
State Table for Sequential Circuit with JK Flip-Flops

| Present State |  | Input <br> $x$ | Next State |  | Flip-Flop Inputs |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B |  | A | B | $J_{A}$ | $K_{\text {A }}$ | $J_{B}$ | $K_{B}$ |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

## Analysis with JK Flip-Flops

$$
\begin{aligned}
& \mathrm{A}(\mathrm{t}+1)=\mathrm{JA}+\mathrm{K}^{\prime} \mathrm{A} \\
& \mathrm{~B}(\mathrm{t}+1)=\mathrm{JB}+\mathrm{K}^{\prime} \mathrm{B}
\end{aligned}
$$

ubstituting the values of JA and KA from the input quations, we obtain the state equation for A :

$$
A(t+1)=B A^{`}+\left(B x^{`}\right)^{`} A=A^{`} B+A B^{`}+A x
$$

he state equation provides the bit values for the column nder next state of $A$ in the state table. Similarly, the state quation for flip-flop $B$ can be derived from the characteristio quation by substituting the values of $\mathrm{J}_{\mathrm{B}}$ and $\mathrm{K}_{\mathrm{B}}$ :

$$
B(t+1)=x^{`} B^{`}+(A \oplus x)^{`} B=B^{\prime} x^{`}+A B x+A^{`} B x^{`}
$$

The state diagram of the sequential circuit is shown in
Fig. 5-19.


Fig. 5-19 State Diagram of the Circuit of Fig. 5-18

## Analysis With T Flip-Flops

## Characteristic equation

$$
\mathrm{Q}(\mathrm{t}+1)=\mathrm{T} \oplus \mathrm{Q}=\mathrm{T}^{`} \mathrm{Q}+\mathrm{TQ}{ }^{`}
$$


(a) Circuit diagram

00/0 : means state is 00 output is 0

(b) State diagram

## Analysis With T Flip-Flops

Consider the sequential circuit shown in Fig. 5-20. It has two flip-flops A and B, one input $x$, and one output $y$. It can be described algebraically by two input equations and an output equation:

Table 5-5

```
    \(\mathrm{T}_{\mathrm{A}}=\mathrm{Bx}\)
    \(\mathrm{T}_{\mathrm{B}}=\mathrm{x}\)
    \(y=A B\)
\(A(t+1)=(B x)^{\prime} A+(B x) A^{\prime}\)
    \(=A B^{\prime}+A x^{\prime}+A^{\prime} B x\)
\(B(t+1)=x \oplus B\)
```

State Table for Sequential Circuit with T Flip-Flops

| Present State |  | Input <br> $x$ | Next <br> State |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | B |  | A | B |  |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 |

## Mealy and Moore Models

## (1)

LISTHTUTIOIS:

- The most general model of a sequential circuit has inputs, outputs, and internal states. It is customary to distinguish between two models of sequential circuits:


## the Mealy model and the Moore model

- They differ in the way the output is generated.
- In the Mealy model, the output is a function of both the present state and input.
- In the Moore model, the output is a function of the present state only.


## Mealy and Moore Models (2)

WISTITINTOIS:

When dealing with the two models, some books and other technical sources refer to a sequential circuit as a finite state machine abbreviated FSM.

- The Mealy model of a sequential circuit is referred to as a Mealy FSM or Mealy machine.
- The Moore model is refereed to as a Moore FSM or Moore machine.


