6) Solve $(D^2 - DD' - 2DD'^2)z = e^{5x+y} + \cos y + \sin(4x - y)$

7) Solve $(D^2 + 2DD' + D'^2)z = e^{x-y} + \sin(x-y) + x^2y$

8) Solve $(D^2 - 2DD')z = e^{2x} + x^2y + 3 + \cos(x-y)$

9) Solve $(D^2 - 2DD' + D'^2)z = x^2y^2e^{x+y}$

10) Solve $(D^2 - D'^2)z = e^{x-y}\, x^2y$.

6) $(D^2 - DD' - 20D'^2)z = e^{5x+y} + \cos y + \sin(4x-y)$

Soln: Replace D by m and D' by 1

The auxiliary eqn is

$$m^2 - m - 20 = 0$$
$$(m-5)(m+4) = 0$$
$$m_1 = 5 ;\ m_2 = -4$$

The roots are real and distinct

$$\therefore CF = f_1(y+5x) + f_2(y-4x)$$

Particular Integral

$$PI = \frac{1}{f(D,D')} e^{5x+y} + \frac{1}{f(D,D')} \cos y + \frac{1}{f(D,D')} \sin(4x-y)$$
$$= PI_1 + PI_2 + PI_3$$

$$PI_1 = \frac{1}{D^2 - DD' - 20} e^{5x+y}$$

Here $a = 5 ;\ b = 1$

$D \to a = 5 ;\ D' \to b = 1$

$$= \frac{1}{25-5-20} e^{5x+y} = \frac{1}{0} e^{5x+y}$$

$$= \frac{1}{2D - D'} e^{5x+y} = \frac{1}{10-1} e^{5x+y}$$

$$PI_1 = \frac{e^{5x+y}}{9}$$

$$PI_2 = \frac{1}{D^2 - DD' - 20} \cos y$$

$\cos(ax+by)$ & $\sin(ax+by)$ –

$D^2 \to -a^2 \mid D'^2 \to -b^2 \mid DD' \to -ab$

Here $a = 0 ;\ b = 1$

$D^2 \to 0$

$D'^2 \to -1$

$DD' \to +0$

$$PI_2 = \frac{1}{D^2 - DD' - 20D'^2} \cos y$$

$$= \frac{1}{0 - 0 + 20} \cos y$$

$$\therefore PI_2 = \frac{1}{20} \cos y$$

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$$PI_3 = \frac{1}{D^2 - DD' - 20D'^2} \sin(4x - y)$$

Here $a = 4$; $b = -1$

$$D^2 \to -a^2 = -4^2 = -16$$

$$D'^2 \to -b^2 = -(-1^2) = -1$$

$$DD' \to -ab = -4(-1) = 4$$

$$PI_3 = \frac{1}{-16 - 4 + 20} \sin(4x - y)$$

$$= \frac{1}{0} \sin(4x - y)$$

$$= \frac{x}{(2D - D')} \sin(4x - y)$$

$$= \frac{x(2D + D')}{(2D - D')(2D + D')} \sin(4x - y)$$

$$= \frac{x(2D + D')}{(2D)^2 - (D')^2} \sin(4x - y)$$

$$= \frac{x(2D + D')}{4D^2 - D'^2} \sin(4x - y)$$

$$= \frac{x(2D + D')}{4(-16) + 1} \sin(4x - y)$$

$$= \frac{x}{-63} 2D \sin(4x - y) + D' \sin(4x - y)$$

$$= \frac{-x}{63} 2\left[\cos(4x - y)(4) + \cos(4x - y)(-1)\right]$$

$$= \frac{-x}{63}\left[8\cos(4x - y) - \cos(4x - y)\right]$$

$$= \frac{-x\cos(4x - y) \cdot 7}{63} = -\frac{x\cos(4x - y)}{9}$$

$\therefore$ The hS is

$$z = f_1(y + 5x) + f_2(y - 4x) + \frac{e^{5x+y}}{9} + \frac{\cos y}{20} - \frac{x\cos(4x - y)}{9}$$

4) $(D^2+2DD'+D'^2)z = e^{x-y} + \sin(x-y) + x^2y$.

Soln: C.F:

The auxiliary eqn is $m^2+2m+1=0$

$(m+1)(m+1)=0$

$m_1=-1$ & $m_2=-1$

The roots are real & equal.

C.F $= f_1(y-x) + x f_2(y-x)$.

P.I

$$P.I = \frac{1}{f(D,D')} F(x,y)$$

$$= \frac{1}{(D^2+2DD'+D'^2)}\left[e^{x-y} + \sin(x-y) + x^2y\right]$$

$$PI_1 = \frac{1}{D^2+2DD'+D'^2} e^{x-y}$$

$D \to 1 \quad D' \to -1$

$$PI_1 = \frac{1}{1-2+1} e^{x-y}$$

$$= \frac{1}{0} e^{x-y}$$

Par diff w.r.to D in Dr & put x in Nr.

$$PI_1 = \frac{x}{2D+2D'} e^{x-y}$$

$$= \frac{x\,\cancel{e^{x+y}}}{2-2} e^{x-y}$$

$$= \frac{x^2 e^{x-y}}{2} \qquad \therefore PI_1 = \frac{x^2}{2} e^{x-y}$$

$$PI_2 = \frac{1}{D^2+2DD'+D'^2} \sin(x-y)$$

Here $a=1$

$b=-1$

$D^2 \to -a^2 \to -1$

$D'^2 \to -b^2 \to -(-1)^2 = -1$

$DD' \to -ab \to +1$

$$= \frac{1}{-1+2-1} \sin(x-y)$$

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$$= \frac{1}{0} \sin(x-y)$$

diff Dr & put x in Nr.

$$= \frac{x}{2D+2D'} \sin(x-y)$$

$$= \frac{x}{2(D+D')} \sin(x-y)$$

$$= \frac{x\,(D-D')}{2\,(D^2-D'^2)} \sin(x-y) = \frac{x}{2} \frac{(D-D')(\sin(x-y))}{[-1-(-1)]}$$

$$= \frac{x}{0} (D-D') \sin(x-y)$$

diff Dr & put x in Nr.

$$= \frac{x^2 (D-D')}{2(2D)} \sin(x-y)$$

$$= \frac{x^2}{4} [D \sin(x-y) - D' \sin(x-y)]$$

$$= \frac{x^2}{4} [\cos(x-y) + \cos(x-y)]$$

$$PI_2 = \frac{x^2}{2} \cos(x-y)$$

$$PI_3 = \frac{1}{D^2+2DD'+D'^2} x^2 y.$$

Binomial formula

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$

$$= \frac{1}{D^2\left[1 + \frac{2DD'}{D^2} + \frac{D'^2}{D^2}\right]} x^2 y.$$

$$= \frac{1}{D^2\left[1 + \left(\frac{2D'}{D} + \left(\frac{D'}{D}\right)^2\right)\right]} x^2 y$$

$$= \frac{1}{D^2}\left[1 + \left(\frac{2D'}{D} + \frac{D'^2}{D}\right)^2\right]^{-1} x^2 y$$

$$= \frac{1}{D^2}\left[1 - \frac{2D'}{D} - \frac{D'^2}{D^2} + \frac{4D'^2}{D^2} + \frac{D'^4}{D^4} \cancel{+} \cdots\right]x^2y$$

$$- \frac{9D'^3}{D^3} \cancel{+} \frac{D'^6}{D^6} + \cdots \Big] x^2y$$

$$= \frac{1}{D^2}\left[x^2y - \frac{2D'}{D}(x^2y) - \frac{D'^2}{D^2}(x^2y) + \frac{4D'^2}{D^2}(x^2y)\right.$$

$$\left. + \frac{D'^4}{D^4}(x^2y) - \frac{9D'^3}{D^3}(x^2y) - 0\right]$$

$$= \frac{1}{D^2}\left[x^2y - \frac{2}{D}(x^2) - \frac{1}{D^2}(0) + 0 + 0 + 0\right]$$

$$= \frac{1}{D^2}\left[x^2y - 2\frac{x^3}{3}\right] = \cancel{\frac{1}{D}\Big[x^2y}$$

$$= \frac{1}{D}\left[\frac{x^3}{3}y - \frac{2x^4}{3 \cancel{6}\times 4}\right] = \frac{1}{D}\left[\frac{x^3y}{3} - \frac{x^4}{6}\right]$$

$$= \frac{x^4y}{12} - \frac{x^5}{30}.$$

$$\therefore PI_3 = \frac{x^4y}{12} - \frac{x^5}{30}.$$

$\therefore$ The general solution is

$$z = f_1(y-x) + xf_2(y-x) + \frac{x^2}{2}e^{x-y}$$

$$+ \frac{x^2}{2}\cos(x-y) + \frac{x^4y}{12} - \frac{x^5}{30}.$$

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8) $(D^2 - 2DD')z = e^{2x} + x^3y + 3 + \cos(x-y)$

Soln: Replace D by m and D' by 1

$m^2 - 2m = 0$

$m(m-2) = 0$

$m = 0 / m = 2$

The roots are real & distinct

C.F

$CF = f_1(y + 0x) + f_2(y + 2x)$

$CF = f_1(y) + f_2(y + 2x)$

Particular Integral

$$PI = \frac{1}{D^2 - 2DD'} e^{2x} + x^2y + 3 + \cos(x-y)$$

$$PI_1 = \frac{1}{D^2 - 2DD'} e^{2x} \quad D \to 2, \; D' \to 0$$

$$= \frac{1}{4-0} e^{2x} = \frac{e^{2x}}{4}$$

$$PI_1 = \frac{e^{2x}}{4}$$

$$PI_2 = \frac{1}{D^2 - 2DD'} x^3y$$

$$= \frac{1}{D^2\left[1 - \frac{2D'}{D}\right]} x^3y = \frac{1}{D^2}\left[1 - \frac{2D'}{D}\right]^{-1} x^3y$$

$$= \frac{1}{D^2}\left[1 + \frac{2D'}{D} + \frac{4D'^2}{D^2} + \cdots\right] x^2y$$

$$= \frac{1}{D^2}\left[x^3y + \frac{2D'}{D}x^3y + \frac{4D'^2}{D^2}x^3y + \frac{8D'^3}{D^3}x^3y + \cdots\right]$$

$$= \frac{1}{D^2}\left[x^3y + \frac{2}{D}x^3 + \frac{4}{D^2}(0) + \frac{8}{D^3}(0) + \cdots\right]$$

$$= \frac{1}{D^2}\left[x^3y + \frac{2}{D}x^3 + \cancel{0}\right]$$

$$= \frac{1}{D^2}x^3y + 2\cdot\frac{x^4}{4} \quad = \frac{1}{D}\left(y\frac{x^4}{4}\right) + \frac{1}{D^2}\frac{x^4}{2}$$

$$= \frac{yx^5}{4\times5} + \frac{x^6}{2\times5\times6} \quad = \frac{yx^5}{20} + \frac{x^6}{60}$$

$$PI_2 = \frac{x^5y}{20} + \frac{x^6}{60}$$

GS $\boxed{z = f_1(y) + f_2(y+2x) + \frac{yx^5}{20} + \frac{x^6}{60}}$

$$PI_3 = \frac{1}{(D^2-2DD')} 3(e^{0x+0y}) = \frac{3x}{2D+2D'} e^{0x+0y}$$

$$= \frac{1}{0} 3e^{0x+0y}$$

$$= 3\frac{x}{0} e^{0x+0y} = 3\frac{x^2}{2} e^{0x+0y}) = \frac{3x^2}{2}$$

$$PI_3 = \frac{3x^2}{2}$$

$$PI_4 = \frac{1}{(D^2-2DD')} \cos(x-y)$$

$D^2 \to -(1)^2 = -1$

$D'^2 \to -(-1)^2 = -1$

$DD' = -(1)(-1) = 1$

$$= \frac{1}{-1-2} \cos(x-y)$$

$$= \frac{-1}{3} \cos(x-y) \quad + \frac{e^{2x}}{4}$$

G.S. $z = f_1(y) + f_2(y+2x) + \frac{yx^5}{20} + \frac{x^6}{60} + \frac{3x^2}{2} - \frac{1}{3}\cos(x-y)$

9) $(D^2 - 2DD' + D'^2) z = x^2y^2e^{x+y}$

Soln: C.F: Replace $D \to m / D' \to 1$

AE is $m^2 - 2m + 1 = 0$

$(m-1)(m-1) = 0$

$m = 1, 1$

The roots are real & equal

$CF = f_1(y+x) + x f_2(y+x)$

Particular Integral

$$PI = \frac{1}{(D^2-2DD'+D'^2)} x^2y^2e^{x+y}$$

e^{ax+by} $a=1 / b=1$ | Replace $D \to D+1$, $D' \to D'+1$

$$= \frac{e^{x+y}}{((D+1)^2 - 2(D+1)(D'+1) + (D'+1)^2)} x^2y^2$$

$a^2 - 2ab + b^2$

$$= \frac{e^{x+y}}{((D+1)-(D'+1))^2} x^2y^2$$

$$= e^{x+y} \frac{1}{(D-D')^2} x^2y^2$$

formula:

$$(1-x)^{-2} = 1+2x+3x^2+4x^3+\dots$$

$$= e^{x+y} \frac{1}{D^2\left[1-\frac{D'}{D}\right]^2} x^2y^2$$

$$= e^{x+y} \frac{1}{D^2}\left[1-\frac{D'}{D}\right]^{-2} x^2y^2$$

$$= e^{x+y}\frac{1}{D^2}\left[1+2\frac{D'}{D}+3\frac{D'^2}{D^2}+4\frac{D'^3}{D^3}+\dots\right]x^2y^2$$

$$= e^{x+y}\frac{1}{D^2}\left[x^2y^2+2\frac{D'}{D}x^2y^2+\frac{3D'^2}{D^2}x^2y^2+\dots\right]$$

$$= e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{2}{D}x^2(2y)+\frac{3}{D^2}x^2(2)+\dots\right]$$

$$= e^{x+y}\frac{1}{D^2}\left[x^2y^2+\frac{4x^2y}{D}+\frac{6x^2}{D^2}\right]$$

$$= e^{x+y}\frac{1}{D^2}\left[x^2y^2+4y\frac{x^3}{3}+6\frac{x^4}{3\times4}\right]$$

$$= e^{x+y}\left[\frac{1}{D^2}(x^2y^2)+\frac{4y}{3}\frac{x^3}{D^2}+\frac{1}{2}\frac{x^4}{D^2}\right]$$

$$= e^{x+y}\left[\frac{y^2x^4}{3\times4}+\frac{4y}{3}\frac{x^5}{4\times5}+\frac{1}{2}\frac{x^6}{5\times6}\right]$$

$$= e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}+\frac{x^6}{60}\right]$$

$$PI = e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}+\frac{x^6}{60}\right]$$

The general solution is $z = CF + PI$

$$z = f_1(y+x)+x f_2(y+x)+e^{x+y}\left[\frac{x^4y^2}{12}+\frac{x^5y}{15}+\frac{x^6}{60}\right]$$