

Linear PDE's of Second order with constant Coefficients

Homogeneous Linear PDE

$$[a_0 D^n + a_1 D^{n-1} D' + a_2 D^{n-2} D'^2 + \dots + a_n D'^n] z$$

where $D \equiv \frac{\partial}{\partial x}$; $D' \equiv \frac{\partial}{\partial y}$ $= F(x, y)$

e.g. $f(D, D') z = F(x, y)$

(x-1) The soln of (2) is $z = CF + PI$

c.f \rightarrow Complementary Function

P-I \rightarrow Particular Integral.

To find CF:

Replace $D \rightarrow m$
 $D' \rightarrow 1$ m given.

Case (i): roots are real and distinct

$$m_1, m_2, \dots$$
$$z = f_1(y + m_1 x) + f_2(y + m_2 x) + \dots + f_n(y + m_n x)$$

Case (ii): any two roots are equal

$$z = f_1(y + m_1 x) + x f_2(y + m_1 x) + f_3(y + m_3 x) + \dots + f_n(y + m_n x)$$

Case (iii): all roots are equal.

$$z = f_1(y + m_1 x) + x f_2(y + m_1 x) + x^2 f_3(y + m_1 x) + \dots + x^{n-1} f_n(y + m_1 x)$$

Particular Integral:

Rules for finding P.I

$$f(D, D')x = F(x, y)$$

$$P.I = \frac{1}{f(D, D')} F(x, y)$$

(i) $F(x, y) = e^{ax+by}$

Replace $D \rightarrow a$ and $D' \rightarrow b$

$$P.I = \frac{1}{f(D, D')} e^{ax+by}$$

$$= \frac{1}{f(a, b)} e^{ax+by}, \quad \text{if } f(a, b) \neq 0.$$

(ii) $F(x, y) = x^m y^n$

$$P.I = \frac{1}{f(D, D')} x^m y^n$$

$$= [f(D, D')]^{-1} x^m y^n$$

Expand $[f(D, D')]^{-1}$ using Binomial Theorem.

1) Solve $[D^2 - 5DD' + 6D'^2]z = 0$.

2) Solve $[D^2 - 6DD' + 9D'^2]z = 0$.

3) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$.

1) Solve $[D^2 - 5DD' + 6D'^2]z = 0$

Soln: The auxiliary eqn is given by

Replace $D \rightarrow m$ and $D' \rightarrow 1$

$$m^2 - 5m + 6 = 0.$$

$$(m-3)(m-2) = 0$$

$$m = 3, 2$$

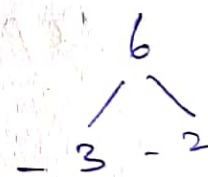
$$m_1 = 3 \text{ and } m_2 = 2$$

The roots are real and distinct

\therefore The complementary function is given by

$$C.F. = z = f_1(y + m_1 x) + f_2(y + m_2 x)$$

$$z = f_1(y + 3x) + f_2(y + 2x).$$



$$(D^2 - 6DD' + 9D'^2)z = 0$$

$$D \rightarrow m \quad D' \rightarrow 1$$

$$m^2 - 6m + 9 = 0$$

$$(m - 3)(m - 3) = 0$$

$$m = 3, 3$$

roots are real & equal

$$2D^2 + 5DD' + 2D'^2 = 0$$

$$D \rightarrow m \quad D' \rightarrow 1$$

$$2m^2 + 5m + 2 = 0$$

example

$$(m + \frac{4}{2})(m + \frac{1}{2}) = 0$$

$$(m + 2)(2m + 1) = 0$$

$$m_1 = -2 \text{ and } m_2 = -\frac{1}{2}$$

roots are real & distinct

$$\text{C.F. } z = f_1(y - 2x) + f_2(y - \frac{1}{2}x)$$

C.F

$$z = f_1(y + 3x) + x f_2(y + 3x)$$

