Inverse z - Transform :

If $z[f(n)] = F(z)$ then $z^{-1}[F(z)] = f(n)$ is called inverse z-transform of $F(z)$.

Eg: $z[a^n] = \frac{z}{z-a}$ | $z\left[\frac{1}{n!}\right] = e^{1/z}$

$\therefore z^{-1}\left[\frac{z}{z-a}\right] = a^n$ | $z^{-1}\left[e^{1/z}\right] = \frac{1}{n!}$.

Methods of finding Inverse z-transforms :

1) Method of Partial Fractions
2) Method of Residues
3) Method Long division Method.

* Partial fraction Method :

1) Find the Inverse z-Transforms of the following:

1) $\frac{z-4}{(z-1)(z-2)^2}$ 2) $\frac{4z^2-2z}{(z-1)(z-2)^2}$

3) $\frac{10z}{z^2-3z+2}$ 4) $\cancel{\frac{z^2+2z}{z^2+2z+4}}$.

1) $z^{-1}\left[\frac{10z}{z^2-3z+2}\right]$

Soln: $F(z) = \frac{10z}{z^2-3z+2} = \frac{10z}{(z-1)(z-2)}$

$\frac{F(z)}{10z} = \frac{1}{(z-1)(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-2)}$

keep $10z$ as such.

$1 = A(z-2) + B(z-1)$

put $z=1$ $A=-1$

put $z=2$ $B=1$

$\therefore \frac{F(z)}{10z} = \frac{-1}{z-1} + \frac{1}{z-2}$

$\therefore F(z) = \frac{-10z}{z-1} + \frac{10z}{z-2}$

$$\therefore Z^{-1}[F(z)] = Z^{-1}\left[\frac{-10z}{z-1}\right] + Z^{-1}\left[\frac{10z}{z-2}\right]$$

$$= -10 Z^{-1}\left[\frac{z}{z-1}\right] + 10 Z^{-1}\left[\frac{z}{z-2}\right]$$

$$= -10(1)^n + 10(2)^n \quad \left(\because Z^{-1}\left(\frac{z}{z-a}\right) = a^n\right)$$

$$\therefore Z^{-1}\left(\frac{10z}{z^2-3z+2}\right) = -10(1)^n + 10(2)^n, \; n \geq 0.$$

ii) $Z^{-1}\left[\frac{4z^2-2z}{(z-1)(z-2)^2}\right]$ (13)

Soln: Let $F(z) = \frac{4z^2-2z}{(z-1)(z-2)^2}$

$$\frac{4z^2-2z}{(z-1)(z-2)^2} = \frac{A}{z-1} + \frac{B}{z-2} + \frac{C}{(z-2)^2}$$

$$4z^2-2z = A(z-2)^2 + B(z-1)(z-2) + C(z-1)$$

put $z=2$

$$16-4 = 0+0+C$$

$$\boxed{12 = C}$$

put $z=1$

$$2 = A(1) \Rightarrow \boxed{A=2}$$

put $z=0$

$$0 = A(-2)^2 + B(-1)(-2) + C(-1)$$

$$0 = 4(2) + 2B - 12$$

$$-2B = -4$$

$$\boxed{B = +2}$$

$$\therefore \frac{4z^2-2z}{(z-1)(z-2)^2} = \frac{2}{z-1} + \frac{2}{(z-2)} + \frac{12}{(z-2)^2}$$

$$\therefore Z^{-1}\left[\frac{4z^2-2z}{(z-1)(z-2)^2}\right] = Z^{-1}\left[\frac{2}{z-1}\right] + Z^{-1}\left[\frac{2}{z-2}\right] + Z^{-1}\left[\frac{12}{(z-2)^2}\right]$$

Problems based on (i) $Z[1] = \frac{z}{z-1}$

(ii) $Z[a^n] = \frac{z}{z-a}$.

1) Find the Z-transform

(i) $Z(k)$ W.K.T $Z(1) = \frac{z}{z-1}$

$Z(k) = Z(k \cdot 1) = k\, Z(1)$

$= k \cdot \frac{z}{z-1}$.

(ii) $Z(-1)^n$ W.K.T $Z[a^n] = \frac{z}{z-a}$.

$Z[(-1)^n] = \frac{z}{z-(-1)}$ Here $a = -1$

$= \frac{z}{z+1}$

(iii) $Z\left[\frac{1}{7^n}\right]$ W.K.T $Z[a^n] = \frac{z}{z-a}$

$Z\left[\frac{1}{7^n}\right] = Z\left[\left(\frac{1}{7}\right)^n\right] = Z\left[\left(\frac{1}{7}\right)^n\right]$

Here $a = \frac{1}{7}$.

$\therefore Z\left[\frac{1}{7^n}\right] = \frac{z}{z-\frac{1}{7}} = \frac{7z}{7z-1}$ //