CANTILEVER:

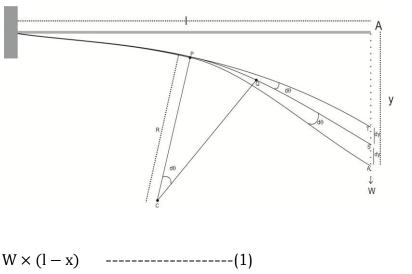
A Cantilever is a beam fixed horizontally at one end and loaded at other end.

Depression of a cantilever loaded at its ends: Theory:

The cantilever OA is fixed at O, its length is 1 and 'W' be the weight loaded at other end. Due to load it moves to OA'.

Let us consider an element PQ of the beam of length dx, at a distance x from fixed end. 'C' be the centre of curvature and R be the radius of curvature.

Due to the load (W) applied at free end, an external couple is created between A and Q, arm of couple is (l - x).



-----(2)

External bending movement = $W \times (l - x)$

Internal bending movement

 $=\frac{\mathrm{YI}}{\mathrm{R}}$

Under equilibrium condition,

External bending movement = Internal bending movement

$$\therefore R = \frac{YI}{W(l-x)} - - - - - (3)$$

From the figure arc length

$$PQ = Rd\theta = dx$$
$$d\theta = \frac{dx}{R} \qquad -----(4)$$

On substituting R Value

$$d\theta = \frac{dx}{YI} W(I - x) \quad -----(5)$$

From ΔQAS

$$\sin\theta = \frac{\mathrm{dy}}{\mathrm{l-x}}$$

On sub (5) in (6) we get,

$$dy = \frac{W(l-x)^2}{YI} dx$$

 \therefore Total depression is by integrating the above within the limit 0 to 1.

$$\therefore y = \frac{W}{YI} \int_0^1 (1-x)^2 dx$$

On solving we get,

$$y = \frac{W}{YI} \cdot \frac{l^3}{3}$$

This equation gives the depression of the cantilever.

Special Cases:

(i) Rectangular cross section. For, $I = \frac{bd^3}{12}$ Depression produced $y = \frac{4Wl^3}{Ybd^3}$

(ii) Circular cross section, For I = $\frac{\pi r^4}{4}$ Depression Produced y = $\frac{4Wl^3}{3\pi r^4 Y}$

'r' is the radius of the circular cross section.