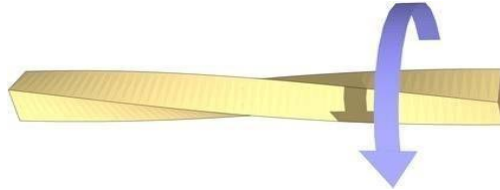




## TORSIONAL STRESS AND DEFORMATION

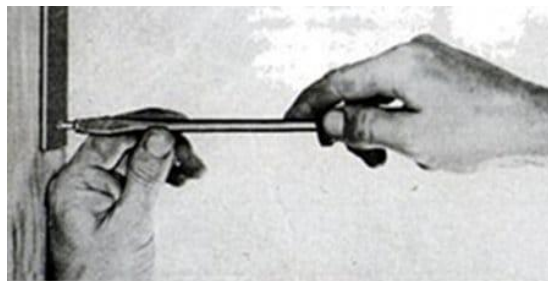
### Torsional Stress and Deformations

*Shear stress is produced about a longitudinal axis of a structural member by the application of twisting couple to the end of the structural member known as*



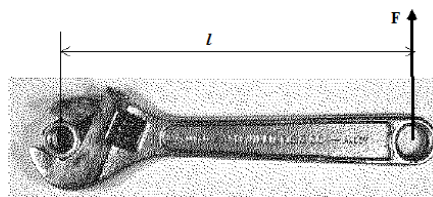
*torsional stress.* Torsion is the twisting of a straight bar when it is loaded by twisting couple or torque. It tends to produce rotation about the longitudinal axes of the bar.

For instant, when we turn a screw driver to produce torsion our hand applies torque “ $T$ ” to the handle and twist the shank of the screw driver.



### Twisting Couple (Torque)

*The twisting of a structural member about its longitudinal axis by two equal and opposite torques is expressed through a certain angle is called twisting couple. The stress is produced in this process is not tensile or compressive, it is said to be*



*shearing or shear stress. The strain is measured by an angle in unit of radians.*

The simple example is that of using a wrench to tighten a nut on a bolt as shown in given figure. If the bolt, wrench, and force are all perpendicular to one another.



## TWISTING COUPLE ON A WIRE

If we have a wire or cylinder, clamped at one end, and twisted at the other through an angle  $\phi$ , about its axis, it is said to be under tension, due to the elasticity of the material of the wire or the cylinder, a restoring couple is set up in it, equal and opposite to the twisting couple.

Consider a cylindrical wire of length  $l$  and radius  $a$ . The cylindrical wire is clamped to a fixed support. This wire is made up of a number of cylindrical tubes (co-axial) whose radii vary from zero to „ $r$ “. Let us consider one such cylinder, as shown in fig (b) with radius „ $x$ “ and thickness „ $dx$ “.

Let  $AB$  be a line on the elementary tube which is parallel to the axis of the tube. Consider a couple applied at the bottom end of the wire, which results in twisting of the wire through an angle  $\phi$ . In the twisted state, the position of  $AB$  will be taken as  $AB'$  as shown in fig 1.6.1

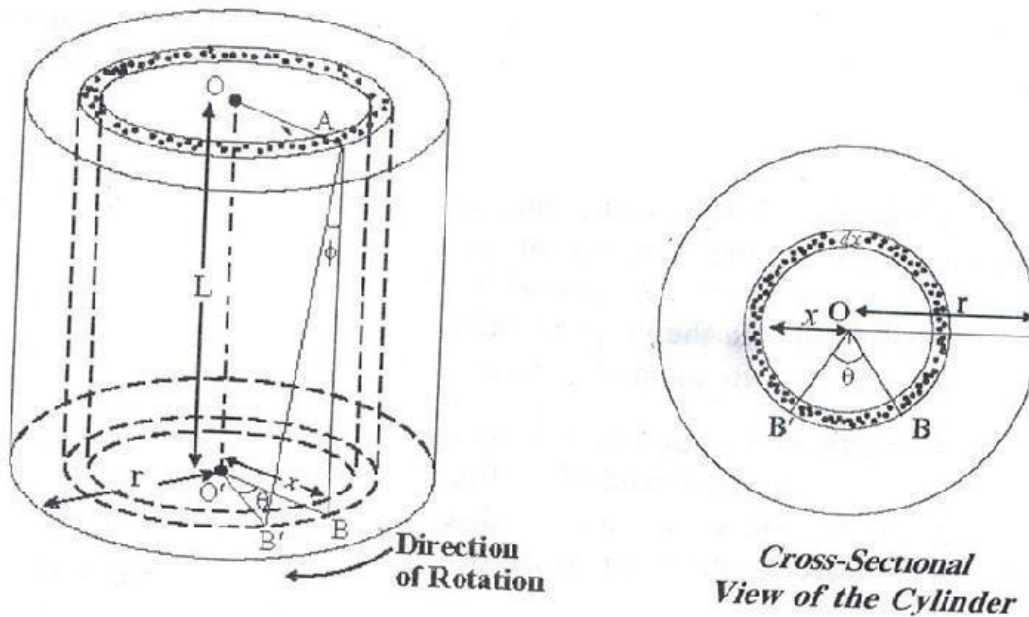


Fig (a)

Fig (b)

Fig 1.6.1 Twisting couple on a wire.

From fig(a)

The angle  $\angle ABB'$   $\approx \frac{BB'}{l}$



.....  
..... (1)



The displacement decreases, as the move from the rim of the cylinder to the Centre. At the Centre, it will become zero. This means that shearing strain is maximum at the rim and minimum at the Centre.

Consider a hollow cylinder along the plane AB and flattened out. Therefore, we get a rectangle OABO'' before twisting and OABB'' after twisting. The angle through which this the hollow cylinder is sheared.

From the cross sectional view of the cylinder

$$BB' = x \dots \dots \dots (2)$$

From equations (1) and (2), we get,

$$l = x$$

$$\text{Shearing strain} = \frac{x}{l} \dots \dots \dots (3)$$

We know that,

$$\text{Rigidity modulus (n)} = \frac{\text{shearing stress}}{\text{shearing strain}}$$

$$\text{Shearing stress} = n \times \text{Shearing strain}$$

$$n \frac{x}{l} \dots \dots \dots (4)$$

Let the area of the elementary tube =  $2x dx$

The shearing force on this area = Shearing stress  $\times$  Area

$$\frac{n \frac{x}{l} \times 2x dx}{2nx^2 dx} \dots \dots \dots (5)$$

The moment of force about the axis of the wire

$$\frac{2nx^2 dx \times x}{2nx^3 dx} \dots \dots \dots (6)$$

The twisting couple applied to the whole wire can be obtained by integrating equation 6

between the limit  $x=0$  and  $x=r$

$$\int_0^r \frac{2nx^3 dx}{l}$$



$$= \frac{2n^r}{l} \int_0^r x^3 dx$$

$$= \frac{2n}{l} \left[ \frac{x^4}{4} \right]_0^r$$

$$C = \frac{1}{2} \frac{n\pi r^4 \theta}{l}$$

..... (7)

If  $\theta = 1$  radian

The twisting couples per unit angular twist of the wire

$$C = \frac{1}{2} \frac{n\pi r^4}{l}$$