Formation of Difference Equations

① Find the difference equation from

$y(n) = (A + nB)\,2^n$.

Given $y(n) = (A+nB)\,2^n \longrightarrow ①$

$y(n+1) = A\,2^{n+1} + (n+1)B\,2^{n+1}$

$y(n+1) = 2A2^n + 2(n+1)B2^n \longrightarrow ②$

$y(n+2) = 4A2^n + 4(n+2)B2^n \longrightarrow ③$

② − ①×2 $\Rightarrow$ $y(n+1) - 2y(n) = 2B2^n \longrightarrow ④$

③ − ②×2 $\Rightarrow$ $y(n+2) - 2y(n+1) = 4B2^n \longrightarrow ⑤$

$$2\left[y(n+1) - 2y(n)\right] = y(n+2) - 2y(n+1)$$

$$\Rightarrow \quad y(n+2) - 4y(n+1) + 4y(n) = 0$$

(2) Find the difference equation from $y_n = A(-2)^n + Bn$.

Given $y_n = A(-2)^n + Bn \longrightarrow (1)$

$$y_{n+1} = A(-2)^{n+1} + B(n+1)$$

$$y_{n+1} = -2A(-2)^n + B(n+1) \longrightarrow (2)$$

$$y_{n+2} = 4A(-2)^n + B(n+2) \longrightarrow (3)$$

$(2) + 2\times(1) \Rightarrow y_{n+1} + 2y_n = B(n+1) + 2Bn$

$$= 3nB + B \longrightarrow (4)$$

$(3) + 2\times(2) \Rightarrow y_{n+2} + 2y_{n+1} = B(n+2) + 2B(n+1)$

$$= 3nB + 4B$$