SNS COLLEGE OF TECHNOLOGY

COIMBATORE-35

AN AUTONOMOUS INSTITUTION

19ECE301 – IMAGE PROCESSING AND COMPUTER VISION

UNIT – V

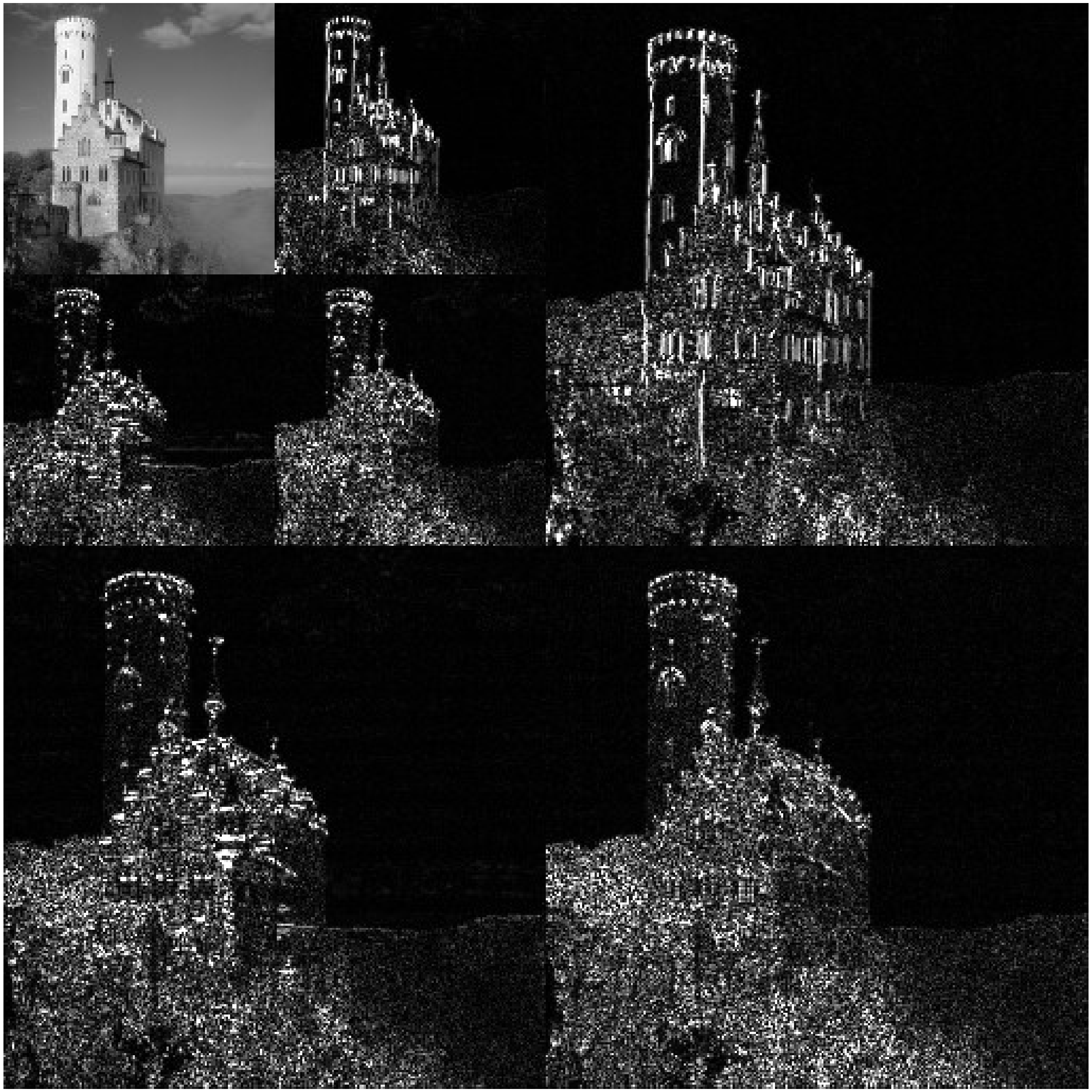
COMPUTER VISION

WAVELETS

WAVELET

A wavelet is a mathematical function applied in digital image processing and compression. Its main aim is to improve the image quality. Also, wavelets can divide signals into time and frequency components. Wavelet transform is the decomposition of a signal to the frequency components.

When choosing a suitable wavelet, it is essential to understand a few basic properties of the wavelets, such as Vanishing moments, Support width, Regularity, Symmetry and Orthogonality.



PROPERTIES OF CWT

- This transform is redundant (undecimated transform). It means that it generates a huge amount of coefficients.
- It is not computationally efficient since it uses a huge amount of coefficients. These coefficients occupy a lot of RAM, thus hindering the execution speed.
- It is shift-invariant. You can shift the signal around before calculating the transform, and the resulting energy will still be the same.
- CWT cannot be implemented with filter banks. It is a drawback.
- ICWT is not much accurate and has less stable numerical computations.

PROPERTIES OF DWT

•DWT has two forms. One is non-decimated, which generates more coefficients, and the other is undecimated, which generates fewer coefficients.

•DWT is more computationally efficient because it generates fewer coefficients.

•The decimated DWT is not shift-invariant as it does downsample, while non-decimated DWT is shift-invariant as it doesn’t use downsampling.

•DWT (decimated and undecimated) can be implemented with the help of filter banks (lower pass filter and high pass filter).

•IDWT is the more accurate and less stable numerical operation with precision.

WAVELET PROPERTIES

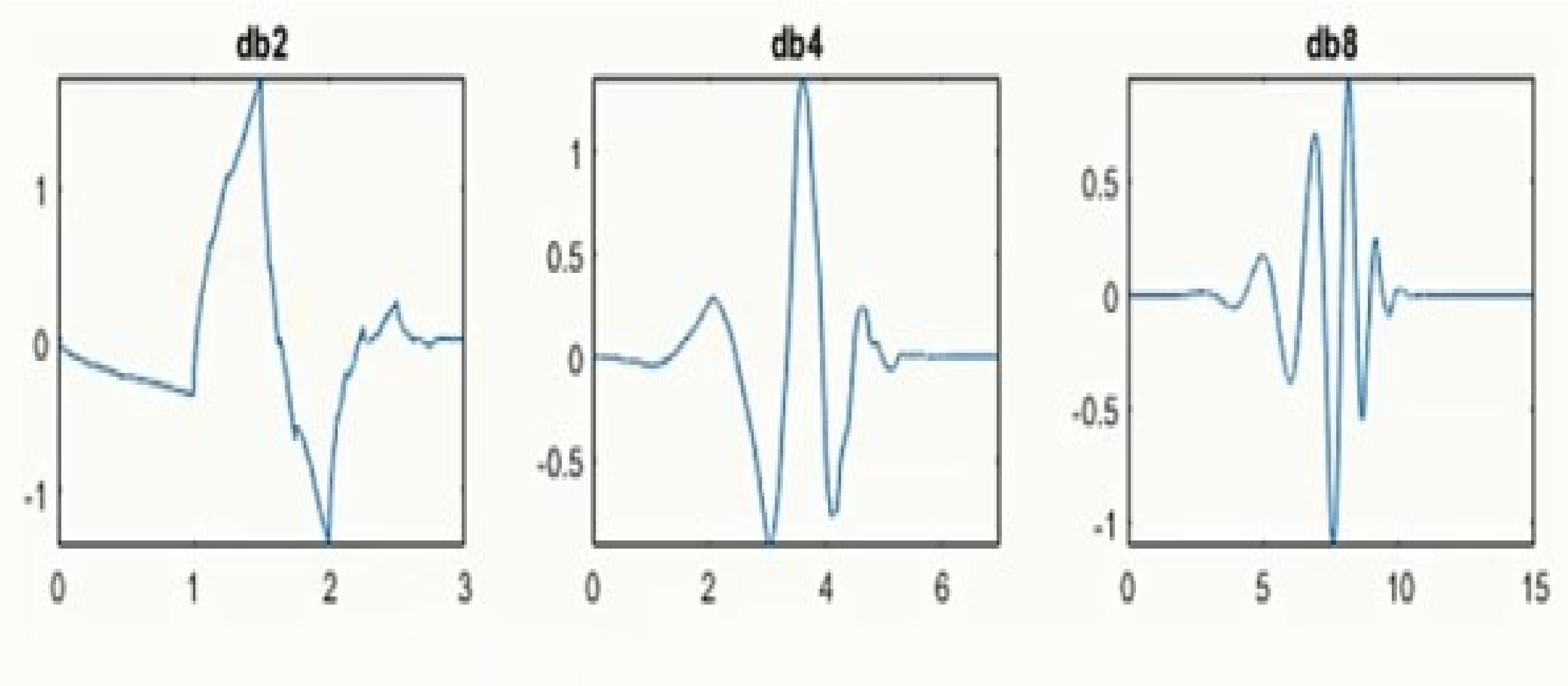
1. VANISHING MOMENTS

The vanishing moment is a criterion about how a function f(t) decays towards infinity. We can estimate the decay rate using the integration below:

$$[\int_{-\infty}^{\infty} t^k f(t);dt]=0$$

Where k is the decay rate.

Also, a wavelet function ψ(t) has N vanishing moments This property shows how fast a wavelet decays with time. The number of vanishing moments and the oscillation of wavelets have a close relationship. As the number of vanishing moments grows, the wavelet oscillation becomes greater.



As we can see, as the number of vanishing moments increases, the number of oscillations also increases. Therefore, the names of many wavelets are derived from the number of vanishing moments.

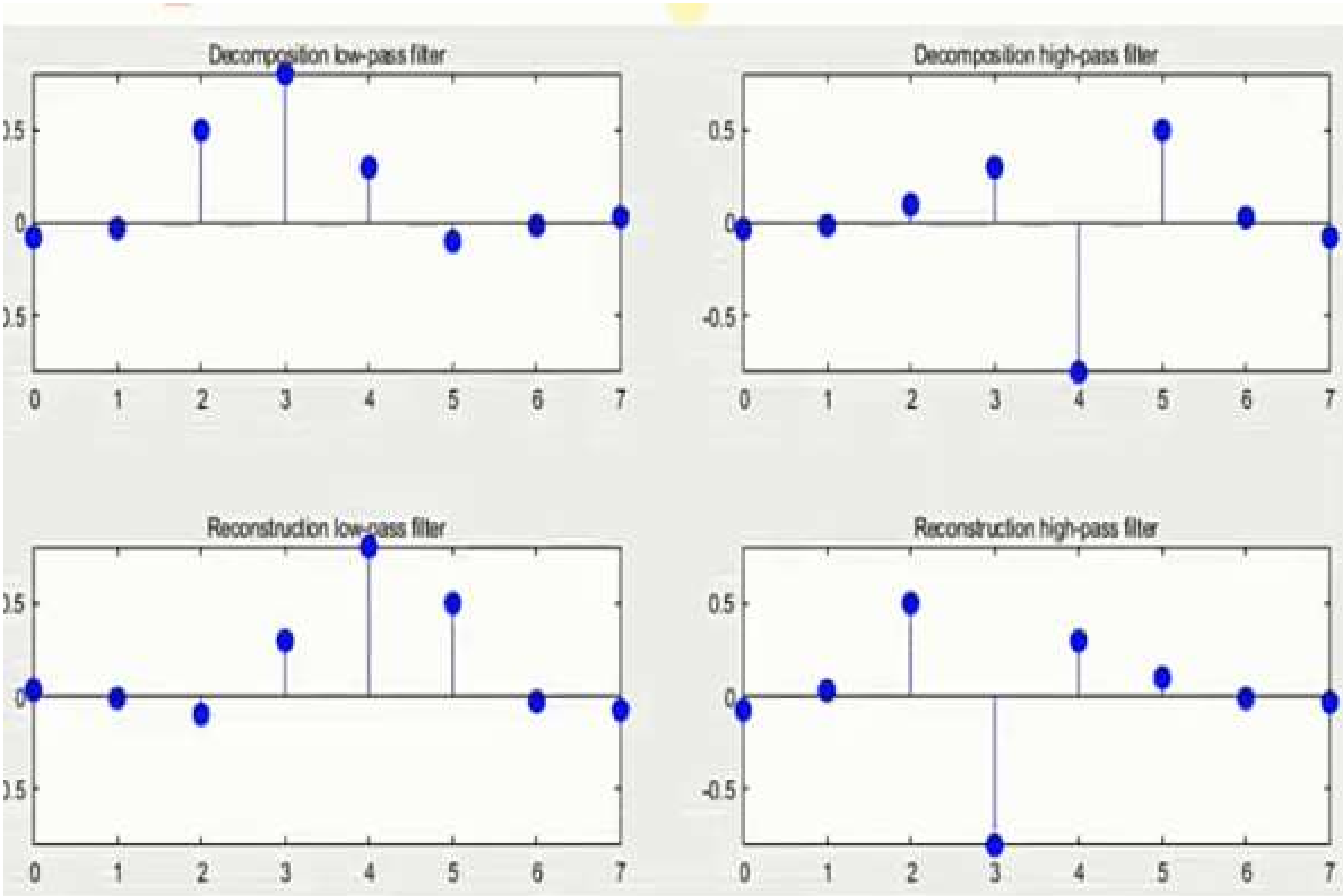
2. SUPPORT SIZE

The size of support indicates the FIR filter's length. If an FIR filter has an N number of samples, the support width is N-1.

For example, haar, dbN, and symN has N number of vanishing moments, filter length of 2N, and the support width is 2N-1.

For coifN, the number of vanishing moments is 2N, and the filter length is 6N.

Below is an example of the symlet4 wavelet:


Decomposition low-pass filter
Decomposition high-pass filter
Reconstruction low-pass filter
Reconstruction high-pass filter
0.5
0
-0.5
0 1 2 3 4 5 6 7

Symlet4 has four vanishing moments, and the filter length is given by 2N. This means that the filter length is 8. As we can see from the figure, there are eight samples (0-7) in all the filters.

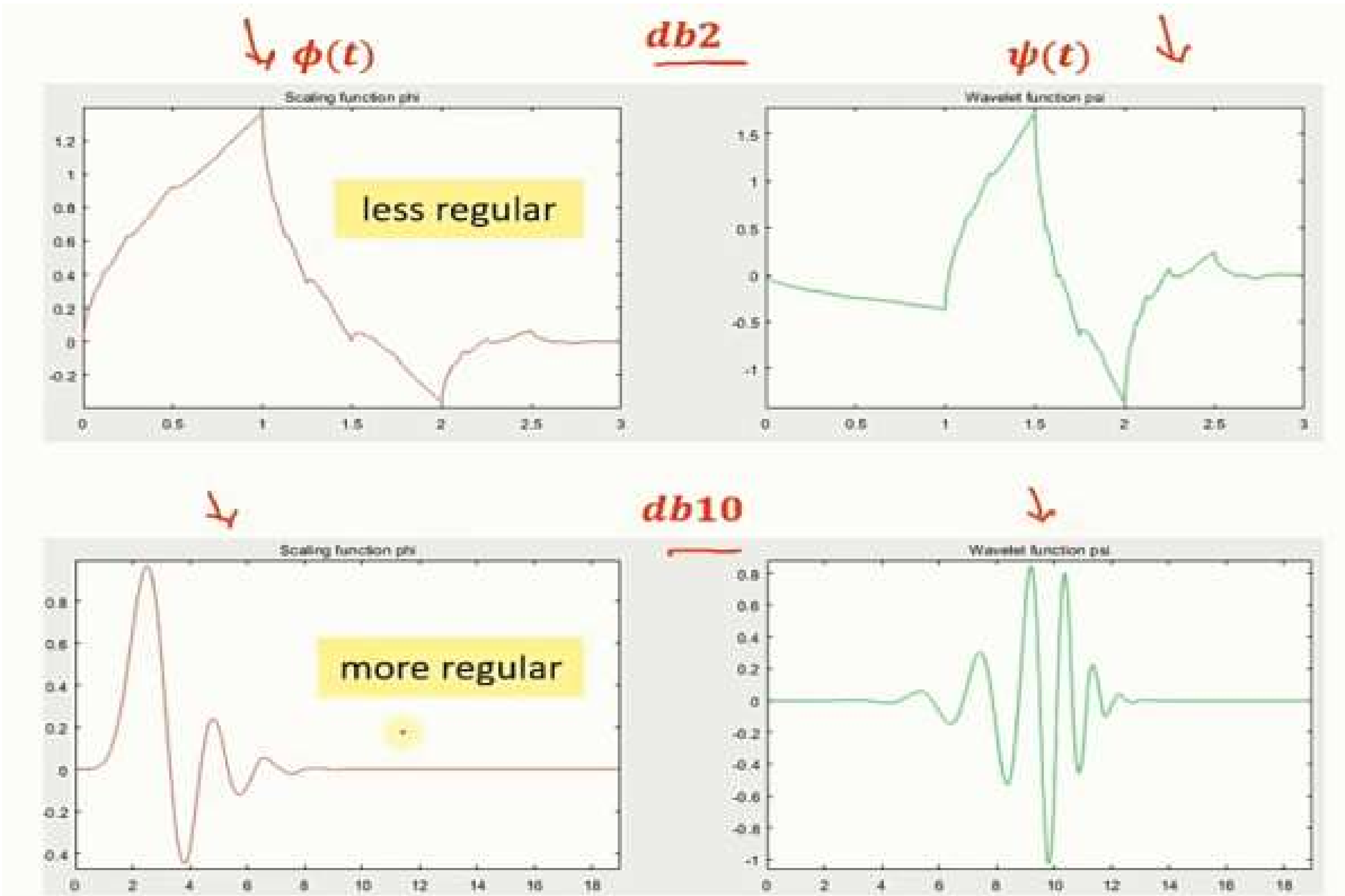
This is how the number of vanishing moments relates to the length of the filters. The more the vanishing moments, the higher the filter’s length.

3. Regularity

Regularity is the number of continuous derivatives a function has.

Intuitively, it can be considered as the measure of smoothness.

A wavelet with more vanishing moments will be more regular. This is shown in the figure below:



db2
φ(t)
ψ(t)
Scaling function phi
Wavelet function psi
less regular
db10
Scaling function phi
Wavelet function psi
more regular

In the figure above, we have two wavelets, db2 and db10. In both of them, we have the wavelet function and scaling function. As we can see, both the wavelet and scaling functions of db10 are smoother than that of db2.

Also, db10 has ten vanishing moments and db2 has two vanishing moments. The higher the vanishing moments, the more regular the wavelet.

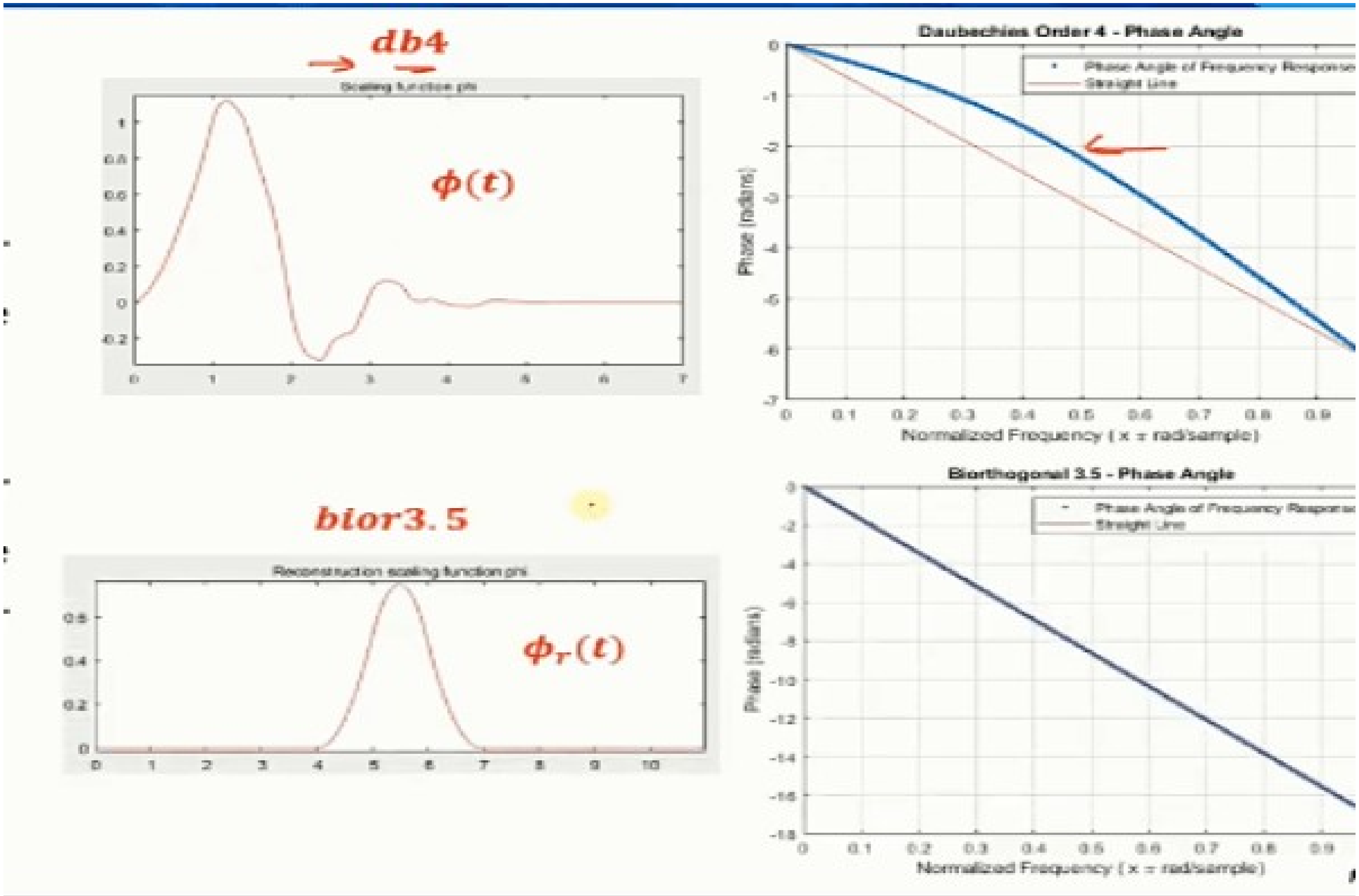
4. Symmetry

This is a property that concerns whether wavelet filter characteristics have a linear phase or not.

Considering this property, db is not symmetric. Therefore, it can be preferred as a non-linear phase.

Sym and coif are near symmetric or almost linear phases. Bior is a linear phase.

We can understand this better in the figure below:


db4
Scaling function phi
$\phi(t)$
Daubechies Order 4 - Phase Angle
Phase Angle of Frequency Response
Straight Line
Phase (radians)
Normalized Frequency (×π rad/sample)
bior3.5
Reconstruction scaling function phi
$\phi_r(t)$
Biorthogonal 3.5 - Phase Angle
Phase Angle of Frequency Response
Straight Line
Phase (radians)
Normalized Frequency (×π rad/sample)

We have used db4 and bior3.5 to clarify this. In the first figure, we represent the scale function of db4. The blue plot is the phase plot in the frequency domain. As we can see, it is not symmetric. However, for the bior3.5, in the frequency domain, the plot is symmetric. This is used in the selection of wavelets for various applications.

VARIOUS APPLICATIONS

1. Wavelet for feature extraction
2. Wavelet for denoising
3. Wavelet for compression
4. Image watermarking
5. Edge detection
6. Ecg signal feature extraction

Thank You!