SNS COLLEGE OF TECHNOLOGY
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COIMBATORE–641 035.
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DEPARTMENT OF BIOMEDICAL ENGINEERING

19BMB302- BIOMEDICAL SIGNAL PROCESSING

III B.E. BME / V SEMESTER

Two Marks with Answers

Academic Year : 2022-2023 (Odd Semester)

UNIT III- IIR FILTER DESIGN

1. What is pre-warping?

   In bilinear transformation, the relation between analog and digital frequencies is nonlinear. This non-linear relationship introduces distortion in frequency axis, when the ‘s’ plane is mapped into ‘z’ plane using bilinear transformation. This effect is known as frequency warping. The pre-warping is performed as follows:

   $$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right),$$

   In above equation, Ω and ω are analog and digital frequencies respectively. T is nothing but a sampling rate. Pre-warping is necessary to eliminate the effect of warping on amplitude response.

2. What are the properties of Chebyshevfilter?
   1. The magnitude response of theChebyshev filterexhibits ripple either in pass band or in
      stop band according to type.
   2. All poles lie on the ellipse.

3. “IIR filter does not have linear phase” – Justify

   A physically realizable and stable IIR filter cannot have linear phase. A linear phase filter must have a transfer function that satisfies the condition.

   $$H(z)=\pm z^{-N}\,H(z^{-1})$$

   where z^{-N} represents a delay of *N* units of time. But if this is case, for every pole inside the unit circle there must be pole outside the unit circle. Hence the filter would be unstable. Therefore, a causal and stable IIR filter cannot have linear phase.

4. Draw the direct form realization of IIR system

   Direct form- I

   Equation:

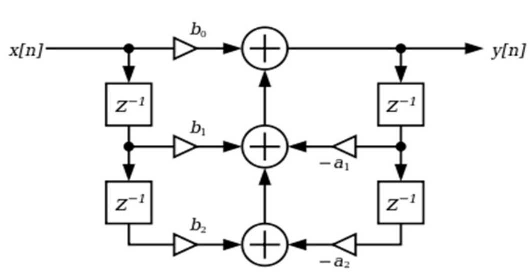
Let us consider an LTI recursive system described by the difference equation.

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$
$$= -a_1 y(n-1) - a_2 y(n-2) \ldots - a_{N-1} y(n-N+1)$$
$$- a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M)$$

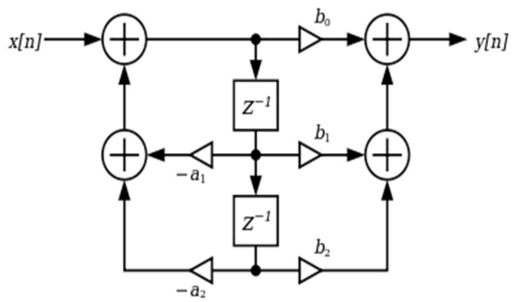
et

$$b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) = w(n).$$

en $y(n) = -a_1 y(n-1) - a_2 y(n-2) + \ldots - a_N y(n-N) + w(n)$



Direct form- II



5. What is meant by bilinear transformation method of designing IIR filter?

   Bilinear transformation is a one to one mapping from the s-domain to the z-domain. That is, the bilinear transformation is a conformal mapping that transforms the j Ω axis into the unit circle in the z plane only once, thus avoiding the aliasing of frequency components. Also the transformation of a stable analog filter result in a stable digital filter as all the poles in the left half of the s plane are mapped inside the unit circle of the z plane. The bilinear mapping is a one to one mapping and it is accomplished when

$$S = \frac{2}{T}\left(\frac{1 - Z^{-1}}{1 + Z^{-1}}\right)$$

6. Compare analog and digital filters.

Analog filter	Digital filter
Constructed using active or passive components and it is described by a differential equation	Consists of elements like adder, Multiplier and delay units and it is described by a difference equation
Analog filter is described by a differential equation.	Digital filter is described by a difference equation.
It processes analog inputs and generates analog output	Processes and generates digital output
The frequency response of an analog filter can be modified by changing the components.	The frequency response can be changed by changing the filter coefficients.

7. Sketch the mapping of s-plane and z-plane in approximation of derivatives. The mapping procedure between S-plane & Z-plane in the method of mapping of differentials is given by

$$H(Z) = H(S)\Big|_{S=\left(\frac{1-Z^{-1}}{T}\right)}$$

   The above mapping has the following characteristics
   - The left half S-plane maps inside a circle of radius ½ centered at Z= ½ in the Z-plane.
   - The right half of S-plane maps into the region outside the circle of radius ½ in the Z-plane.
   - The j Ω-axis maps onto the perimeter of the circle of radius ½ in the Z-plane.

8. What are the properties of impulse invariant transformation?
   - All the poles in left half s-plane where σ< 0, those pole map to inside the unit circle.
   - All the poles in right half s-plane where σ> 0, those digital pole outside the unit circle.

9. Mention the properties of Butterworth filter.
    - The Butterworth filters are all pole designs.
    - At the cut-off frequency Ωc the magnitude of normalized Butterworth filter is $1/\sqrt{2}$.
    - The filter order 'n' completely specifies the filter and as the value of N increases the magnitude response approaches the ideal response.

10. Write the transfer equation to convert low pass filter into band stop filter.

$$s \rightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{\Omega_l(\Omega_u - \Omega_l)}{-{\Omega_1}^2 + \Omega_u \Omega_l}$$

$$B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-{\Omega_2}^2 + \Omega_u \Omega_l}$$

11. What is meant by aliasing?
    W/hen the sampling frequency is less than twice of the highest frequency content of the signal, then the aliasing is frequency domain takes place. In aliasing, the high frequencies of the signal mix with lower frequencies and create distortion in frequency spectrum.

12. Compare the impulse invariant and bilinear transformations.

S. No	Impulse Invariant Transformation	Bilinear transformation
1	It is many-to-one mapping	It is one-to-one mapping
2	The relation between analog and digital frequency is linear	The relation between analog and digital frequency is non-linear
3	To prevent the problem of aliasing the analog filters should be band limited	No problem of aliasing and so the analog filters need not be band limited
4	The magnitude and phase response of analog filter can be preserved by choosing low sampling time or high sampling frequency	Due to the effect of warping, the phase response of analog filters cannot be preserved. But the magnitude response can be preserved by pre-warping

13. Given low pass transfer function $H_a(s)=\frac{1}{s+1}$. Find the high pass transfer function having a cutoff frequency 10 rad/sec.
    For high pass filter in $H_a(s)$ s should be replaced by,

$$s \rightarrow \frac{\Omega_c}{s} = \frac{10}{s}$$

$$Ha(s) = \frac{1}{\frac{10}{s} + 1} = \frac{s}{10 + s}$$

14. What is Butterworth approximation?
    In Butterworth approximation, the error function is selected such that the magnitude is maximally flat in the origin (i.e., at Ω = 0) and monotonically decreasing with increasing Ω.

15. Why impulse invariant method is not preferred in the design of IIR filters other than low pass filter?

In this method the mapping from s plane to z plane is many to one. i.e. ,all the poles in the s plane between the intervals $(2k-1)\pi/T$ to $(2k+1)\pi/T$ .Thus there are an infinite number of poles that map to the same location in the z plane, producing an aliasing effect. Due to spectrum aliasing the impulse invariant method is inappropriate in designing high pass filters. That is why the impulse method is not much preferred in the design of IIR filters other than low pass filter.

16. Write the steps in designing chebyshev filter.

1. Find the order of the filter N.
2. Find the value of major and minor axis.
3. Calculate the poles.
4. Find the denominator function using the above poles.
5. The numerator polynomial value depends on the value of N. If N is odd: put s=0 in the denominator polynomial. If N is even put s=0 and divide it by $(1+\varepsilon)^{½}$

17. Write down the steps for designing a Butterworth Low pass filter.

1. From the given specifications find the order of the filter N.
2. Round off it to the next higher integer.
3. Find the transfer function *H(s)* for the value of N
4. Find the cut off frequency Ω_C.
5. Find the transfer function $H_a(s)$ by substituting s by $\frac{s}{\Omega c}$ value in H(s).

18. Distinguish between Butterworth and Chebyshev filter.

S.No.	Butterworth Filter	Chebyshev Type-I Filter
1	All pole design, the poles lie on a circle in S-plane.	All pole design, the poles lie on an ellipse in S-plane.
2	The magnitude response is maximally flat at the origin and monotonically decreasing function of Ω.	The magnitude response is equiripple in pass band and monotonically decreasing in the stop band.
3	The normalized magnitude response has a value of 0.707 at the cutoff frequency Ω_C.	The normalized magnitude response has a value of $\frac{1}{\sqrt{1+\varepsilon^2}}$ at the cutoff frequency ΩC.
4	Only few parameters has to be calculated to determine the transfer function.	A large number of parameters has to be calculated to determine the transfer function.

19. What are the main advantages and disadvantages of bilinear transformation?

Advantages of bilinear transformation

1. The bilinear transformation is one –to-one mapping.
2. There is no aliasing
3. Stable continuous systems can be mapped in to realizable, stable digital systems.

Disadvantages of bilinear transformation

- The mapping is highly non-linear producing frequency compression at high frequencies.
- Neither the impulse response nor the phase response of the analog filter is preserved in a digital filter obtained by bilinear transformation.

20. Convert the given analog transfer function $H(s) = \frac{1}{(s+6)}$ into digital by impulse invariant method

Using impulse invariant transformation,

$$\frac{1}{(s+a)} \Rightarrow \frac{1}{(1 - e^{-a}\ Z^{-1})}$$

Here, a= - 6 and Assume T=1 sec

$$H(z) = \frac{1}{(1 - e^{-6}Z^{-1})}$$

21. What are the different types of structures for realization of IIR systems?

The different types of structures for realization of IIR system are.

1. Direct – form I structure,
2. Direct – form II structure,
3. Transposed Direct – form II structure,
4. Cascade form structure,
5. Parallel form structure,
6. Lattice – ladder structure.

22. What are the disadvantages of direct form realization?

Direct form-I

1. It uses separate delays for both input and output
2. It requires M+ N+ 1 multiplication, M+N additions and M+N+1 memory locations.

Direct form-II

1. It has internal overflows
2. It is necessary to scale the input before applying into filter section.

23. What is the relation between digital and analog frequency in impulse invariant transformation?

The relation between analog and digital frequency in impulse invariant transformation is given by digital frequency $W = \Omega T and\ \Omega = W/T$

Where Ω is analog frequency and W is digital frequency.

24. What is the relation between digital and analog frequency in bilinear transformation?

The relation between analog and digital frequency in bilinear transformation is given by digital frequency $W = 2tan^{-1}(\Omega T)\ and\ \Omega = \frac{2}{T}\tan\left(\frac{w}{2}\right)$

Where Ω is analog frequency and W is digital frequency.