SNS COLLEGE OF TECHNOLOGY

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Department of Biomedical Engineering

Course Name: Control Systems

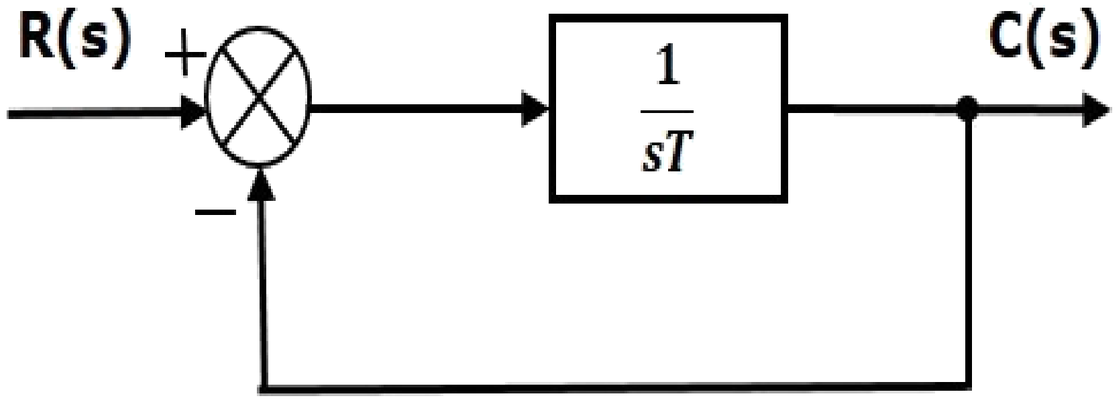
III Year : V Semester

Unit III –Frequency Response

Topic : M & N Circles

Introduction

- The study of closed loop frequency response is every useful as it enables us to use the second order correlations between frequency response and time response.



Frequency Domain Specifications

Usually the specifications in frequency domain are:

- Resonance Peak
- Resonant Frequency
- Bandwidth
- Cutt-off rate
- Gain margin and Phase margin

The maximum value of M and the frequency at which it occurs are important figures of merit.

Constant M Circles

- Consider any point G(jw)=x+jy, on the polar plot. The closed loop response is

$$G(j\omega) = X + jY$$

where X and Y are real quantities. Then M is given by

$$M = \frac{|X + jY|}{|1 + X + jY|}$$

and M^2 is

$$M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2}$$

Hence

$$X^2(1 - M^2) - 2M^2X - M^2 + (1 - M^2)Y^2 = 0$$

$$X^2 + \frac{2M^2}{M^2 - 1}X + \frac{M^2}{M^2 - 1} + Y^2 = 0$$

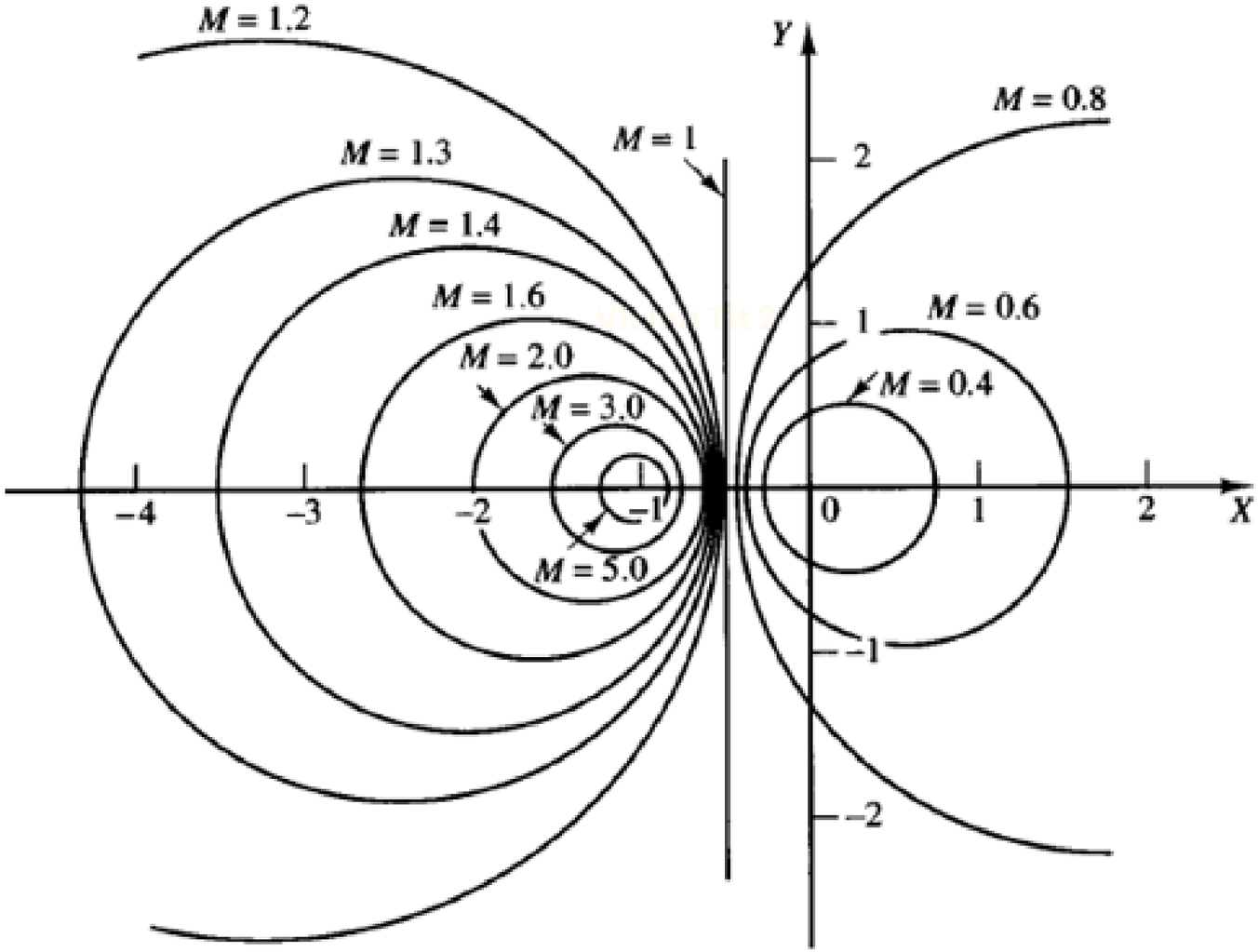
If the term $M^2/(M^2 - 1)^2$ is added to both sides of this last equation, we obtain

$$\left(X + \frac{M^2}{M^2 - 1}\right)^2 + Y^2 = \frac{M^2}{(M^2 - 1)^2}$$

$$x_0 = -\frac{M^2}{M^2 - 1} \; ; \; y_0 = 0$$

- The above equation is the eqationn of circle with centre & radius $r_0 = \frac{M}{M^2 - 1}$

Constant M Circles



Constant N Circles

- Consider any point G(jw)=x+jy, on the polar plot. The closed loop response is

$$\angle e^{j\alpha} = \left| \frac{X + jY}{1 + X + jY} \right.$$

the phase angle α is

$$\alpha = \tan^{-1}\left(\frac{Y}{X}\right) - \tan^{-1}\left(\frac{Y}{1+X}\right)$$

If we define

$$\tan \alpha = N$$

then

$$N = \tan\left[\tan^{-1}\left(\frac{Y}{X}\right) - \tan^{-1}\left(\frac{Y}{1+X}\right)\right]$$

Since

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

we obtain

$$N = \frac{\frac{Y}{X} - \frac{Y}{1+X}}{1 + \frac{Y}{X}\left(\frac{Y}{1+X}\right)} = \frac{Y}{X^2 + X + Y^2}$$

or

$$X^2 + X + Y^2 - \frac{1}{N}Y = 0$$

The addition of (¼) + 1/(2N)² to both sides of this last equation yields

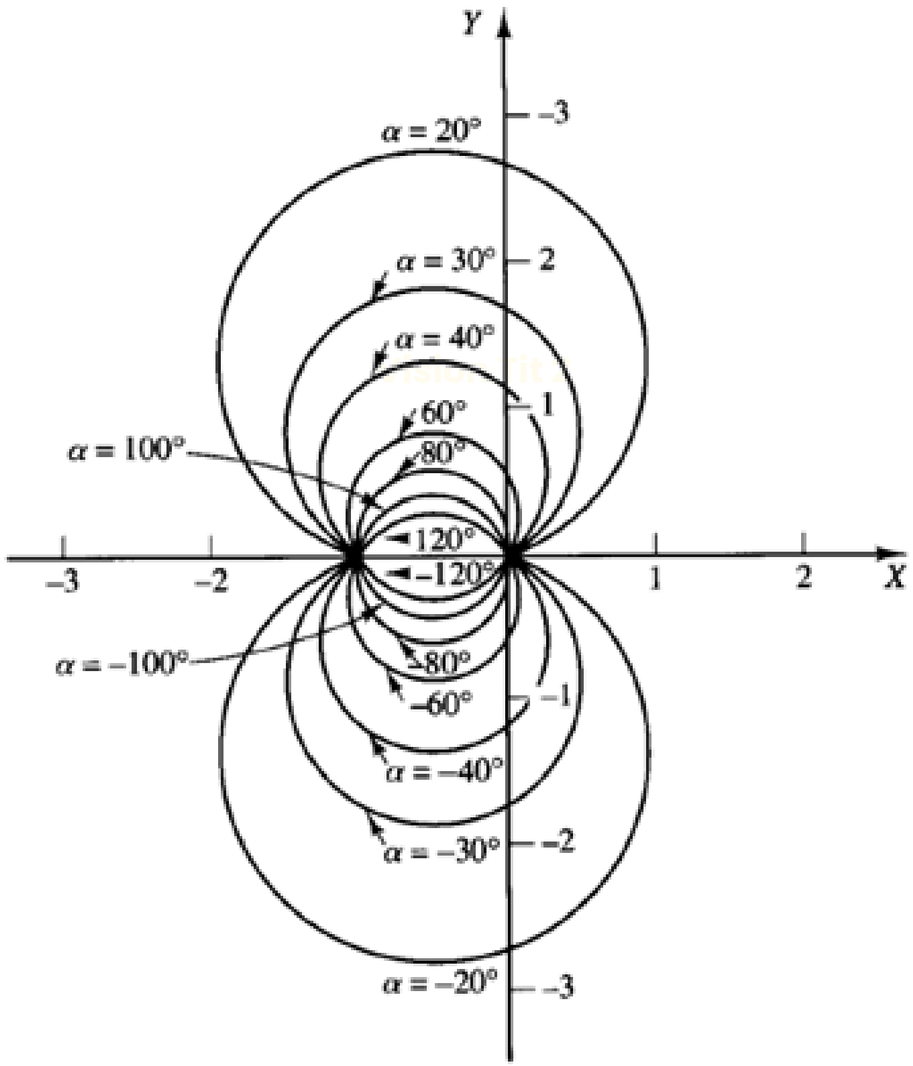
$$\left(X + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$x_0 = -1/2\,;\; y_0 = 1/2N$$

$$r_0 = \frac{1}{2N}\left(N^2 + 1\right)^{1/2}$$

- The above equation is the eqn of circle with centre & radius

Constant N Circles



Nicholas Chart

- The chart consisting of the M and N loci in the log magnitude versus phase diagram is called the Nichols chart.
- The critical point (-1+j0) is mapped to the Nichols chart as the point (0db, -180^{o})
- The Nichols chart is symmetric about the -180^{o} axis. The M & N loci repeat for every 360^{o}.
- The Nichols chart is useful for determining the frequency response of the closed loop from the open loop.

Nicholas Chart