UNIT-I BASICS & STATICS OF PARTICLES

SHORT ANSWERS

Syllabus

Introduction – Units and Dimensions – Laws of Mechanics – Lame's theorem, Parallelogram and triangular Law of forces – Vectors – Vectorial representation of forces and moments – Vector operations: additions, subtraction, dot product, cross product – Coplanar Forces – Resolution and Composition of forces – Equilibrium of a particle – Forces in space – Equilibrium of a particle in space –Equivalent systems of forces – Principle of transmissibility – Single equivalent force.

1. Define the term Mechanics

Mechanics is defined as the branch of physical science that deals with the state of rest or state of motion, under the action of forces, is termed as Mechanics.

2. What are the types of mechanics?

Mechanics can be divided in to two types. They are

- ✓ Statics.
- ✓ Dynamics.

Dynamics is further divided in to:

- ✓ Kinematics.
- ✓ Kinetics.

3. Define Kinematics

Kinematics is defined as the study of the relationship between displacement, Velocity and acceleration without considering the forces which cause the motion.

4. Define Kinetics

Kinetics is defined as the study of motion of the bodies with consideration of forces involved on it.

5. Define the term Particle

Particle is defined as a body whose shape and dimensions are considered as negligible. The mass is concentrated at a point. Example. A ball kept on a table

6. What is meant by a rigid body?

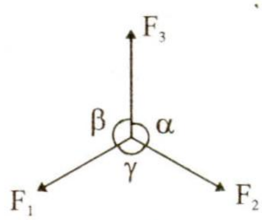
A rigid body is a body having a mass and dimension. It does not deform when force is applied.

Example: Road roller of radius 'r' and weight 'W' rolls on a road.

7. State Lami's Theorem

Lami's theorem states that, if three forces acting on a particle, keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces and the constant of prortionality is the same. Mathematically it is represented as

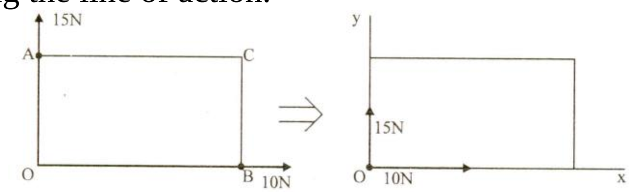
$F1/ \sin\alpha = F2 / \sin\beta = F3 / \sin\gamma$



Lami's Theorem

8. State the Principle of transmissibility

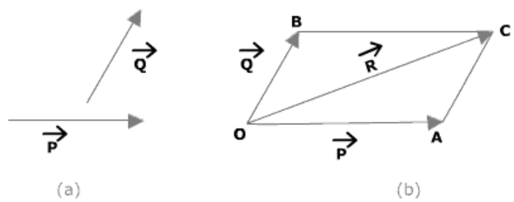
The effect of a force will not be changed, if the point of application is shifted anywhere along the line of action.



Principle of transmissibility

9. Define Parallelogram law of force

The parallelogram law states that, when two forces (P, Q) represent the two sides of a parallelogram, then the diagonal(R) will be the resultant force**.**

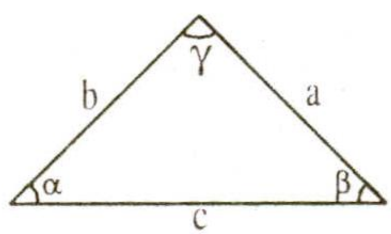


Parallelogram law

10. State the Law of triangle

If a,b,c are the sides of a triangle and α, β, γ be the angle between the sides, then the sine law states that

a / Sin α = b / Sin β = c / Sin γ



Triangle Law

11. What are the primary and secondary dimensions?

Dimensions which are independent of others dimensions are termed as basic or primary dimensions. The basic dimensions are (i) length (ii) mass and (iii) time.

Length (L): Length is a primary dimension which describes the physical size of a body quantitatively. The unit of length is ‘m’.

Mass (m) : The quantity of matter contained in a body is known as mass of the body. It is a scalar quantity is a property of matter. The property of each body manifesting itself to the amount of gravitational pull is called mass. The SI unit of mass is ‘kg’.

Time (t) : Time is a primary dimension which describes the . The SI unit for time is ‘Sec’

Secondary dimensions:

The Dimensions which are derived from the basic dimensions like length, mass and time are called secondary units. When the physical characteristics of a body are designed in terms of basic dimensions by the use of suitable definitions, such quantities are called secondary dimensions.

Ex: (i) Velocity (v) = (distance traveled / time taken for the distance traveled)
Unit for velocity is ‘m / s’

(ii) Acceleration (a) = Rate of change of velocity
Unit for velocity is ‘m / s^2’

12. What are the fundamental principles and laws of mechanics?

The fundamental principles of Mechanics are the

(i) Newton’s first law of motion
(ii) Newton’s second law of motion
(iii) Newton’s third law
(iv) Gravitational law of attraction
(v) Principle of transmissibility of forces
(vi) Parallelogram law of forces
(vii) Sine law
(viii) Cosine law
(ix) Lami’s Theorem
(x) Polygon law

13.State three Laws of Newton of motion

Newton’s First Law:

A body continuous to remain at rest or in motion in a straight line with uniform velocity unless it is acted on by external force to change its state of rest or uniform motion.

Newton’s second Law

The acceleration of a particle is proportional to the resultant fore acting on it and is in the direction of the force applied. The rate of change of momentum of a body under force is directly proportional to the force applied on the body and change of momentum takes place in the direction of the applied force.

F= m x a

Where,

F – Force applied in 'N'

m – Mass of the body in 'kg'

a – Acceleration of the body due to the force 'F' in 'm / s^2'

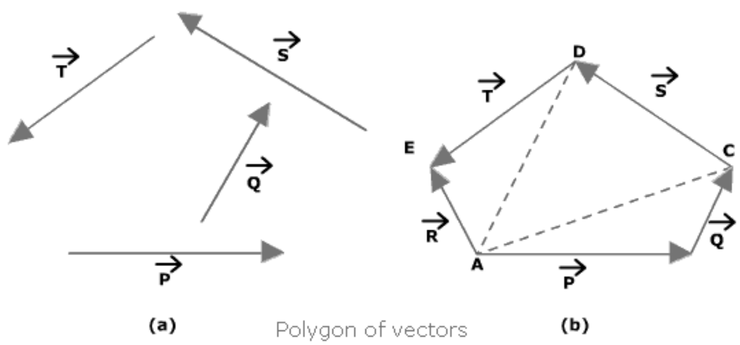
Newton's third Law

To Every action there is equal and opposite reaction. The forces of action and reaction between interaction bodies are equal in magnitude and line of action but opposite in direction.

14. State Polygon Law

polygon law

The law states that if a number of coplanar concurrent forces are represented in magnitude and direction by the sides of a polygon taken in an order, then their resultant force is represented by the closing side of the polygon taken in the opposite order



Polygon of vectors

15. State the gravitational law of attraction

This law states that 'Two particles of mass $\mathbf{m_1}$ and $\mathbf{m_2}$ are attracted towards each other along the line connecting them with a force whose magnitude 'F' is proportional to the product of their masses $(\mathbf{m_1 x\, m_2})$ and inversely proportional to the square of the distance (r^2) between them.

$$F = G\left(\frac{m_1\, x\, m_2}{r^2}\right)$$

Where,

F - Force of attraction between the two particles

m_1 – Mass of one particle

m_2 – Mass of the other particle

r - Distance between the two particles

G - Constant of gravitation

16. Define moment of a force about a point

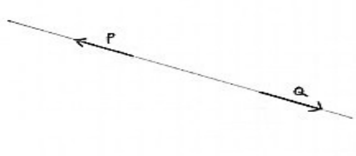
Moment of a force is defined as the product of a force and the perpendicular distance of the line of action of the force from a point**.**

17. What is meant by coplanar concurrent force system?

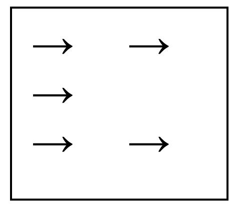
The system in which the forces act on a single plane and they intersect at a common point is called coplanar concurrent force system.

18. What is meant by coplanar and collinear forces?

- ✓ Forces acting on a single plane are known as coplanar forces.
- ✓ Forces acting on a common line of action are known as collinear forces



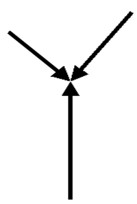
Collinear force



coplanar force

19. What is meant by concurrent forces system?

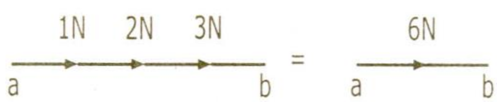
Concurrent force system is that all the line of action of forces is passing through single point.



Concurrent force system

20. What are resultant forces?

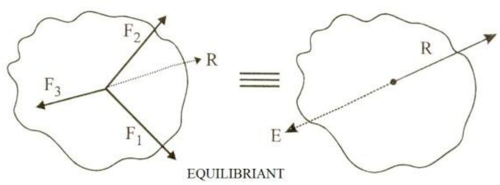
If a number of forces acting on a particle simultaneously are replaced by a single force, which could produce the same effect as produced by the given forces, that single force is called resultant force. It is an equivalent force of all the given forces.



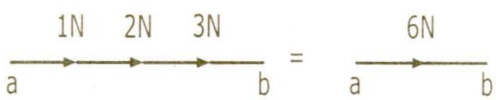
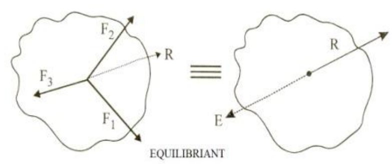
Resultant force

21. Define equilibriant

Defined as a single force having same magnitude with that of the resultant force and acting in the opposite direction, so as to make the body at rest or in equilibrium.



22. Differentiate resultant and equilibrium

S.No	RESULTANT	EQUILIBRIANT
1	It is the single force which is equivalent to the given system of forces on a body	It is a force which brings the body to equilibrium and it has same magnitude as that of resultant and in opposite direction to it.
2	e.g 	e.g 

23. State the conditions for the equilibrium of a coplanar system of forces.

$\sum$ $\mathbf{F_x = 0}$ **(**algebraic sum of all the forces in the horizontal direction is 0)

$\sum$ $\mathbf{F_y = 0}$ **(**algebraic sum of all the forces in the vertical direction is 0)

$\sum$ $\mathbf{F_M = 0}$ **(**algebraic sum of all the moments of the system is 0)

24. Define vector quantity

The quantities possessing magnitude and direction are called vector quantity

Examples: velocity, acceleration

25. Define scalar quantity

The quantities possessing only magnitude and not the direction is called scalar quantity.

Examples: mass, time, length

26. Distinguish between particle and rigid body

S.No	PARTICLE	RIGID BODY
1	A body of infinitely small volume and is considered to be concentrated at a point.	A rigid body is the one which retains its shape and size, if subjected to some external forces
2	Here mass is negligible	body having a mass and dimension
3	e.g atom	e.g Road roller

27. if A = 2i – 3j -3k , B = 4i + 3j – 5k, C= 2i + 4j – 3k, find the vector 2A –B +2C in terms of i ,j,k and its magnitude.

Ans :

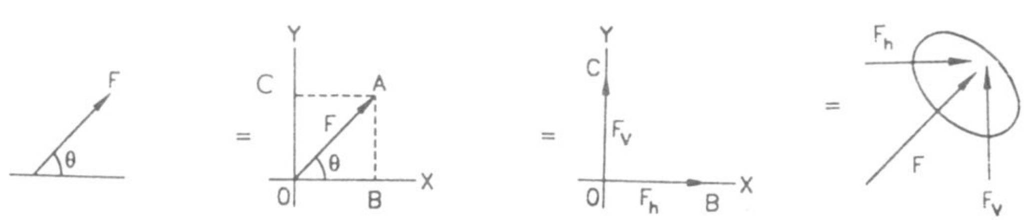
$$\vec{A} = 2i - 3j - 3k$$
$$\vec{B} = 4i + 3j - 5k$$
$$\vec{C} = 2i + 4j - 3k$$
$$2\vec{A} + \vec{B} + 2\vec{i} = 2(2i - 3j - 3k) + (4i + 3j - 5k) + 2(2i + 4j - 3k)$$
$$= 4i - 6j - 6k + 4i + 3j - 5k + 4i + 8j - 6k$$
$$= 12i + 5j - 17k$$
$$\text{Magnitude} = \sqrt{12^2 + (5)^2 + (-17)^2}$$
$$= 21.4$$

Results:

Magnitude = 21.4 units

28. What are resolutions of a force?

Splitting up a force into components along the fixed reference axes is called resolution of a force.

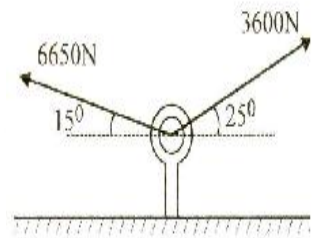


Resolution of force

29.Distinguish between units and dimensions? Give an example?

S.No	UNITS	DIMENSIONS
1	Length (m), Mass (Kg) and Time (t) are the fundamental units	The qualitative description of physical variable is known as dimension
2	They are classified into basic units e.g length ‘m’, supplementary units – angle (α), derived unit force – N	Usually we follow MLT System of dimension. Example: for velocity – LT^{-1} Momentum – MLT^{-1}

30. Two wires attached to a bolt in a foundation as shown in figure. Determine the pull exerted by the bolt on the foundation.



Solution

Resolve the forces into components and

$\Sigma F_x = 3600 \cos 25 - 6650 \cos 15 = -3160.6987$ N

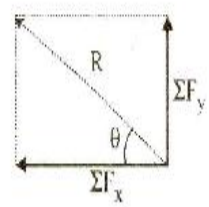
$\Sigma F_y = 3600 \sin 25 + 6650 \sin 15 = 3242.572$ N

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

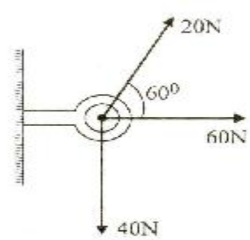
$$= \sqrt{3160.6987^2 + 3242.572^2}$$

$$= \mathbf{4528.167\ N}$$

$$\theta = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right] = \mathbf{45.73^\circ}$$



31. Three wires exert the tensions indicated on the ring as shown in the figure. Assuming as concurrent system. Determine the force in a single wire to replace the three wires



Solution

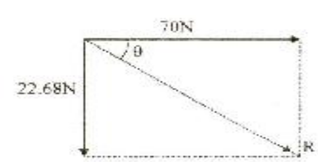
Sum of Horizontal forces $\Sigma F_x = 60 + 20 \cos 60 = 70$ N

Sum of vertical forces $\Sigma F_y = 20 \sin 60 - 40 = -22.68$ N

$\therefore$ Resultant $R = \sqrt{70^2 + 22.68^2} = \mathbf{73.582\ N}$

$$\theta = \tan^{-1}\left[\frac{22.68}{70}\right]$$

$$= \mathbf{17.952^\circ}$$



32. Write the classification of force systems.

I) Coplanar force system

i. Co-planar collinear force system
ii. Co-planar concurrent force system
iii. Co-planar parallel force system
   a) Like parallel force system
   b) Unlike parallel force system
iv. Co-planar non-concurrent, non parallel force system.

II) Non -Coplanar force system

i. Non-coplanar concurrent force system
ii. Non-coplanar parallel force system
iii. Non-coplanar non-concurrent non-parallel force system

33. A force $\vec{F} = \left(8.25\,\vec{i} + 12.75\,\vec{J} - 18\,\vec{K}\right)N$ acts through the origin what is the Magnitude of the force and the angles it makes with X, Y and Z axis (June 2005)

Solution

$$F = \sqrt{F_X{}^2 + F_Y{}^2 + F_Z{}^2}$$

$$= \sqrt{8.25^2 + 12.75^2 + (-18)^2}$$

Magnitude of force F = 23.551 kN

$$\theta_x = Cos^{-1}\left[\frac{F_X}{F}\right]$$

$$= Cos^{-1}\left[\frac{8.25}{23.551}\right]$$

$\theta x = 69.5^o$

Similarly $\theta y = 57.2^o$ & $\theta_Z = 139.9^o$

34. Two forces 60 N and 65 N act on a screw at our angle of 25^o and 85^o from the base, Determine the Magnitude and direction of their resultant (July 99)

Solution:

Magnitude of Resultant force

$$\sum H = 60Cos25 + 65Cos85$$

= 60 N

$$\sum V = 60Sin\,25 + 65Sin85$$
$$=90 \text{ N}$$

$$R = \sqrt{\sum H^2 + \sum V^2}$$

$$R = \sqrt{60^2 + 60^2}$$

Direction

$$\alpha = tran^{-1}\left(\frac{\sum V}{\sum H}\right)$$
$$= tran^{-1}\left(\frac{90}{60}\right)$$
$$= 56.31^o$$

35. Find the Resultant of an 800 N force acting towards eastern direction and a 500 N force acting towards North eastern directions
(November / December 2003)

Solution:

Let R be the resultant force using parallelogram law of forces,

$$R = \sqrt{P^2 + Q^2 + 2PQCos\theta}$$
$$= \sqrt{800^2 + 500^2 + (2 \times 800 \times 500 \times Cos45)}$$
$$\boxed{R = 1206.51N}$$

Let α be the direction of resultant force made with eastern direction

Then $\alpha = \tan^{-1}\left[\frac{QSin\theta}{P + QCos\theta}\right]$

$$= \tan^{-1}\left[\frac{500Sin45}{800 + 500Cos45}\right]$$

$$= \tan^{-1}\left[\frac{353.55}{800 + 353.55}\right]$$

$$\boxed{\alpha = 17.04^o}$$

36. A man has a mass of 72 Kg is standing on a board inclined 20^o with the horizontal. Find the component of mans weight

a. Perpendicular to the plane of the board

b. Parallel to the plane of the board. (May 2003)

Solution:

Component perpendicular to the board

$$= 72 \times 9.81 \times Cos 20^o$$

F_V = 663.72N.

Component parallel to the board

$$= 72 \times 9.81 \times Sin 20^o$$

F_h = 241.57N

37. The lie of action of a 100 N force F passes through the points A (2, 5,8) m and B(7,2,6)m. The direction of the force is form A to B . Express F interms of the unit vectors I, J, and K (November 2002)

Solution:

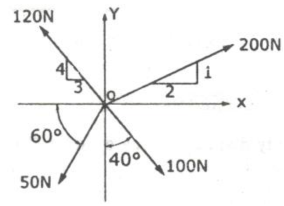
$$\vec{F} = F.\lambda_{AB}$$

$$=100\left[\frac{(7-2)i+(2-5)J+(6-8)K}{(7-2)^2+(2-5)^2+(6-8)^2}\right]$$

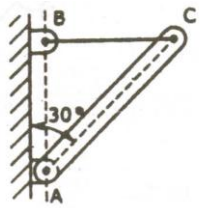
= 81.1 I – 48.66 J – 32.46 K

Part-B

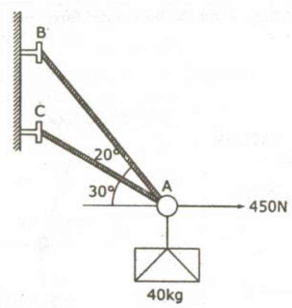
1. A system of four forces acting on a body is shown in figure. Determine the resultant force and its direction.



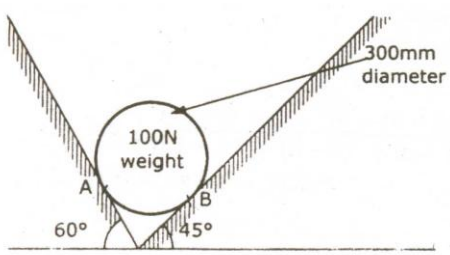
2. The bar AC, 10 m long supports a load of 6000 N as shown below. The cable BC is horizontal and 5m long. Determine the forces in the cable and the bar.



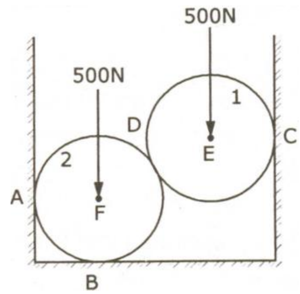
3. Determine the tension in cables AB and AC required to hold the 40 kg crate shown in figure below.



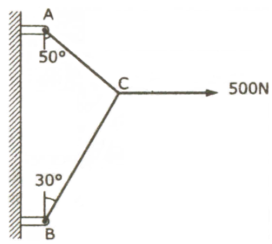
4. Determine the reactions at A and B from the figure given below.



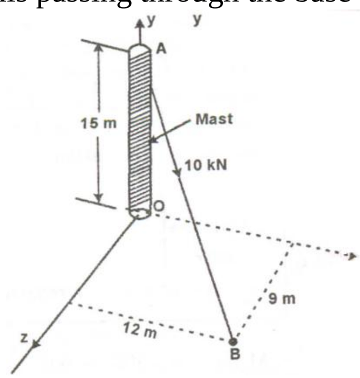
5. Two spheres each of weight 500N and of radius 100 mm rest in a horizontal channel of width of 360 mm as shown in figure below. Find the reactions on the points of contact A, B and C. Assume all the surfaces of contact are smooth.



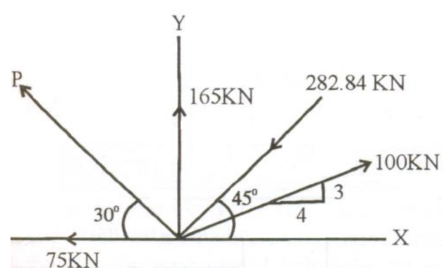
6. Two cables are tied together at C and are loaded as shown in figure below. Determine the tension in the cable AC and BC



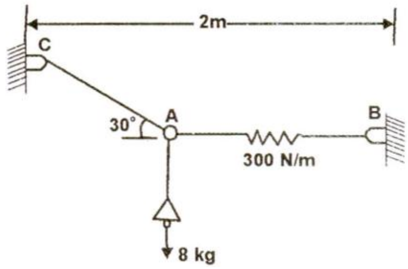
7. A tension T of magnitude 10 KN is applied to the cable attached to the top A of rigid mast and secured to the ground at B as shown in figure. Determine the moment of the tension T about the Z- axis passing through the base O.



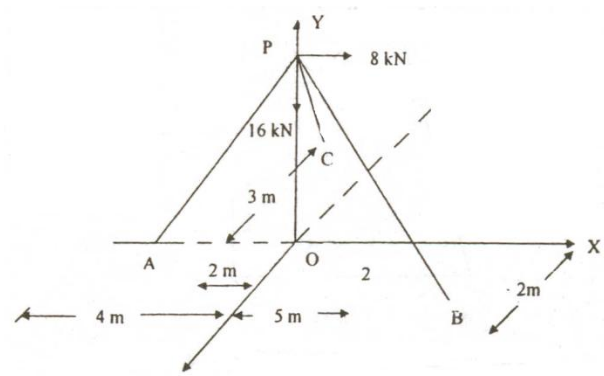
8. The algebraic sum of horizontal components of all forces acting on a particle as shown in figure is -324.904 N. Find the magnitude of P and resultant force.



9. Determine the length of cord AC in the figure. so that the 8 Kg lamp is suspended in the position shown. The underformed length of the spring AB is I_{AB} =0.4 m, and the spring has a stiffness of k_{AB} = 300 N/m



10. A tripod is acted upon by forces at P as shown in the figure. Determine the forces in the legs of tripod if the legs rest on ground at A, B and C whose coordinates with respect to O are as shown in the figure. The height of P above the origin is 10 m



UNIT- II EQUILIBRIUM OF RIGID BODIES
Two Mark Question & Answers

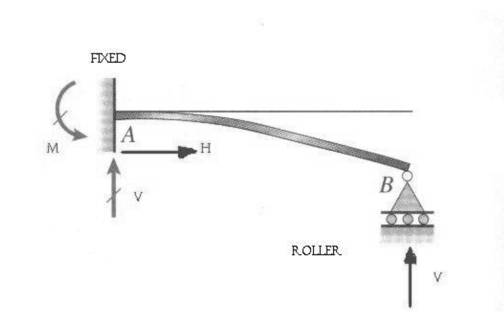
Syllabus

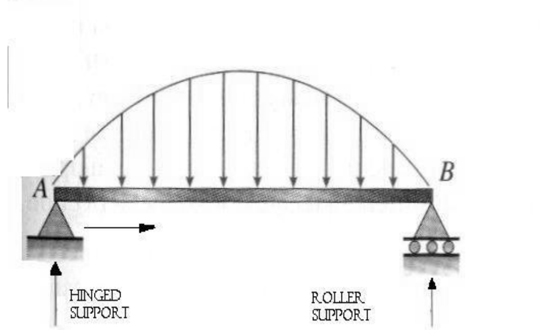
Free body diagram – Types of supports and their reactions – requirements of stable equilibrium – Moments and Couples – Moment of a force about a point and about an axis – Vectorial representation of moments and couples – Scalar components of a moment – Varignon's theorem – Equilibrium of Rigid bodies in two dimensions – Equilibrium of Rigid bodies in three dimensions – Examples

1. **State varignon's theorem**
   The algebraic sum of the moments of all the forces about any point is equal to the moment of the resultant force about the same point.
2. **What is meant by equilibrium of a particle**
   When the resultant of a number of forces acting on a particle is zero, the particle is in equilibrium.

3. **State equilibrium forces**
   The set of forces where the resultant is zero and bring the body to its original position . Such a force is called equilibrium forces

4. **Write the equations which express the conditions for the equilibrium of a rigid body?**
   $\sum F_x = 0$ $\quad$ $\sum M_x = 0$
   $\sum F_y = 0$ $\quad$ & $\quad$ $\sum M_y = 0$
   $\sum F_z = 0$ $\quad$ $\sum M_z = 0$
5. **What are the different types of support?**
   a. Roller support $\quad$ b. hinged support $\quad$ c. fixed support





6. Define Concentrated or point load

Concentrated or point load

Any load acting at a point on a point on a load, is known as point load. In actual practice, it is not possible to apply a load at a point (ie. at a mathematical point) as it must have some contact area. But this area in comparison to the length of the beam is very small (or area is negligible).

7. Define Uniformly Distributed Load (UDL)

If a beam is loaded in such a way, that each unit length of the beam carries same intensity of the load, then that type of load is known as uniformly distributed load.

For finding the reactions the total uniformly distributed load is assumed to act at the Centre of Gravity (C.G.) of the load.

8.Define Uniformly Varying Load (UVL)

If a beam is loaded in such a way, that each unit length of the beam carries same intensity of the load, then that type of load is known as uniformly distributed load.

For finding the reactions the total uniformly distributed load is assumed to act at the Centre of Gravity (C.G.) of the load

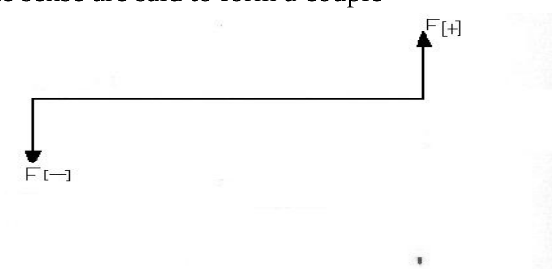
9. What are statically determinate?

A structure which can be completely analysed by static conditions of equilibrium alone say

$\sum H = 0$, $\sum V = 0$, $\sum M = 0$ are called statically determinate structure

10. **Define couple**

Couple is defined as two forces having the same magnitude , parallel lines of action and opposite sense are said to form a couple



11. **Define moment of a couple**

Moment of a couple is defined as the product of force and arm of the couple.

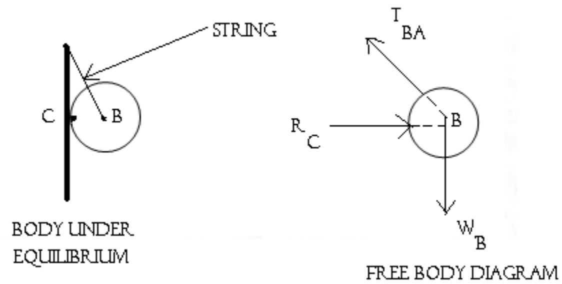
Moment of a couple = Force x arm of the couple

12. **State the difference between moment and a couple**

COUPLE	MOMENT
The couple is pure turning effect which may move anywhere in its own plane without change of its effect on the body	Moment of a force include a description of the reference axis about which the moment is taken

13. **What is free body diagram?**

The sketch showing all the forces both external forces and reactions and moments acting on the body and isolated from surrounding body is called free body diagram.



14. **State two force principle?**

    If a body is subjected to two forces, then the body will be in equilibrium if these two forces are collinear, equal and opposite. i.e. $F_1 = F_2$

15. **State three force principle**

    If a body is subjected to three forces, then the body will be in equilibrium if the the resultant of any two forces is equal, opposite and collinear, with the third force

16. **State four –force principle**

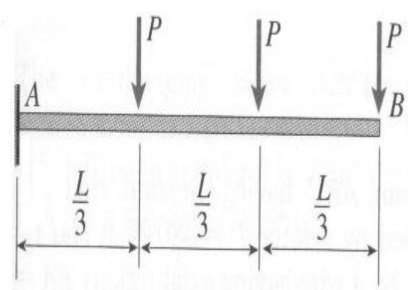
    If a body is subjected to force forces, then the body will be in equilibrium if the resultant of any two forces is equal, opposite and collinear, with the resultant of the other two.

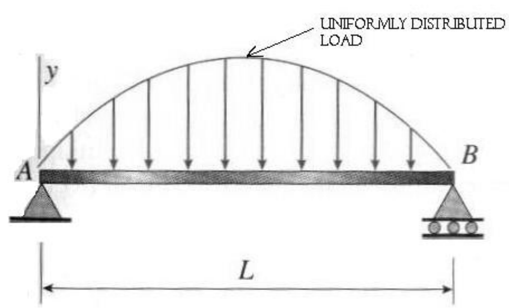
17. **What are the types of loads?**

    a) Point load

    b) Uniformly distributed load

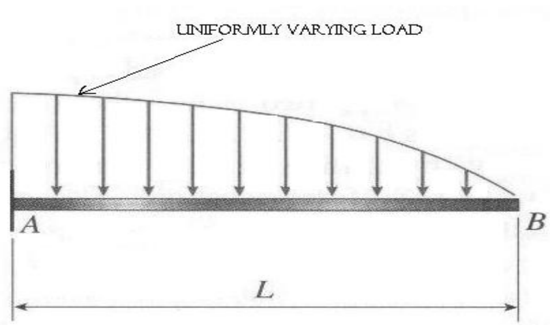
    c) Uniformly varying load





. POINT LOAD

UNIFORMLY DISTRIBUTED LOAD



UNIFORMLY VARYING LOAD

.

18. **What are the types of equilibrium**
    - ✓ Stable equilibrium
    - ✓ Unstable equilibrium
    - ✓ Neutral equilibrium

19. **What is meant by stable equilibrium?**
    A body is said to be in equilibrium when it comes back to its original position after it has been given a small displacement. This is known as stable equilibrium

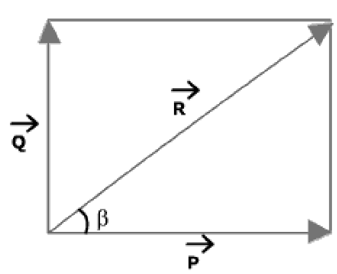
20. **What is meant by Unstable equilibrium**

If a body does not return back to its original position and moves further apart after being slightly displaced from its rest position, the body is said to be in unstable equilibrium. This condition arises when the additional force causes to move apart from its rest position.

21. **What is meant by Neutral equilibrium**

If a body occupies a position and remains at rest in this position after being slightly displaced from its rest position, the body is said to be in neutral equilibrium

22. **State triangular law**
    The law stats that if two forces acting at a point are represented by the two sides of a triangle, taken in order , then their resultant force is represented by the third side taken in opposite order.



Triangular law

23. **A simply supported beam AB of span 6m carries point loads of 3kN and 6kN at a distance of 2m and 4m from the left end A. Find the reactions at A and B.**

Given data:

The given system is as shown in fig.

Span of the beam = 6m

R_A = Reaction at A

R_B = Reaction at B.

Solution:

As the beam is in equilibrium, the moments of all forces about any point should be zero.

Now, taking moment of all forces about A, and equating the resultant moment to zero, we get

$(R_B X6)-(3X2)-(6X4) = 0$

$6R_B = 6+24=30$

R_B = **5 kN.**

Also for equilibrium, $\sum F_y = 0$

$R_A + R_B = 3+6 = 9$

$R_A = 9 - R_B = 9-5 =$ **4 kN.**

Result:

Reaction at point A = R_A = **4 kN.**

Reaction at point B = R_B = **5 kN.**

24. **A force of 500 N forms angles 60°, 45°s and 120° respectively with X, Y and Z axis, write the force in vector form.**

Given data:

F = 500 N, $\theta_x = 60^o$, $\theta_y = 45^o$ and $\theta_Z = 120^o$

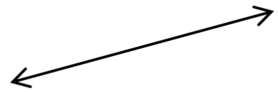
$$\vec{F} = FCos\theta_{xi} + FCos\theta_{yJ} + FCos\theta_{ZK}$$

$$= (500Cos60^o)i + (500Cos450^o)J + (500Cos120^o)K$$

$$\vec{F} = 250i + 353.55J - 250K$$

25. A force of Magnitude 750 N is directed along AB where A is (0.8, 0, 1.2) m and B is (1.4, 1.2, 0) m write the vector form of the force.

Solution:



B (1.4, 1.2, 0)

A (0.8, 0,1.2)

$$\hat{F} = F\lambda$$

$$= F \frac{(x_2 - x_1)i + (y_2 - y_1)j + (Z_2 - Z_1)K}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (Z_2 - Z_1)^2}}$$

$$= 750\left[\frac{(1.4-0.8)i + (1.2-0)j + (0-1.2)K}{(1.4-0.8)^2 + (1.2-0)^2 + (0-1.2)^2}\right]$$

$$= 750\left[\frac{0.6i + 1.2j - 1.2K}{\sqrt{0.6^2 + 1.2^2 - 1.2^2}}\right]$$

$$= 750\left[\frac{0.6i + 1.2j - 1.2K}{1.8}\right]$$

$$= 416.67\left[\frac{(0.6i + 1.2j - 1.2K)}{1.8}\right]$$

$$= 416.67(0.6i + 1.2j - 1.2k)$$

$$= 250i + 500J - 500k$$

26. A force $\vec{F} = 700i + 1500J$ is applied to a belt A. Determine the Magnitude of the force and the angle it forms with the horizontal (May 2004)

Solution:

Magnitude $F = \sqrt{(Fx)^2 + (Fy)^2}$

$$= \sqrt{700^2 + 1500^2}$$

$$\boxed{F = 1655.29}$$

Angle of the force with horizontal

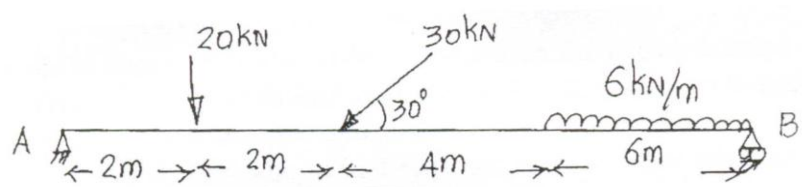
$$\theta x = Cos - 1\left[\frac{Fx}{F}\right]$$

$$= Cos - 1\left[\frac{700}{1655.29}\right]$$

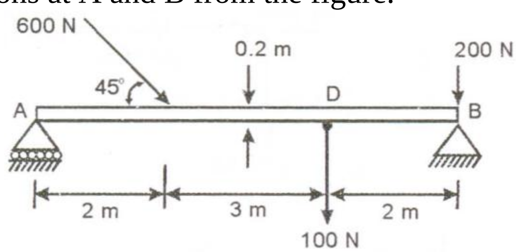
$$\boxed{\theta_x = 64.98^o}$$

Part-B

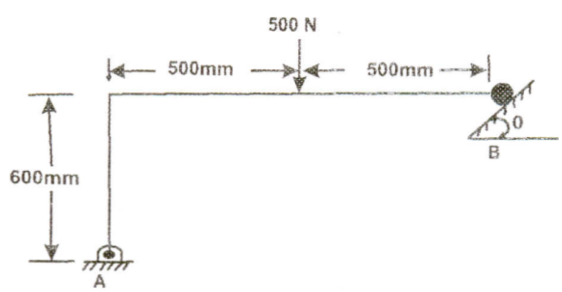
1. Determine the reactions at supports A and B of the simply supported beam shown in figure.



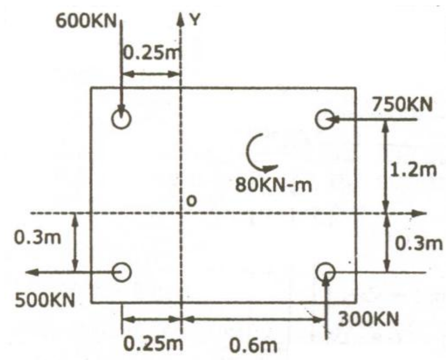
2. Determine the reactions at A and B from the figure.



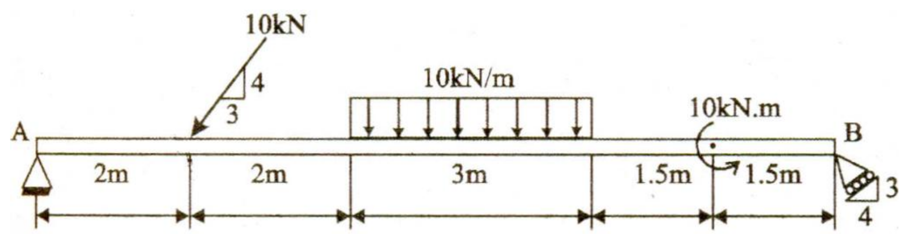
3. A frame supported at A and B is subjected to a force of 500 N as shown in figure. Compute the reactions at the support points for the cases of Θ = 0^o, Θ = 60^o and Θ = 90^o



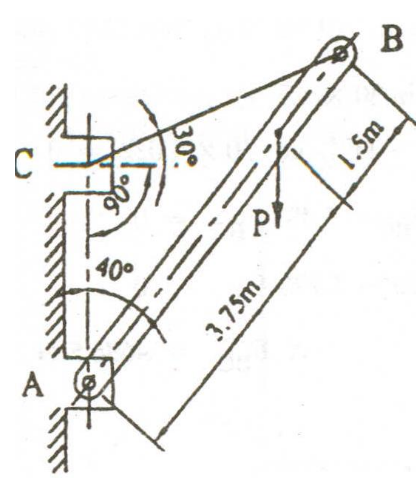
4. Four forces and a couple are applied to a rectangular plate as shown in figure below. Determine the magnitude and direction of the resultant force –couple system. Also determine the distance x from O along x-axis where the resultant intersects.



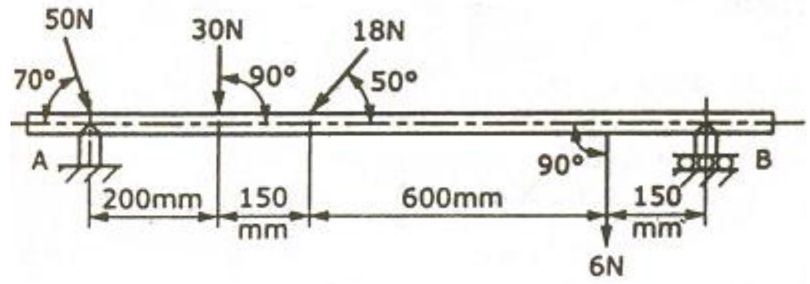
5. Find the reaction at the supports A and B for a beam as shown in figure.



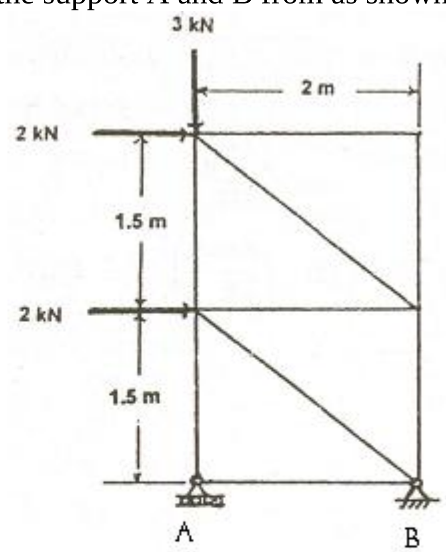
6. A Load P of 3500 N is acting on the boom, which is held by cable BC as shown in the Figure. The weight of the boom can be neglected.
   (a)Draw the free body diagrams of the boom
   (b) Find the tension in cable BC
   (c) Determine the reaction of A.



7. Determine the reaction at the support A and B from as shown figure.



8. Determine the reaction at the support A and B from as shown figure

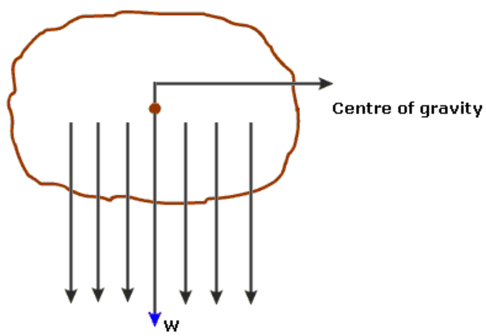


Syllabus

Determination of Areas and Volumes – First moment of area and the Centroid of sections – Rectangle, circle, triangle from integration – T section, I section, Angle section, Hollow section by using standard formula – second and product moments of plane area – Rectangle, triangle, circle from integration – T section, I section, Angle section, Hollow section by using standard formula – Parallel axis theorem and perpendicular axis theorem – Polar moment of inertia – Principal moments of inertia of plane areas – Principal axes of inertia - Mass moment of inertia – Derivation of mass moment of inertia for rectangular section, prism, sphere from first principle – Relation to area moments of inertia.

1. **Define Centre of Gravity**

   Centre of gravity is defined as an imaginary point at which the entire weight of the body is assumed to act.



$$x = \frac{W_1 X_1 + W_2 X_2 + W_3 X_3 + ...}{W_1 + W_2 + W_3 + ...}$$

$$y = \frac{W_1 Y_1 + W_2 Y_2 + W_3 Y_3 + ...}{W_1 + W_2 + W_3 + ...}$$

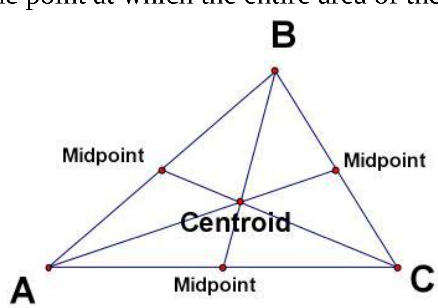
Where,

W_1 , W_2 ….are weight of different components of composite solid.

X_1 , X_2 ….and Y_1 , Y_2 .....distances of their respective C.G from the reference axis in X and Y direction

2. **Define Centroid**

   Centroid is defined as the point at which the entire area of the body is assumed to be concentrated.



$$x = \frac{A_1 X_1 + A_2 X_2 + A_3 X_3 + ...}{A_1 + A_2 + A_3 + ...} \; or \; \frac{V_1 X_1 + V_2 X_2 + V_3 X_3 + ...}{V_1 + V_2 + V_3 + ...} \; or \; \frac{L_1 X_1 + L_2 X_2 + L_3 X_3 + ...}{L_1 + L_2 + L_3 + ...}$$

$$y = \frac{A_1 Y_1 + A_2 Y_2 + A_3 Y_3 + ...}{A_1 + A_2 + A_3 + ...} \; or \; \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3 + ...}{V_1 + V_2 + V_3 + ...} \; or \; \frac{L_1 Y_1 + L_2 Y_2 + L_3 Y_3 + ...}{L_1 + L_2 + L_3 + ...}$$

Where,

A_1 , A_2 ….are cross sectional area of different components of composite solid.

V_1 , V_2 …. are volume of different components of a body.

L_1 , L_2 …. are length of different parts of lines.

X_1 , X_2 ….and Y_1 , Y_2 …..distances of their respective C.G from the reference axis in X and Y direction

3. **When will the centroid and centre of gravity coincide?**
When gravitational field is uniform and parallel. i.e. if the body is homogenous or density of the body is uniform throughout the body then the centre of gravity and centroid are coincide.

4. **Define moment of inertia of a body**
Moment of inertia (I) about an axis is the algebraic sum of the products of the elements of mass and the square of the distance of the respective element of mass from the axis.

$$\mathbf{I = \sum m_1 r_1^2}$$

5. **Define centroid axis**
Centroidal axis is defined as that axis which passes through the centre of gravity of the body or through the centroid of an area

6. **Define radius of gyration**
Radius of gyration of a body is defined as the distance from axis of reference to where the whole mass of a body is assumed to be concentrated so as to not alter the moment of inertia about the given axis.Radius of gyration k = $\sqrt{I / A}$, where I = Moment of inertia, A = Total area of the plane

7. **Differentiate centre of gravity and centroid**

S.No	Centre of gravity(C.G)	Centroid(G)
1	Centre of gravity is defined as an imaginary point at which the entire weight of the body is assumed to act	Centroid is defined as the point at which the entire area of the body is assumed to be concentrated
2	It is a physical property of a body	It is a geometric property of body
3	It has both area and mass	It has only area and no mass
4	It refers to three dimensional body	It refers to one, two dimensional body

4	It is applied to solids.E.g. wire, rod	It is applied to area. E.g. line, area, volume

8. State pappus and guldinus theorems

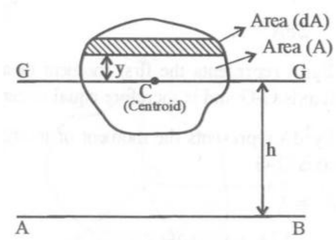
Theorem –I: The area of a surface of revolution is equal to the product of length of the generating curve and the distance travelled by the centroid of the curved line while the surface is being generated

Theorem-II: The volume of a body of revolution is equal to the product of the generating area and the distance travelled by the centroid of the area while the body is being generated.

9. State parallel axis theorem

Moment of inertia of a plan area which is parallel to centroidal axis about an axis is equal to the sum of the moment of inertia about centroidal axis and the product of area and square of the distance between the two parallel axes.

$$I_{AB} = I_G + Ah^2$$



Where I_{AB} = moment of inertia about parallel axis x-x

I_G=moment of inertia about its centroidal axis G-G

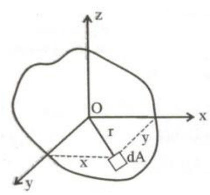
A= Area of the body

h= distance between the axes GG and XX

10. **State perpendicular axis theorem or Polar moment of inertia**.

Moment of inertia of an area about two mutually perpendicular axes passing through its centroid is equal to the sum of the moment of inertia about two mutually perpendicular axes passing through centroid and in the plane of body by considering the body of area.

$$I_{ZZ} = I_{XX} + I_{YY}$$



11. What are principal axes?

Principal axes are the set of mutually perpendicular axes where the product of inertia is zero.

12. What are principal moment of inertia?

Moment of inertia of a section about principal axes are called as principal moment of inertia.

13. Define polar moment of inertia

Polar moment of inertia is defined as the moment of inertia of the lamina or plane about the axis perpendicular to the plane of the section. It is denoted by Ixx or J

14. Define mass moment of inertia

The mass moment of inertia about an axis is the product of elemental mass and the square of the distance between the mass centre of the elemental mass and the axis.

Mass moment of inertia =$\int x^2$ dm

15. What is section modulus?

The modulus of section of a figure is the quantity obtained by dividing the moment of inertia of the figure about its C.G by the distance of the extreme fibre from the centroidal axis

16. How many centre of gravity a body has?

Only one.

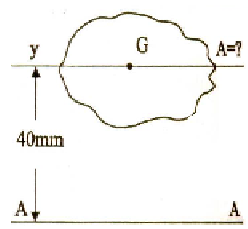
.

17. **Write the formula for principal axes?**

Principal axes are a set of mutually perpendicular axes where the product of inertia is zero. The position of the axes are given by

$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$

18. **Given that M.I I_{xx} =500 cm^4 and M.I about an axis AA which is parallel to centroidal xx axis at a distance of 40 mm is 900 cm^4. Determine the area?**



<u>Given data</u>

$I_{xx} = I_G$ = 500 cm^4

I_{AA} = 900 cm^4

Height h = 40mm = 4cm

<u>Solution</u>

Moment of inertia $I_{AA} = I_G + Ah^2$

$$900 = 500 + A \times 40^2$$

Area (A) = 25 cm^2

19. **Write the formula for principal moment of inertia?**

Moment of inertia of a section about principal axes are called as principal moment of inertia and is given by the equation

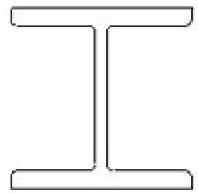
$$I_{max}, I_{min} = \frac{I_{xx} + I_{yy}}{2} \pm \sqrt{\left(\frac{I_{xx} - I_{yy}}{2}\right)^2 + I_{xy}^2}$$

20. What is meant by moment of inertia?

The property which gives the measure of resistance to bending in the case of plane area or plates is known as moment of inertia of the area

21.What is axis of symmetry?

The axis about which similar configuration exist with respect to shape, size and weight on either side is known as axis of symmentry

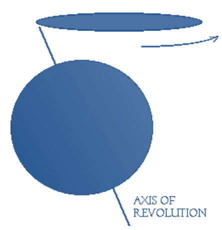


Column

I section is symmetrical about both x-axis and y –axis

22.What is axis of revolution?

The fixed axis about which a plane curve or plane area is rotated is known as axis of revolution



22. Define centre of mass.

Center of mass is the point where the entire mass of a body is assumed to be concentrated. The position of centre of mass depends upon the shape and density of the body. The mass centre may (or may not) necessarily lie within the boundary of the body. Each body has only one mass centre for all positions of the body. The mass centre for a composite solids made up of different materials is given by,

$$x = \frac{\rho_1 V_1 X_1 + \rho_2 V_2 X_2 + \rho_3 V_3 X_3 + ...}{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...}$$

$$y = \frac{\rho_1 V_1 Y_1 + \rho_2 V_2 Y_2 + \rho_3 V_3 Y_3 + ...}{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + ...}$$

Where,

ρ, V , are density and volume of different components of composite solid.

X_1 , X_2 ….and Y_1 , Y_2 …..distances of their respective C.G from the reference axis in X and Y direction

23. Using the theorems of Pappus, determine the surface area and volume of a right circular cone, with radius of base r and height h.

Surface area = Length of the curve x Distance traveled by the centroid during revolution

$= l \times 2\pi y = l \times 2\pi (r/2)$

$A_S = \pi r l$

Volume = area of the generating area × distance traveled by the centroid during one revolution.

$= \frac{1}{2} hr \times 2 \times \pi \times r/3$

$V = \pi r^2 h/3$

24. Write the expression to find the co ordinates of centroid by integration method?

for plane figure $\bar{x} = \dfrac{\int x_s dA}{\int dA}$; $\bar{y} = \dfrac{\int y_s dA}{\int dA}$

for solid figure , $\bar{x} = \dfrac{\int x_m dm}{\int dm}$; $\bar{y} = \dfrac{\int y_m dm}{\int dm}$

25. Under what condition do the following coincide? a). Center of mass and \ Centre of gravity b). center of gravity and centroid of area .

A. centre of gravity coincides with the centre of mass if the gravitational forces are taken to be uniform and parallel.

B. If a lamina (plate) has uniform thickness and is homogeneous, its centre of gravity coincides with the centroid if the area.

26. What is axis of revolution and axis of symmetry?

Axis of revolution

The fixed axis about which a plane curve(may be an arc, straight line) or plane area is rotated is known as axis of revolution.

Axis of symmetry

The axis about which similar configuration exist with respect to shape , size an weight on either side is known as axis o symmetry. It may be horizontal, vertical or inclined.

27. State the salient properties of principal axes.

1) If the given area has an axis of symmetry passing through a point, then the axis must be principal axis need not necessarily be an axis of symmetry.
2) The properties of principal axes hold good for any point located inside or outside the given area.
3. If the given point coincides with the centroid of an area, the two principal axes of the area about the centroid are known as the principal centroidal axes of the area.

28. **What do you understand from principle moments of inertia**

The perpendicular axes about which "Product of inertia" is zero are called 'principal axes' and the moment of inertia with respect to these principal axes are called as "Principal moments of inertia"

The maximum moment of inertia about principal axes is called Major principal moment of inertia.

The minimum moment of inertia about principal axes is called Major principal moment of inertia.

29. **Define mass moment of inertia of a solid body**

The moment of inertia of solid figure is called mass moment of inertia. For plane figure, masses are assumed to be negligible and hence area of plane figures are taken to find the moment of inertia, whereas for solid figures masses are considered.

Mass moment of inertia of any solid = Thickness x Density x Area Moment of Inertia of the solid about the same axis

$(I_{xx})_{mass} = (I_{XX})_{area} \times M$

Where,

M- Mass of the solid

$(I_{xx})_{mass}$ –Mass moment of inertia

$(I_{XX})_{area}$ - Area moment of inertia

30. A rectangle has a width of 2m and height of 3m. Find its product of inertia about a set of coordinate axes passing through its top left corner and parallel to its sides.

$$I_{xy} = xydxdy$$

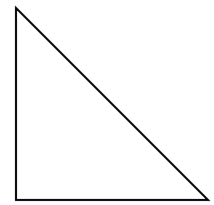
$$\text{here } I_{xy} = \int_0^2 \int_{-3}^0 xydxdy = \frac{-2^2 \times 3^2}{4} = -9 \text{ mm}^4$$

31 Find the product of inertia of a right angled triangle with respect to x and y axes are shown in fig

$$I_{xy} = \frac{b^2 h^2}{24}$$

$$= \frac{20^2 \times 40^2}{24}$$

$$= 26.667 \times 10^3 mm^4$$



32 semi circular area having radius 100 mm is located in the xy plane such that its. Diametrical edge coincides with the y axis. Determine the x- coordinate of its centroid.

$$x = \frac{4r}{3\pi}$$

$$x = \frac{4 \times 100}{3\pi}$$

$$= 42.44 \text{ mm}$$

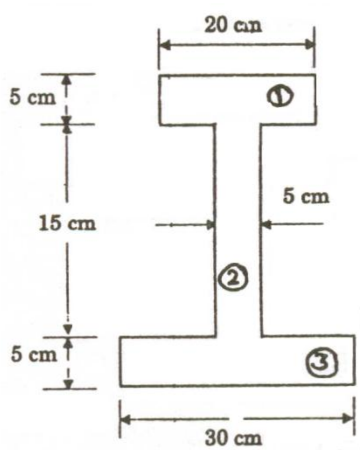
33 State the relationship between the second moment of area and mass moment of inertia for a thin uniform plate.

Mass moment of inertia of thin plate about any axis

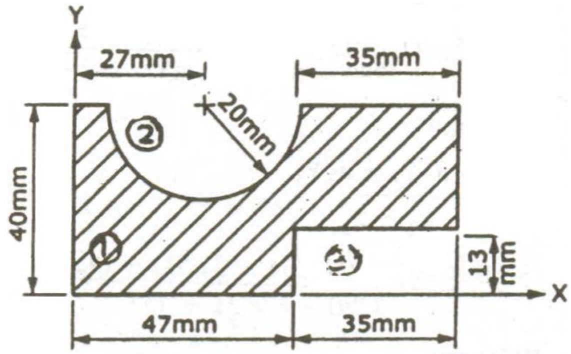
= thickness × density × second moment of area of the plate about the same axis.

PART-B

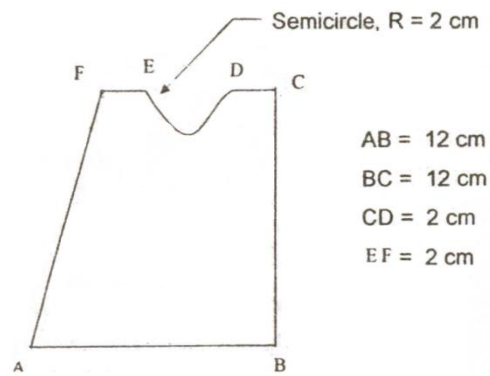
1. Determine the centroid of the cross-sectional area of an unequal I –section shown below.



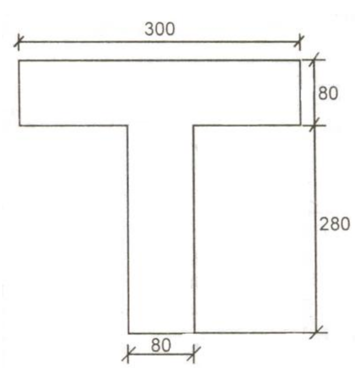
2. Find the centroid of the shaded area shown below.



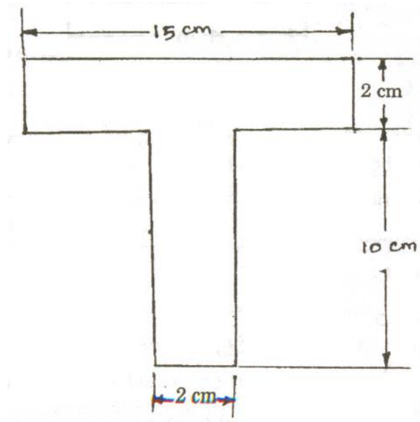
3. Find the moment of inertia of a plane area as shown in figure about its centroidal X- axis



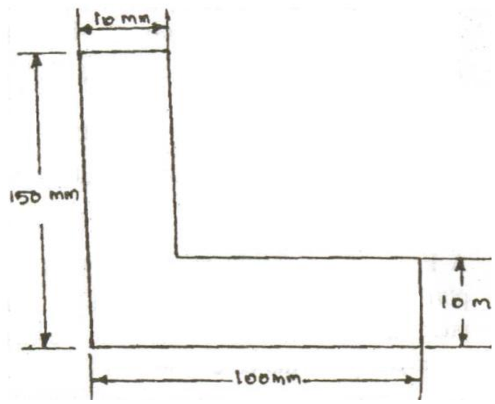
4. Find the polar moment of inertia of a T-section shown in figure about an axis passing through its centroid. Also find the radius of gyration with respect to the polar axis (dimensions are in mm)



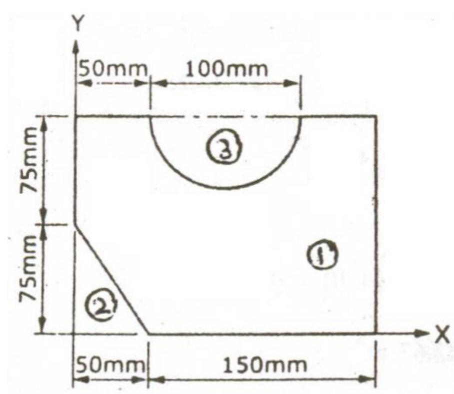
5. Find the centre of gravity of the T-section shown in figure



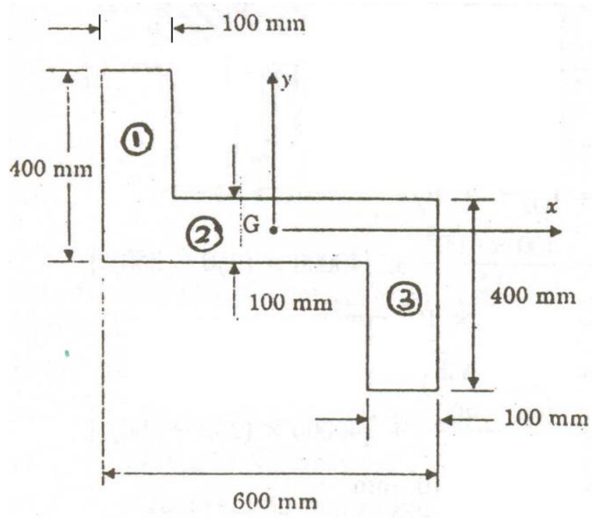
6. Find the principal moment of inertia of the L section about the centroidal axis shown in the figure.



7. For the plane area shown in the figure, determine the area, moment of inertia and radius of gyration about the x- axis



8 Determine the moment of inertia of beam, cross –sectional area shown below about the x and y centroidal axes.



UNIT-IV DYNAMICS OF A PARTICLE

Syllabus

Displacements, Velocity and acceleration, their relationship – Relative motion – Curvilinear motion – Newton's law – Work Energy Equation of particles – Impulse and Momentum – Impact of elastic bodies

1. **State D' Alembert's principle**.
   D' Alembert's principle states that the system of forces acting on a body in motion is in dynamic equilibrium, with the inertia force of the body.

2. **What is general plane motion?**
   When the motion of a rigid body is said to be general plane plane motion when the body undergoes a combination of translation and rotation. E.g. All planets revolving around the sun

3. **What is plane motion?**
   When a particle moves in a plane either in a straight or in a curve is known as plane motion of a particle.

4. **What are the types of plane motion?**

✓ Rectilinear motion
✓ Curvilinear motion

5. **What is uniform motion?**
   If the velocity of a body does not change with time, then the motion is called as uniform motion.

6. **Define coefficient of restitution**
   The ratio of the relative velocity of their separation after collision to the relative velocity of their approach before collision is known as coefficient of restitution. (e)

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

7 **State Newton's law of collision of elastic bodies**

When two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach.

$$v_2 - v_1 = e\,(u_1 - u_2)$$

8 **State the principle of work and energy**

It is states that when a particle moves from position S_1 to S_2 under the action of a force F, the change in kinetic energy of the particle is equal to the work of the force F

i.e Work done by all the forces acting on various = change in total Kinetic Energy of
Particles of body particles forming the body

9. State the equation to determine the escape velocity of a satellite to escape from earth's gravitational field.

Equation for escape velocity V (e) = $\sqrt{2gR}$.

Where 'g' – acceleration due to gravity in m/s^2

'R'- Radius of earth in 'm'

10. What are motion curves?

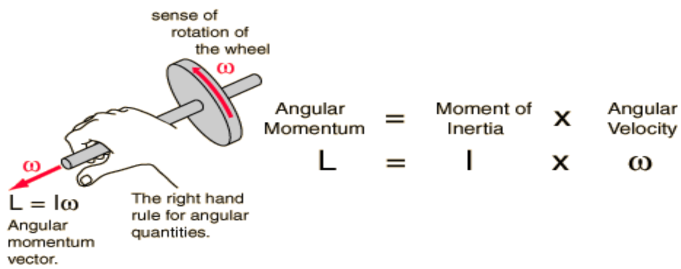
The path described by a particle or a rigid body with respect to time is called motion curve.

11. Define linear momentum

Linear momentum of a particle at any instant is the product of its mass and its velocity at that instant. It is denoted by' p= mv' Unit – Nsec

12. Define angular momentum

Angular momentum of a rigid body at any instant is the product of mass moment of inertia of the rigid body and its angular velocity at that instant. It is denoted by L= Iw



13. What is dynamic equilibrium?

The resultant of system of forces acting on a body in motion and the inertia force of the body is zero is known as dynamic equilibrium.

14. State the law of conservation of momentum

It states that, if the resultant force acting on a particle is zero, then the linear momentum of the particle remains constant.

I.e. Final momentum = initial momentum where u = initial velocity

$\mathbf{m_1u_1 + m_2 u_2 = m_1v_1 + m_2 v_2}$ v = final velocity

15. State the difference between kinematics and kinetics

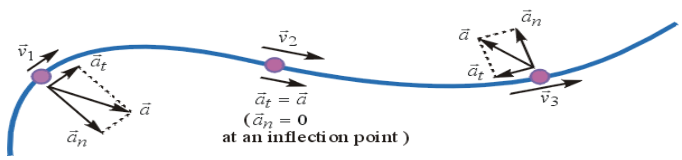
S.No	Kinematics	kinetics
1	It is the study of a body in motion without considering forces that cause the motion	It is the study of a body in motion by considering forces that cause the motion
2	It is dealt with the relationship between velocity, acceleration, time etc. of the bodies without any reference to cause of motion	It is used to predict the motion of a body caused by given force or to determine the force required to produce a given motion.

16. Define instantaneous centre of a rotation

Instantaneous centre of a rotation is a point identified within a body where the velocity is zero and it rotates about an axis perpendicular to the plane of body and passing through any point at any instant of time, then the point is called instantaneous centre of a rotation

17. Define curvilinear motion?

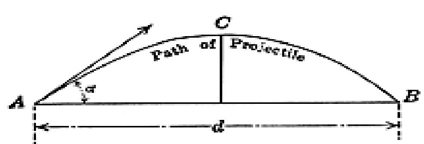
Curvilinear motion is defined as when a particle moves in a curved path is said to have a curvilinear motion.



Curvilinear motion

18. What is a projectile?

When an object is thrown upwards at some angle with respect to the earth's surface, it moves in a curved path in the atmosphere and finally it returns to the ground at some other point. The path traced by the particle in air is known as trajectory of the particle and the particle is called projectile. The path traced by the particle is parabolic.



19. Define rectilinear motion

When the particle moves along a straight line the motion of the body is called rectilinear motion.

20. Define impact

The phenomenon of collision of two bodies which occurs in a very small interval of time and during which the two bodies exert a very large force on each other is called an impact

21. State the difference impulse and momentum

S.No	impulse	momentum
1	Impulse is equal to the change in momentum	Momentum is the quantity of motion and is equal to the impulse of the force acting during same interval of time
2	It is given by the product of impulsive force and time of application of that force	It is given by the product of mass and velocity of the body (L = mv)

22. What is meant by elastic impact?

If there is no loss of kinetic energy during collision then it is known as elastic impact. In elastic impact both momentum and kinetic energy conserved.

23. What are the two types of impact?

- ✓ Indirect impact
- ✓ Oblique impact.

24. What are conservative forces?

If work of a force in moving particle between two positions is independent of path followed by the particle and can be expressed as a change in its potential energy, then such a force is called a conservative force.

25. What is meant by relative velocity?

Relative velocity is defined as the rate of change of relative position of one particle with respect to the other.

26. What is rigid body?

A rigid body is defined as body which does not deform under the action of forces

27. What do you mean by translation?

When a rigid body is in translation, all the points of the body have the same velocity and acceleration at any given instant. Translation can be rectilinear or curvilinear.

28. Define rotation

Rotation about a fixed axis is defined as the angular motion about the axis and it follows the principle that the entire particles move in a circular path about the axis of rotation and the all lines in the body rotate through the same angle.

29. What is relative motion?

When a motion of the body is evaluated with respect to another moving body, then the motion is said to be relative motion.

30. What is uniform circular motion?

When a particle moves along a circular path with constant speed (v) then the motion is called uniform circular motion.

31. What is spring constant?

K=F/x is called spring constant is defined as the force required for the unit deformation of the spring.

32 . Define the term Dynamics.

It is the branch of mechanics, which deals with the analysis of bodies in motion. Dynamics is divided into two parts.

a) Kinematics
b) Kinetics

Kinematics is the study of geometry of motion. It is used to relate displacement, velocity, acceleration and time without reference to the forces causing the motion.

Kinetics is the study of the relation existing between the forces acting on the body, the mass of the body and the motion of the body.

33. What are the different types of motion?

A body is said to be in motion, if it changes its position with respect to a reference point.
The type of motion is defined by the path traversed by it.
The motion of the body is classified into

I. Translation
II. Rotation

I) Translation:

When a straight line drawn on the moving body remains parallel to its original position at the given time then it is called as Translation. Translation is classified into two types,

(i) Rectangular motion

If the path traced by a point on a body is a straight line, the motion of the body is called the rectilinear or translation.

Ex: (i) A stone falling vertically downward
(ii) A ball thrown vertically upward

Rectilinear motion is classified into (i) motion with uniform acceleration (ii) motion with variable acceleration

(ii) Curvilinear motion

If the path traced by a point on a body is a curved line, the motion of the body is called as curvilinear motion.

Ex: (i) A car moving along a turning

***II*) Rotation**

If the path traced by a point on a body is a circle then the motion of the body is called a pure rotation.

Ex: (i) Rotating shafts in machines

34. What are the Characteristics of Kinematics?

a. Displacement (S)
b. Speed
c. Velocity (v)
d. Acceleration (a)
e. Average Velocity
f. Average Speed
g. Instantaneous Velocity

35. Give Mathematical expression for velocity and acceleration.

Let s=Distance traveled by a particle in a straight line
t=Time taken by the particle to traveled this distance‘s’

Velocity, $v=\frac{ds}{dt}$

Acceleration, $a=\frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$

$=\frac{d^2 s}{dt^2}$

36. Give the Equations of motion in a straight line.

Velocity, V=u+at

Displacement, $S=ut+\frac{1}{2}at^2$

$V^2=u^2+2as$

Let u=Initial velocity
v=Final velocity
s=Distance travelled by the particle
t=Time taken by the particle, to change from u to v.
a=Acceleration of the particle.

37. A motorist traveling at a speed of 18km/hr, suddenly applies the brakes and comes to rest after skidding 75m. Find the time required for the car stop.

Given Data:

Initial velocity u =18km/hr

$=\frac{18*1000}{3600}=5m/s$

Final velocity v=0;
Distance traveled s=75m.

Let us find the acceleration first.
Using the relation,
$V^2=u^2+2as$
$0=5^2+(2 \times a \times 75)$
Acceleration a= - 0.167m/s^2. (Negative sign shows retardation)

Using the relation, v=u + at
0=5+(-0.167 x t)
Time required to stop the car (t)=29.94sec

Result:
Time required to stop the car (t)=29.94sec

38. A body was thrown vertically down from a tower and travels a distance of 3m in the 5th second of its flight. Find the initial velocity of the body.
In downward motion, height travelled in nth second.

Distance traveled in nth second = h^{nth} =u+ $\frac{g}{2}(2n-1)$

Where,
u-Initial velocity in m/sec
n- Number of sec

$3 = u + \frac{9.81}{2}((2\ X\ 5)-1)$

u= - 41.14m/s

Negative sign shows that the body was initially thrown in the opposite direction.
Result:
Initial velocity u = - 41.14m/s

39. A bus beginning to move with an acceleration of 0.3m/s^2. A man who is 12m behind the bus starts running at 3m/s to catch the bus. After how many seconds will the man able to catch the bus.

Let the motion of man and bus starts simultaneously from B and A respectively and the man catch the bus at C, distance of x from A.

Motion of bus(moving with acceleration)
Initial velocity u=0; Acceleration (a) =0.3m/s. Time taken to travel the distance (S) = t sec
Distance traveled = S

Using the equation,
Distance traveled S=ut+$\frac{1}{2}at^2$

$S=\frac{1}{2}\ X\ 0.3\,t^2$

$S=0.15t^2$

Motion of man (moving with constant velocity)
Initial velocity u=3m/s; Distance traveled =(S + 12)m
But
S + 12=Uniform velocity X time
(Where t is the time taken by the man to catch the bus)

$S + 12 = 3t$
$(S-3t) + 12 = 0$
Substitute $S = 0.15t^2$,
t=14.47sec (or) 5.528sec

Result:
t=14.47sec (or) 5.528sec

40. What are the meaning of Projectile, Angle of projection and Velocity of projectile?

Projectile

A particle projected in space at an angle to the horizontal plane is called a projectile.

Angle of projection

The angle to the horizontal at which the projectile is projected is called angle of projection. It is denoted by α.

Velocity of projectile

The velocity with which the projectile is thrown into space is called the velocity of projectile.

41. Define Trajectory, Time of flight and Range.

Trajectory

The path described by the projectile is called Trajectory.

Time of flight

Time of flight is the total time taken by the projectile from the instant of projection and the projectile hits the plane again.

Range

Range is the distance along the plane between the point of projection and the point at which the projectile hits the plane at the end of its journey.

42. What is the Equation of projectile?

Equation of trajectory, Vertical upward distance traveled $S_y = u\sin\alpha\, t - \frac{1}{2}gt$

Horizontal range traveled $S_x = u\cos\alpha t$

$$S_y = S_x\tan\alpha - \frac{1}{2}\frac{gs^2}{u^2}[1 + \tan^2\alpha]$$

43. What is the expression of Maximum height attained in path of projectile?

Time to reach maximum height $(t_1) = u\sin\alpha / g$

Total time of flight, $t = 2u\sin\alpha / g$

Horizontal range, $R = u^2\sin 2\alpha / g$

Maximum range, $R_{max} = u^2 / g$, when $\boldsymbol{\alpha = 45^\circ}$

Maximum height, $h = u^2\sin^2 2\alpha / 2g$

44. Define the Law's of motion

Newton's first Law of motion

'Every body continues to be in its state of rest or of uniform motion motion in a straight line unless and until it is acted upon some external force force to change that state"

Newton's second law of motion

"The rate of change of momentum is directly proportional to the impressed force and it takes place in the direction of the force".

Rate of change of momentum = mass x Acceleration

$$m(v-u) / t = \mathbf{m} \; x \; \mathbf{a}$$

Where,

m- Mass of the body	in	'kg'
v- Final velocity of the body	in	'm/sec'
u- Initial velocity of the body	in	'm/sec'
a – Acceleration due to gravity	in	'm /sec^2'
t – Time taken to change the velocity from 'u' to 'v'	in	'sec'

Newton's third law of motion

To every action, there is always an equal and opposite reaction.

45. A body of mass 4kg is at rest. What force should be applied to move it to a distance of 12m in 4 sec?

Given:

m=4kg. s=12m. t=4sec. u=0

Find

The force required to move the body to a distance of 12m in 4 sec

Solution:

Using the equation,

$$S=ut+\frac{1}{2}at^2$$

$$12=0+ (\frac{1}{2}a \; X \; 4^2)$$

$$a=\frac{12}{8}m/s^2$$

The force required to move, F = ma

$$= 4 \text{ X} \frac{12}{8}m/s^2$$

F = 6 N.

Result:

The force required to move the body to a distance of 12m in 4 sec = 6 N.

46. State D'Alembert principle.

D'Alembert principle

This principle states that that "A system of forces acting on a body in motion is in Dynamic equilibrium with the inertia force of the body. It is an application of Newton's second law.

$$\sum F = ma$$
$$\sum F - ma = 0$$

Where,

F – applied force in 'N'

- 'ma' is called inertia force in 'N'

47. Define Law of Conservation of Energy

The law of conservation of energy states that " Energy can neither be created nor destroyed, but it may change its from and may get converted into another form of energy" ie the total energy possessed by a body remains same.

48. Define (i) Work (ii) Energy (iii) Power

Work

Work is defined as the product of force acting on a body and displacement of the body due to the force acting on it.

W= F x s

Where, W- Wokdone in 'N-m'

F – Force acting on the body in 'N'

S – Displacement of the body in 'm'

Energy

The capacity of doing work is known as energy

The energy may be of

(i) Potential energy = mgh

(ii) Kinetic energy = ½ mv^2

(iii) Strain energy

(iv) Mechanical energy

Power

The rate of doing work is known as power

Unit is 'W' = N-m /Sec

49. Define Law of Conservation of linear momentum

"If there is no external force acting on the system, (two or more bodies) then the total linear momentum of the system remains constant"

Initial momentum = Final momentum

Sum of the momentum of two bodies before impact =
Sum of the momentum of the bodies after impact.
(Direction to be strictly considered)

$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

50. Explain Conservation of Linear momentum and motion of bullet and gun

Conservation of Linear momentum

Sum of the momentum of two bodies before impact =
Sum of the momentum of the bodies after impact.
(Direction to be strictly considered)

$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$

Motion of bullet and gun:

Let u=Velocity of bullet; m=Mass of bullet;
V=velocity of the gun; M=Mass of the gun;

Initial momentum before firing = 0
Total final momentum = mu + MV

As per law of Conservation of Linear momentum

Initial momentum before firing = Final momentum after firing

0= mu + MV

Velocity of the gun and bullet after firing $V=-\left(\frac{mu}{M}\right)$

51. What are the Types of impact?

1) **Direct impact:**
In direct impact, the velocities of the two colliding bodies, before collision are collinear with the line of impact.

2) **Oblique impact:**
In oblique impact, the velocities of the two colliding bodies, before collision are not collinear with the line of the impact.

52. Explain Period of restitution and co-efficient of restitution.

Period of restitution:
The time elapsed from the instant of initial to the maximum deformation is known as **period of deformation**

Period of deformation

The time the elapsed from the instant of maximum deformation to the instant of just separation is known as period of restitution.

Co-efficient of restitution

"The ratio of the magnitude of restitution impulse to the magnitude of deformation impulse is known as co-efficient of restitution". It is denoted by the letter 'e'.

$$\text{Co-efficient of restitution} = \frac{\text{Impulse during restitution}}{\text{Impulse during deformation}}$$

$$\text{Co-efficient of restitution} = \frac{\int R\,dt}{\int P\,dt} = \frac{V_2 - V_1}{U_1 - U_2}$$

Where, U_1, U_2 -Initial velocities (Before impact) of the two elastic body

V_1, V_2 -Final velocities (after impact) of the two elastic body

53. State Newton's Law of collision of elastic bodies

It states that, " for two colliding bodies, their relative velocity of separation bears a constant ratio to their relative velocity of approach"

$$\frac{V_2 - V_1}{U_1 - U_2} = a\ constant = e = Coefficient\ of\ restitution\ or\ coefficient\ of\ elasticity$$

54. A particle of mass 2kg moving with a velocity of 2m/s hits a second particle of mass 4kg at rest. The first particle comes to rest. Find the velocity of the second particle after impact and the co-efficient of restitution.

As per Conservation of Linear momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(2 \times 2)+(4 \times 0)=(m_1 \times 0) + (4 \times v_2)$

$4 = 4 \times v_2$

Velocity of the second particle after impact, v_2=1 m/s.

Co-efficient of restitution, $e = \frac{v_2 - v_1}{u_1 - u_2}$

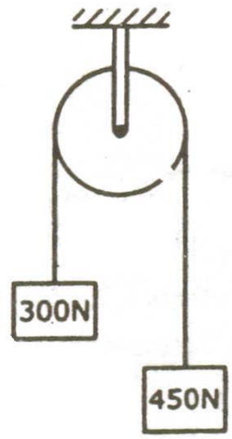
$$e = \frac{1-0}{2-0} = \frac{1}{2}$$

Result

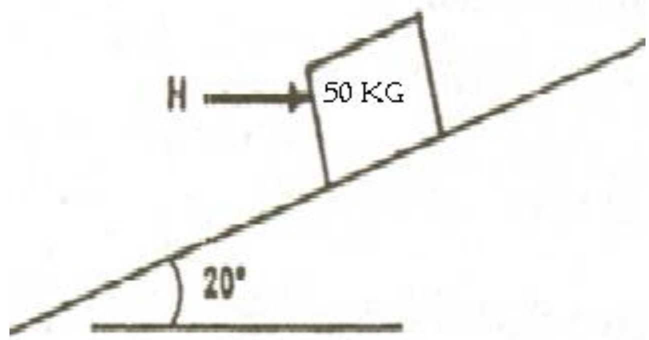
Co-efficient of restitution, e = 0.5

PART-B

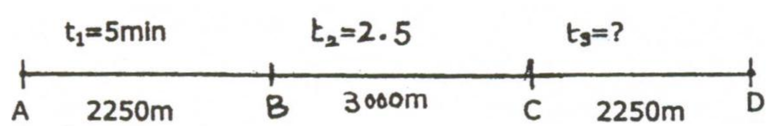
1. Two bodies weighing 300N and 450N are hung to the ends of a rope passing over an ideal pulley.with what acceleration the heavier body comes down? What is the tension in the string?



2. What horizontal force is needed to give the 50 Kg block an acceleration of 3 m/sec^2 up the 20^{0}plane? Assume a coefficient of friction between the block and the plane is 0.25



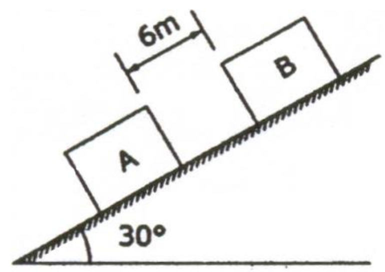
3. A train is traveling from A to 0 along the track shown in fig. Its initial velocity at A is zero. The train takes 5 min to cover the distance AB, 2250 m length and 2.5 minutes to cover, the distance BC, 3000 m in length, on reaching the station C, the brakes are applied and the train stops 2250 m beyond, at 0 (i) Find the retardation on CD, (ii) the time it takes the train to get from A to 0, and (iii) its average speed for the whole distance.



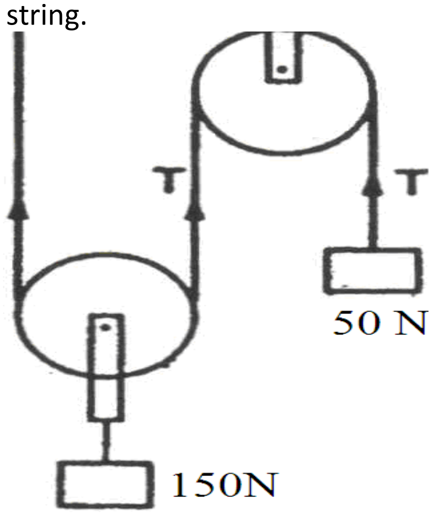
4. The position of the particle is given by the relation S=1.5e _9t^2 -22.5t+60, where S is expressed in meters and t in seconds. Determine (i) the time at which the velocity will be zero (ii) the position and distance traveled by the particle at that time (iii) the acceleration of the particle at that time and (iv) the distance traveled by the particle from t = 5s to t = 7s.

5. A particle is projected with a initial velocity of 12m/s at an angle a with the horizontal. After sometime, the position of the particle is observed by its x and y distances of 6m and 4m respectively from the point of projection. Find the angle of projection.

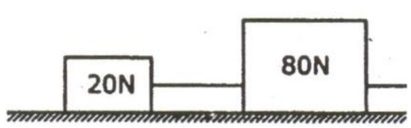
6. Two Blocks A and B of weight 100 N and 200 N respectively are initially at rest on a 30^0 inclined plane as shown in figure. The distance between the blocks is 6 m. The co efficient of friction between the block A and the plane is 0.25 and that between the block B and the plane is 0.15. If they are released at the same time, in what time the upper block (B) reaches the Block (A).



7. Two blocks of weight 150 Nand 50 N are connected by a string and passing over a frictionless pulley as shown in figure. Determine the acceleration of blocks A and B and the tension in the string.



8. Two weights 80 N and 20 N are connected by a thread and move along a rought horizontal plane under the action of a force 40 N, applied to the first weight of 80 N as shown in figure. The coefficient of friction between the sliding surfaces of the wrights and the plane is 0.3. Determine the acceleration of the weights and the tension in the thread using work-energy equation.



9. A stone is thrown form the top of a building 30 m high with an initial velocity of 30 m/s directed vertically upward. On its way down, the stone just misses the edge of the roof. Determine (i) the velocity 'v' and elevation of the stone above the ground level at any time 't'o (ii) The time needed for the stone to reach its maximum height. (iii) maximum height above the ground level (iv) velocity when it reaches the ground. (v) Position and velocity when t = 6 seconds.

10. A stone is projected with a speed of 30 m/s at an angle of elevation of 55°.
    Find its velocity (i) after two seconds (ii) at highest point of its path (iii) at a height of 5 m. find the time interval between the two points at which the stone attains the speed of 25 m/s.

UNIT-IV FRICTION AND ELEMENT OF RIGID BODY DYNAMICS

TWO MARKS

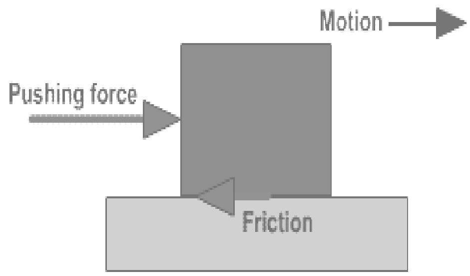
Syllabus

Frictional force – Laws of Coloumb friction – simple contact friction – Rolling resistance – Belt friction.

Translation and Rotation of Rigid Bodies – Velocity and acceleration – General Plane motion.

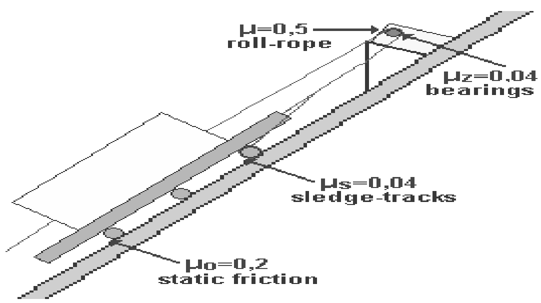
1. Define Friction

   The resistance to lateral motion when one attempt to slide the surface of one object over another surface is called **friction or traction**



2. Define static friction

   Static friction between two bodies is the tangential force which opposes the sliding of one body relative to the other.



3. Define limiting friction

   Limiting friction 'F_m' is defined as the maximum external force developed at the contact area when motion is impending (about to occur).

4. Define coefficient of friction

   The ratio of limiting friction to the normal reaction is known as coefficient of friction.

   $$\text{Coefficient of friction } (\mu) = \frac{\text{Limiting friction } (F_m)}{\text{Normal reaction } (N_R)} \quad \textbf{(}\text{Or)} \quad F_m = \mu\, N_R$$

   Where μ = Constant

5. Define Angle of repose

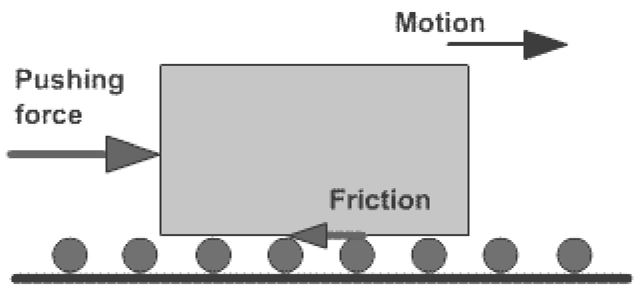
   Angle of repose (α) is defined as the angle to which an inclined plane may be raised before an object resting on it will move under the action of the force of gravity.

6. Define kinetic friction
   Kinetic friction is defined as the tangential force between the two bodies after motion begins

7. What is dry friction
   The friction that exists between two forces with no lubrication used, that friction is called dry friction or solid friction. I.e. Dry friction resists relative lateral motion of two solid surfaces in contact.

8. What is sliding friction?
   Sliding friction is the friction experienced by a body when it slides over another body.

9. What is rolling friction?
   Rolling friction is the friction experienced by a body when it roller over the other.



10. Fluid friction?

    When a solid object is in contact with a fluid, such as a liquid or gas, and a force is applied to either the object or to the fluid, there is a friction force that resists the motion.

    E.g. water flowing through a hose, an airplane flying through the atmosphere and oil lubricating moving parts.

11. What is a fluid?

    A fluid is a substance that continuously deforms or flows under action of gravity is known as fluid. All gases are fluids, but not all liquids are fluids.

12. What is impending motion?
    The motion is said to be impending if the applied forces are such that the body is just about to slide.
13. Give the equation of dynamic equilibrium
    P – ma = 0 is called dynamic equation of equilibrium, in this 'ma' is an imaginary force which is applied in the opposite direction of motion.(F = ma is Newton's II- law)
    Where P = external force, m = mass of the body, a = acceleration of the body.

14 State D'Alembert's principle

The system of forces acting on a body in motion is in dynamic equilibrium with the inertia force of the body.

15. State the laws of dry friction
    - ✓ The direction of frictional force on a surface is such as to oppose the tendency of one surface to slide relative with other.
    - ✓ The total frictional force is independent of area of contact between two surfaces and depends on the nature of surface of contact.
    - ✓ The magnitude of limiting friction bears constant ratio to the normal reaction between two forces.
    - ✓ Frictional force is never greater than the force required to prevent motion.

16. What are the different types of friction?
    - ✓ dry friction
    - ✓ fluid friction
    - ✓ lubricated friction
    - ✓ skin friction
    - ✓ internal friction

17. What is skin friction?

    Skin friction is a component of drag, the force resisting the motion of a solid body through a fluid.

18. What is rolling resistance?

    When one body is made to roll freely over another body, a resistance is developed in the opposite direction is known as rolling resistance.

19. What is coefficient of rolling resistance?

    $b = P.\,r / W$ where, r = radius of wheel, P = horizontal force, W = self weight.
    Co -efficient of rolling resistance 'b' is horizontal distance of point of resistance measured from centre of wheel is known as coefficient of rolling resistance.
    Unit: mm

20. What are the types of simple contact friction?

    (i)Ladder friction (ii) Belt friction
    (iii)Wedge friction (iv) Screw friction

21. **What are the application areas of friction?**

The application areas of friction are,

i) Moving of a body on a horizontal plane

ii) Moving of a body on a inclined plane

iii) Ladder friction

iv) Wedge friction

v) Screw friction

vi) Belt friction

vii) Bearing friction

viii) Wheel friction

22. **What do you mean 'Wedge friction'?**

A Wedge is a piece of metal or wood in the shape of prism whose cross section is a triangle or trapezoid. It is used for lifting heavy loads and tightening of fits. The wedge friction is same as the friction in inclined plane. The problems on wedges are basically the problems of equilibrium on inclined planes. Therefore problems on wedge friction may be solved either by equilibrium method or by Lami's theorem.
Where,

W-Weight of the body which is to be moved by using wedge
P-Force applied to the wedge to move the body
μ_1 – Co efficient of friction between the body and wedge
μ_2 – Co efficient of friction between the body and floor or fixed surface

Reactions an forces on the wedge can be calculated by

(i) Resolving forces on the body, into horizontally and vertically and applying equilibrium conditions ($\Sigma F_x = 0$) and ($\Sigma F_y = 0$)

(ii) Resolving forces on the wedge, into horizontally and vertically and applying equilibrium conditions ($\Sigma F_x = 0$) and ($\Sigma F_y = 0$)

23. **When will a body placed on an inclined plane will be in equilibrium? What are the equilibrium conditions for a ladder?**

A body placed on an inclined plane will be in equilibrium if,

(i) The algebraic sum of the resolved parts of the forces acting on the body along the supporting plane is equal to zero
(ii) The algebraic sum of the resolved parts of the forces acting on the body perpendicular to the supporting plane is equal to zero.

<u>The equilibrium conditions for a ladder are</u>

(i) The algebraic sum of the resolved parts of the forces acting on the ladder along the horizontal plane should be equal to zero ($\Sigma F_x = 0$)
(ii) The algebraic sum of the resolved parts of the forces acting on the ladder along the vertical plane should be equal to zero. ($\Sigma F_y = 0$)

(iii) The algebraic sum of the moments of forces acting on the ladder about any point in their plane should be equal to zero. ($\Sigma M = 0$)

24. **What is belt friction? What are the equilibrium conditions for solving belt problems?**

Belt friction is the frictional force developed between the belt and pulley. The belt friction is used for raising a load, transmitting power and applying brakes to stop the motion. The relation between the tensions on the two sides of the belt, the angle of lap , and coefficient of friction,
For flat belt

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Where T_1 – Tension in the tight side ; T_2 – Tension in the tight side
μ - Co efficient of friction ; θ – Angle of lap of the belt over the pulley

For V-belt

$$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}(\frac{\beta}{2})}$$

Where T_1 – Tension in the tight side ; T_2 – Tension in the tight side
μ - Co efficient of friction θ – Angle of lap of the belt over the pulley
β – Angle between the two surfaces of contact

The equilibrium conditions for solving belt problems are,

(i) The algebraic sum of the resolved parts of the tensions acting on the belt along the tangential direction to the pulley should be equal to zero ($\Sigma F_T = 0$)
(ii)The algebraic sum of the resolved parts of the tensions acting on the belt along the normal to the tangential plane should be equal to zero. ($\Sigma F_N = 0$)

25. **What is rolling resistance or wheel resistance ?**

$$\text{Rolling friction} = P = \frac{W \times a}{r}$$

Where,

P- Additional force required to overcome the rolling friction which is equal to the rolling friction between the wheel and the surface.
W – Weight of the wheel roller
a - Forward length of deformation (or) Co efficient of rolling resistance (or) Horizontal distance between the reaction point and line of action of Weight
r - Radius of the wheel

<u>The three concurrent forces</u> acting on the roller are

(i) Weight of the wheel 'W'(vertical)
(ii) Applied force equal to the frictional force 'P'(Horizontal)
(iii) Resultant force of friction 'R'

26. **A body of weight 100N is placed on a rough horizontal plane. Determine the co efficient of friction , if a horizontal pull of 20N is applied on the body.**

Solution

The conditions for the equilibrium of a rigid body placed on a rough horizontal plane are

(i) The algebraic sum of the forces along the horizontal plane should be equal to zero. ($\Sigma F_x= 0$)

(ii) The algebraic sum of the forces vertical to the horizontal plane should be equal to zero. ($\Sigma F_y = 0$)

Applying the condition ($\Sigma F_x = 0$)

$P = F$

$20 = \mu R_N$ -----(1)

Applying the condition ($\Sigma F_Y = 0$)

$W = R_N$

$100 = R_N$ ------(2)

By substituting (2) in (1)

$20 = \mu \times 100$

$\mu = 0.2$

Result

The co efficient of friction $\mu = 0.2$

27. **A body of weight 500N is placed on a rough horizontal plane. Determine frictional force 'F' if the co efficient of friction is 0.3, and a horizontal pull of 'P' applied on the body is20^0 inclined to the horizontal plane.**

Solution

The conditions for the equilibrium of a rigid body placed on a rough horizontal plane are

(i) The algebraic sum of the forces along the horizontal plane should be equal to zero. ($\Sigma F_x = 0$)

(ii) The algebraic sum of the forces vertical to the horizontal plane should be equal to zero. ($\Sigma F_y = 0$)

Resolving the applied force into horizontally and vertically and

Applying the condition ($\Sigma F_x = 0$)

$P \cos 20^0 = F$

$P \cos 20^0 = 0.3 \ R_N$ -----(1)

Applying the condition ($\Sigma F_Y = 0$)

$R_N + P \sin 20^0 = 500$

$R_N = 500 - P \sin 20^0$ 'N' ------(2)

By substituting (2) in (1)

$P \cos 20^0 = 0.3 \ (500 - P \sin 20^0)$

$0.94 P = 150 – 0.102 P$

$0.94 P + 0.102 P = 150$

$1.0426 P = 150$

$P = 143.87$ 'N'

$F = P \cos 20^0$ 'N'

$F = 143.87 \times \cos 20^0$

F = 135.19 'N'

Result

The frictional force developed F = 135.19 'N'

28. **A belt embraces an angle of 200 degree over the surface of a pulley of 500mm diameter. If the tight side tension of the belt is 2.5 Kn. Find out the slack side tension of the belt. The coefficient of friction between the belt and the pulley can be taken as 0.3.**

Given data:

Tight side tension $T_2 = 2.5$ kN
Contact angle $\theta = 200$ degree $= 200 \times \pi / 180$
μ - Co efficient of friction = 0.3

Solution

$$\frac{T_1}{T_2} = e^{\mu \theta}$$

Where T_1 – Tension in the tight side ; T_2 – Tension in the tight side
μ - Co efficient of friction ; θ – Angle of lap of the belt over the pulley

$$T_1 = \frac{T_2}{e^{\mu \theta}}$$

$$T_1 = 0.877 \quad 'kN'$$

Result

The slack side tension of the belt $T_1 = 0.877 \quad 'kN'$

29. **A vehicle weighs 50kN having a wheel of diameter 50cm, is rolling on a road with coefficient of rolling resistance of 1.15mm. Compute the rolling resistance for the wheel and the friction angle.**

Given

Weight of the vehicle	W = 50 kN
Wheel diameter	D = 100 cm
(i.e) Wheel radius	r = 500 mm
Coefficient of rolling resistance	a = 1.15 mm

Solution:

(i) Rolling resistance $P = \frac{W \ x \ a}{r}$

$$P = \frac{50 \ x \ 10^3 \ \ x \ 1.15}{500} = 115 \ N$$

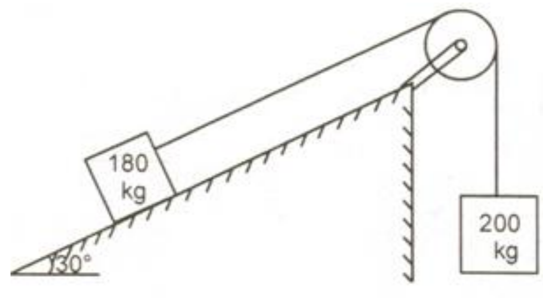
(ii) Friction angle $\theta = \tan^{-}\left(\frac{a}{r}\right) = an^{-}\left(\frac{1.15}{500}\right) = 0.1317^0$

Result

(i) The Rolling resistance = 115 N
(ii) The Friction angle = 0.1317^0

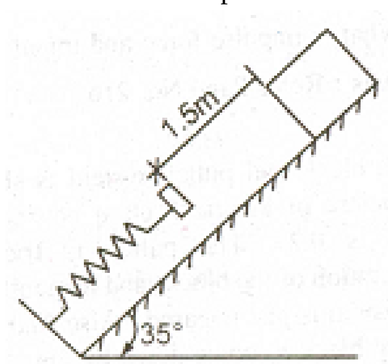
PART-B

1 A block and pulley system is shown in figure. The coefficient of kinetic friction between the block and the plane is 0.25. The pulley is frictionless. Find the acceleration of the blocks and the tension in the string when the system is just released. Also find the time required for 200 Kg block to come down by 2m.

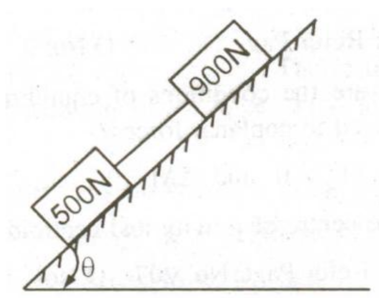


2 A block of mass 50 kg slides down a 35^0 incline and strikes a spring 1.5 m away from it as shown

The maximum compression of the spring constant is 1 KN/m , find the coefficient of kinetic friction between the block and the plate.

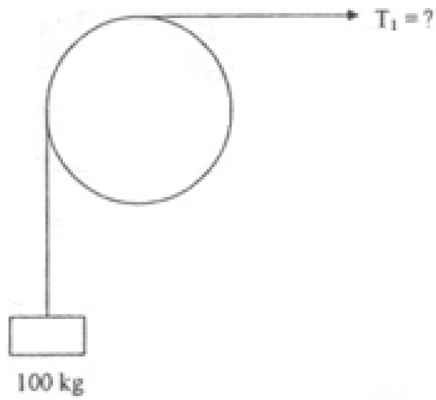


3 Two blocks of weight 500 N and 900 N connected by a rod are kept on an inclined plane as shown in figure. The rod is parallel to the plane. The coefficient of friction between 500 N block and the plane is 0.3 and that between 900 N block and plane is 0.4. Find the inclination of the plane with the horizontal and the tension in the rod when the motion down the plane is just about to start.

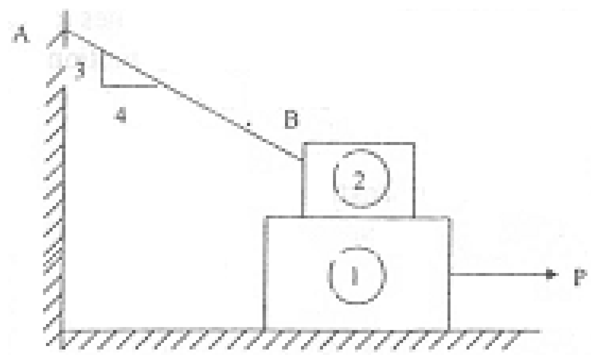


4 A 100 Kg mass is lifted by applying a horizontal force by a belt drive, rolling on a cylinder of 150 mm diameter as shown in figure. Calculate

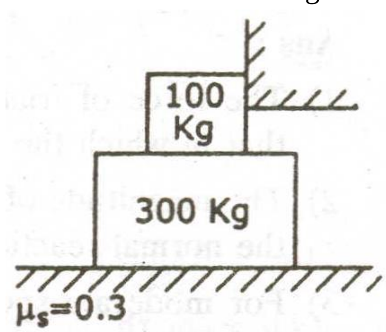
(i)The necessary force (ii) torque at the cylinder surface (iii) power transmitted. The coefficient of friction is 0.20



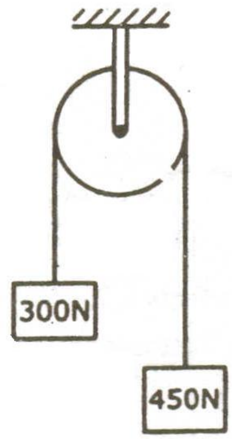
5 A block of weight W_1 = 1290 N rests on a horizontal surface and supports another block of weight W2 = 570 N on top of it as shown in figure. Block of weight W2 is attached to a vertical wall by an inclined string AB. Find the force P applied to the lower block that will be necessary to cause the slipping to impend. The coefficient of friction between 1 and 2 is 0.25 and the coefficient of friction between 1 and the horizontal surface is0.40



6 If the coefficient of friction between all surfaces is 0.3 what is the horizontal force required to get the 300 Kg block shown below moving to the right?



7 Two bodies weighing 300N and 450N are hung to the ends of a rope passing over an ideal pulley.with what acceleration the heavier body comes down? What is the tension in the string?



8 What horizontal force is needed to give the 50 Kg block an acceleration of 3 m/sec^2 up the 20^0plane? Assume a coefficient of friction between the block and the plane is 0.25